Proceedings of the Institute of Acoustics

ACOUSTIC OPTIMISATION USING FINITE AND BOUNDARY ELEMENTS

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1. INTRODUCTION

Finite and boundary element analysis have many applications in the audio industry. These include analysis of the magnetic field around the voicecoil, cone breakup, radiation from a drive unit, diffraction by cabinet edges, positioning of speakers in a small room, design of listening rooms or studios and many others. The computational power of PCs continues to increase permitting wider application to more complex problems and/or higher frequencies. This benefits the engineer, who is under constant pressure to shorten the design cycle. It is natural to ask whether computer methods can not only assess a particular design, but also determine the design itself. This is considered in this paper, where attention is restricted to acoustics, and horn shape in particular. However the range of potential applications is far wider and includes all application areas listed above.

2. OPTIMISATION

In order to optimise some variable parameters $\alpha_1, \alpha_2, ..., \alpha_k$, an objective function g and some constraints must be selected. In the example below the parameters are the radii of some points on the cross section, which are used to define the shape using a spline fit. For loudspeaker design, possible parameters could include the fillet radius on the cabinet edge, the dimensions of the cabinet, the material properties of the surround, ... The objective function is used for comparing designs. Lower values correspond to better designs. This might be taken as negative the on axis response (trying to maximize output), some measure of smoothness (trying to produce a uniform response with frequency) or some weighted sum of pressure amplitudes at selected points (trying to control the directivity pattern). Alternatively, as is often used in the automotive and aerospace industries, the objective function could be the weight . There are also usually constraints which need to be satisfied, restricting the problem to feasible solutions. If the weight is minimized then these are likely to be that certain stresses do not exceed required values for specified loading conditions. In the example below the constraints are simply bounds on the parameters.

There are many optimisation methods [1]. Most methods require the gradient of the objective function,

$$\underline{\nabla} g = \begin{Bmatrix} \dfrac{\partial g}{\partial \alpha_1} \\ \vdots \\ \dfrac{\partial g}{\partial \alpha_k} \end{Bmatrix} \tag{1}$$

to be evaluated. In the method of steepest descent the direction $-\underline{\nabla} g$ is used for a one dimensional line search. At the minimum of the line search the gradient is re-evaluated and the procedure repeated. The procedure is usually stopped when the gradient is sufficiently small. There are many improved methods, such as the conjugate gradient algorithm, which use previous derivative information to devise a better search direction, on the assumption that the objective function is locally quadratic.

The method of steepest descent and conjugate gradient algorithm will tend to home in on the nearest minimum, and as such are probably only appropriate for tuning an existing design, rather than looking for a radical new design. There are techniques, such as the method of simulated annealing, which search for the global minimum.

3. PREVIOUS WORK

The current author has done some optimisation using a coupled finite/boundary element analysis method, for sonar array design, ref [2],[3]. An active sonar array can be adjusted by moving the position of the projectors. Design constraints often inevitably force close spacing of elements. Thus neighbouring transducers are strongly coupled through the dense fluid medium, and it is not possible to use simple theoretical models, based non-interacting directional point sources. Configurations can be accurately modelled with a coupled finite/boundary element formulation. The parameters were perturbations of the elements in the array from their initial configuration. The objective function was constructed to minimise the back to front ratio, i.e. so that, hopefully, very little energy is radiated in the back direction, whereas much energy is radiated in the forward direction. In this case it was possible to evaluate the gradient of the objective function, equation (1), as a resolution, using the original factorisation of the system equations. This is both accurate and fast, but needed very specific program modification. Figure 1 shows some of the results from ref [3].

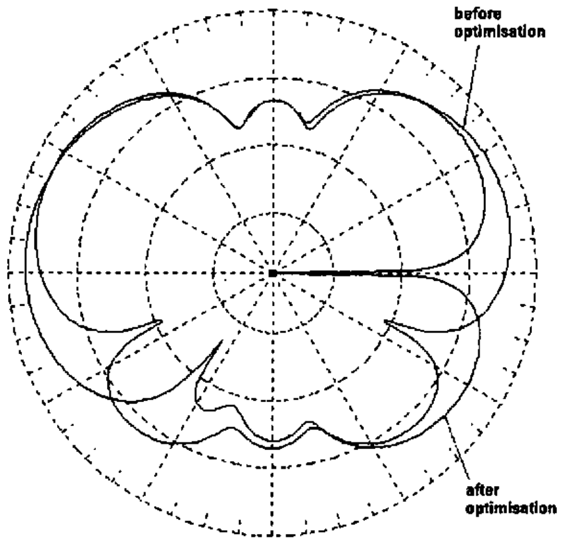

Figure 1 : *optimisation of simple sonar array from ref [3].*

The optimisation procedure has increased the power radiated in the forward direction (left) and reduced the power radiated in the backwards direction (right). A dramatic decrease in the ratio of the directivities has been achieved. However the very narrow null in the backwards direction is not practically useful. A differently constructed objective function attempting to minimise the power radiated over a small solid angle may have produced a better design.

This illustrates the principle that it is crucially important to get the definition of the objective function right.

Optimisation techniques have been used in the audio industry, for improving horn shape, ref [4] Geaves and Henwood. They used a purely boundary element based approach for modelling the acoustic domain, whereas the work below uses a hybrid acoustic finite element / boundary element approach. They consider radiation into full 3D space, whereas the problem below models a horn embedded in a baffle, radiating into a half space. They used a different optimisation technique, WRSM (weighted random search method), an evolutionary approach which should avoid being trapped by local minima. Their analyses produced some gains in efficiency and smoothness of response, indicating that optimisation is a useful technique in the audio industry.

4. CURRENT APPROACH

The current optimisation work, as with the sonar array work above by the same author, is based on the PAFEC VibroAcoustics program. In the previous work the objective function gradient evaluation is done in the kernel of the FE/BE analysis program, which was both accurate and efficient, but application specific. A different approach was adopted for the current work, where there is a controlling executive program, which performs an analysis by starting a child process. The objective function is evaluated by postprocessing the analysis results. The derivatives of the objective function are evaluated by simple differencing, using e.g.

$$\frac{\partial g}{\partial \alpha_1} \approx \frac{g\left(\alpha_1 + \frac{\Delta\alpha_1}{2}, \alpha_2, \ldots, \alpha_k\right) - g\left(\alpha_1 - \frac{\Delta\alpha_1}{2}, \alpha_2, \ldots, \alpha_k\right)}{\Delta\alpha_1} \tag{2}$$

The tolerances, $\Delta\alpha_i$, and permitted range for each of the optimisation parameters is input data into the program. The method is applicable to models which have parameterised data files. With this approach it is easy to incorporate objective functions based on pressure, displacement, velocity, stress, natural frequency, … . The parameters can be radii, lengths, material properties, plates thickness … . If more complicated constraints, other than simple upper and lower limits on parameters, are required, then the objective function could be modified with additional penalty function contributions, increasing exponentially with the extent to which a constraint is violated. It is of course necessary that the objective function should be differentiable, since the optimisation technique used requires the gradient to be evaluated. The method of steepest descent and conjugate gradient algorithm were both programmed up. The latter was found to be far superior and all the results presented below were computed using this method.

5. TEST PROBLEM AND RESULTS

The code written was given some simple testing on a single parameter optimisation problem, a rigid piston, of variable radius, vibrating in an infinite baffle at a fixed frequency. The pressure amplitude at a point on axis was minimised by adjusting the piston size. This was checked against a simple analytical formula, predicting zero pressure for certain piston sizes.

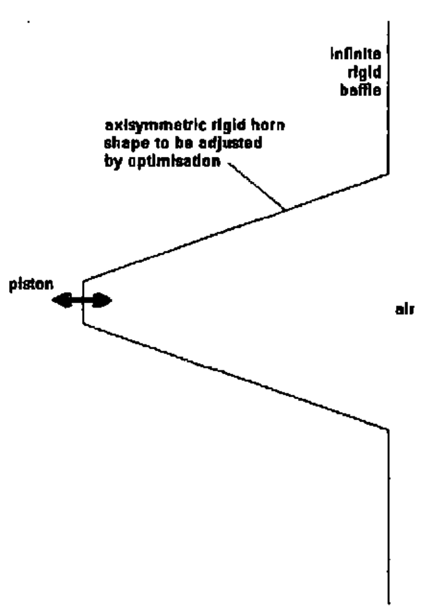

Figure 2 : horn optimisation problem

A more interesting optimisation problem, is illustrated in figure 2. An axisymmetric horn of axial length 0.18m has radius 0.0125m at one end, where unit amplitude acceleration is applied by a rigidly moving piston, and radius 0.075m at the other end, which is mounted in an infinite rigid baffle. The shape of the cross section is to be varied to improve the performance over the frequency range 2000 Hz to 4000 Hz. The performance was taken to be the level of output, the smoothness of response or a combination of both factors for a point at 1 metre on axis. The properties of air were taken as $c = 340ms^{-1}$ and $\rho = 1.2kgm^{-3}$.

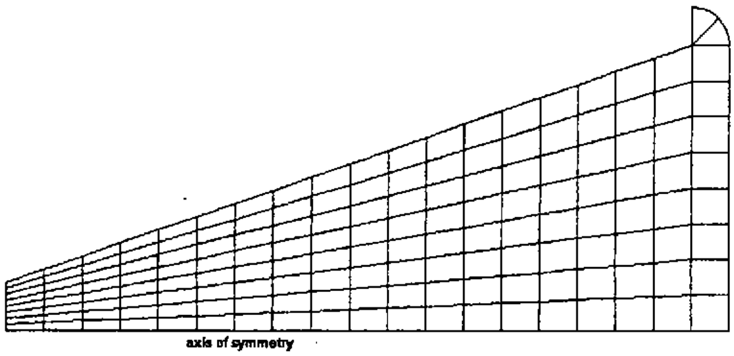

Figure 3: *initial mesh used for horn optimisation*

An axisymmetric model was used. The air inside the horn and extending a small distance out into the half space was modelled with 154 quadratic acoustic finite elements, shown in figure 3. At the left hand end a prescribed acceleration was applied.

The right hand side of the mesh was coupled to an acoustic boundary element, modelling the remainder of the half space. The curve defining the shape of the horn was defined as a natural spline with 3 interior points. The initial configuration was for a straight line, as in figure 3. The system was analysed for excitation from 2000 Hz to 4000 Hz in steps of 50 Hz. The optimisation procedure above was applied for three different objective functions.

$$g_1 = -\sum_{i=1}^{k} |p(f_i)|^2 \quad (3)$$

$$g_2 = \sum_{i=1}^{k} |p(f_i)|^4 - \frac{1}{k}\left(\sum_{i=1}^{k} |p(f_i)|^2\right) = \mathrm{var}\left(|p|^2\right) \quad (4)$$

$$g_3 = \frac{\sum_{i=1}^{k} |p(f_i)|^4 - \frac{1}{k}\left(\sum_{i=1}^{k} |p(f_i)|^2\right)}{\sum_{i=1}^{k} |p(f_i)|^4} \quad (5)$$

In all cases f_i runs through the frequencies analysed and p is the pressure at 1 metre on axis. g_1 is constructed to maximise the output, over the frequency range. g_2 is for making the output smoother and g_3 is attempting to do both. After 6 iterations the objective function gradient had reduced by 2 orders of magnitude for g_1 and g_2 but only one order of magnitude for g_3. In all cases the optimisation parameters stayed strictly within the permitted ranges. The final horn shapes are shown in figure 4. A comparison of pressure at 1 metre on axis against frequency is shown in figure 5. It is again clear that it is crucially important to get the objective function definition correct to achieve what is required. Optimisation using g_1 increased the overall on axis response, but not at all frequencies, and did have the side effect of making the response less smooth. It is interesting to note that the horn meets the baffle perpendicularly in this case. Perhaps the optimisation method 'decided' that the best way to achieve maximum on axis response is to have a plane wave emerging from the mouth of the horn!

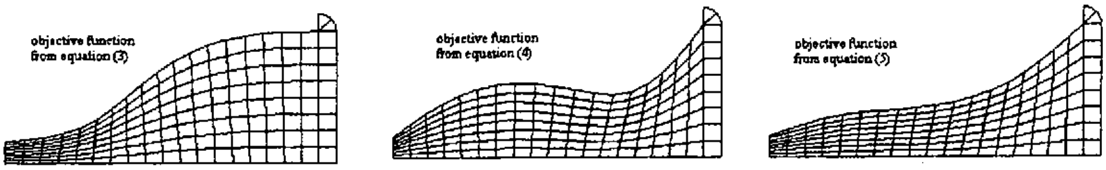

Figure 4 : *optimisation shapes for different objective functions*

Optimisation using g_2 produced a smoother response, particularly in the upper part of the frequency range, but reduced the overall level of output. Optimisation using g_3 produced a slightly smoother response, but at the cost of a reduced level of output. The shape, apart from the initial section near the throat, looks similar to an exponential curve.

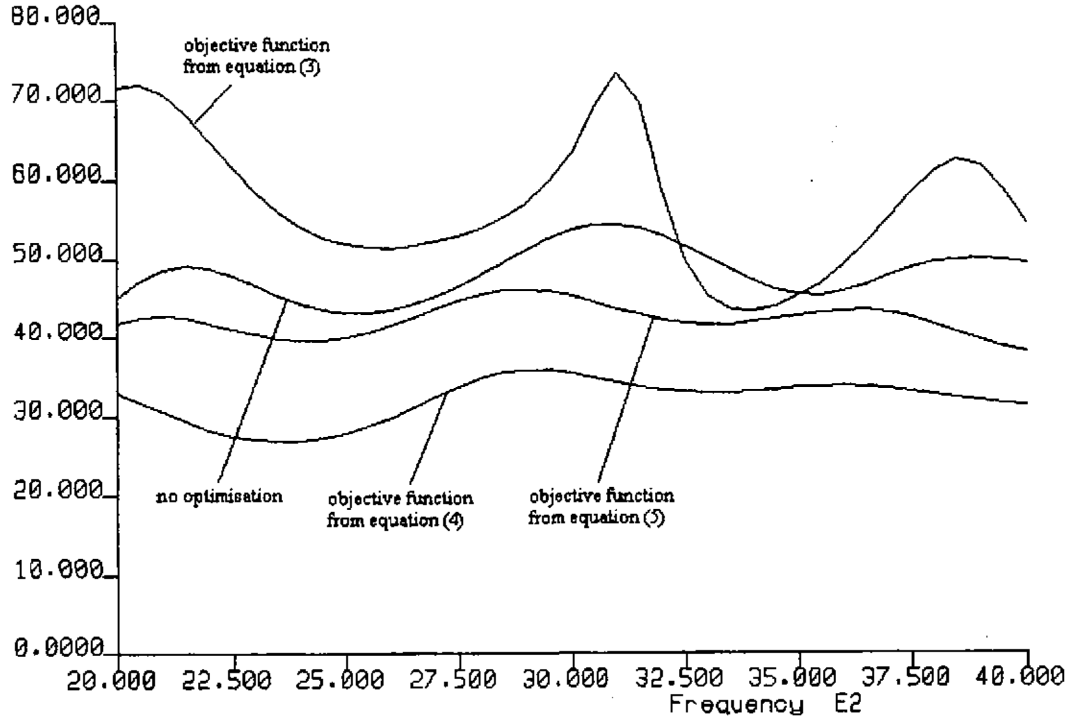

Figure 5 : *pressure on axis before and after optimisation*

6. CONCLUSIONS AND FURTHER WORK

The optimisation procedures tested seem quite capable of improving a single requirement, e.g. output or smoothness of response. Further investigation in constructing the objective function would be helpful for more complex design requirements.

The optimisation scheme would benefit from the inclusion of other methods such as genetic algorithms or simulated annealing, to search for a global minimum. The optimisation scheme written is applicable to a wide range of problems. It has been demonstrated on horn design. It could be used, for instance, to design the speaker cabinet, containing a cone/dustcap/surround drive unit, or to adjust the position of speakers in a small room. In principle it would be possible to define an objective function measuring the level of agreement between some experimental measurements and some computations. This could be minimised with respect to some unknown material parameters.

It is concluded that optimisation techniques could be of great benefit in the audio industry.

7. REFERENCES

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