ANALYSIS OF THE EFFECT OF MANUFACTURING TOLERANCES ON THE RADIATED FIELD FROM AN ARRAY OF SONAR TRANSDUCERS

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1. INTRODUCTION

Coupled finite/boundary element modeling is an established tool for determining the pressure radiated from an array of sonar transducers. Accurate predictions can be made for a given transducer specification, reducing the need for costly experiments. Generally, it is assumed that the transducers deployed within an array are identical, however in practice, the transducers may exhibit a range of performances due to manufacturing tolerances. Manufacturing tolerances may include variations in the dimensions and material properties of the transducers' component parts and it is important to determine their affect on overall array performance. A numerical method has been developed to address this. The proposed method starts with an in vacuo natural frequency calculation using a structural finite element mesh. A number of modes are then selected. A second fully coupled analysis includes a modal model of each transducer and a boundary element mesh of the wet surface of the array. The acoustic degrees of freedom are eliminated, reducing down to a small complex dense set of equations, relating to the modal contribution factors. The sensitivity of modal response and radiated pressure due to changes in the modal dynamic stiffness matrix (and indirectly the in vacuo natural frequencies of the transducers) can now be determined.

2. MODAL EQUATIONS

For a structural FE mesh with stiffness matrix [S] and mass matrix [M] the modes can be computed by solving the eigenvalue problem.

$$[S]\{u\} = \omega^2[M]\{u\} \tag{1}$$

Let the (j)th eigenvalue be ω_j with associated eigenvector $\{u_j\}$. Then

$$[S]\{u_j\} = \omega_j^2 [M]\{u_j\} \tag{2}$$

The eigenvectors can be scaled and hence it can be assumed that they are orthonormal with respect to the mass matrix, i.e.

$$[U]^T [M][U] = [I] \tag{3}$$

where [U] is a matrix where the (j)th column is $\{u_j\}$ and [I] is the unit matrix. With this normalization it follows that

$$[U]^T [S][U] = \begin{bmatrix} \omega_1^2 & 0 & 0 & 0 & 0 \\ 0 & . & 0 & 0 & 0 \\ 0 & 0 & \omega_j^2 & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & \omega_n^2 \end{bmatrix} \tag{4}$$

When using modal representations it is often mathematically convenient to use a modal damping assumption, i.e. such that $[U]^T[C][U]$ is a diagonal matrix. If γ_j is the critical damping ratio for the (j)th mode then

$$[U]^T [C][U] = \begin{bmatrix} 2\gamma_1\omega_1 & 0 & 0 & 0 & 0 \\ 0 & . & 0 & 0 & 0 \\ 0 & 0 & 2\gamma_j\omega_j & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 2\gamma_n\omega_n \end{bmatrix} \tag{5}$$

The modal vectors span the space of all possible displacements. However for vibration at low frequencies only the geometrically most simple mode shapes, which are the lowest, have an effective contribution, unless there is excitation varying on a very small length scale. Thus it may be advantageous to represent the structural behaviour as a linear combination of the lowest few modes, i.e. assume that

$$\{u\} = \alpha_1\{u_1\} + \alpha_2\{u_2\} + \ldots + \alpha_k\{u_k\} \tag{6}$$

where $k \ll n$. In the text below [U] will now be taken to have k columns and be composed of the first k eigenvectors.

3. COUPLING TO ACOUSTIC BOUNDARY ELEMENT

If the elastic structure is in contact with an acoustic medium, modelled with the boundary element method, then the fully coupled equations take the form

$$\begin{bmatrix} [S]+i\omega[C]-\omega^2[M] & [T]^T \\ -\omega^2\rho[G][E]^T & [H] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{p_I\} \end{Bmatrix} \quad (7)$$

using the notation of ref [1].

Making the substitution

$$\{u\} = [U]\{\alpha\} \quad (8)$$

and premultiplying the structural equation by $[U]^T$ results in

$$\begin{bmatrix} [D] & [U]^T[T]^T \\ -\omega^2\rho[G][E]^T[U] & [H] \end{bmatrix} \begin{Bmatrix} \{\alpha\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} [U]^T\{F\} \\ \{P_I\} \end{Bmatrix} \quad (9)$$

where [D] is a diagonal matrix with the (j)th diagonal term being $\omega_j^2-\omega^2+2i\gamma_j\omega\omega_j$. Equation (9) can be solved to determine the modal contribution factors $\{\alpha\}$ and surface pressures $\{p\}$. The displacements $\{u\}$ can then be determined from equation (8).

If the system to be analysed contains several repeated structural components which are not coupled together except via the fluid, such as an array of identical sonar transducers, then equation (1) needs only be solved for a single component. If it is necessary to have an accurate model of one particular component, but take into account the iteraction effects with other radiating/scattering systems then an ordinary structural FE mesh could be used for the part which needs accurate modelling and the other components can be modelled with modal substructures, partitioning the structural part of the equations and using a combination of equations (7) and (9).

The methodology above works well if the system is excited by structural forces, but if a structural freedom has a defined response, as e.g. in voltage excitation of a transducer, then this cannot be easily incorporated into equation (9). This can be achieved however by partitioning the structural matrices as

$$[S] = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix}, \quad [M] = \begin{bmatrix} [M_{11}] & [M_{12}] \\ [M_{21}] & [M_{22}] \end{bmatrix}$$

(10)

where the subscript 1 is for degrees of freedom with unknown displacement (or voltage) and known force (or charge) and 2 is for degrees of freedom where the response is defined. The eigenvalue problem would be solved using the matrices [S_{11}] and [M_{11}], i.e. the short circuit case, with both electrodes earthed, for transducers. Equation (9) then becomes

$$\begin{bmatrix} [D_{11}] & [U_1]^T([S_{12}]-\omega^2[M_{12}]) & [U_1]^T[T_1]^T \\ ([S_{21}]-\omega^2[M_{21}])[U_1] & [S_{22}]-\omega^2[M_{22}] & [T_2]^T \\ -\omega^2\rho[G][E_1]^T[U_1] & -\omega^2\rho[G][E_2]^T & [H] \end{bmatrix} \begin{Bmatrix} \{\alpha_1\} \\ \{u_2\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} [U_1]^T\{F_1\} \\ \{F_2\} \\ \{p_I\} \end{Bmatrix} \quad (11)$$

The enforced values can then be applied in a straightforward manner, giving

$$\begin{bmatrix} [D_{11}] & [U_1]^T[T_1]^T \\ -\omega^2\rho[G][E_1]^T[U_1] & [H] \end{bmatrix} \begin{Bmatrix} \{\alpha_1\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} [U_1]^T(\{F_1\}-([S_{12}]-\omega^2[M_{12}])\{u_2\}) \\ \{p_I\}+\omega^2\rho[G][E_2]^T\{u_2\} \end{Bmatrix} \quad (12)$$

If the pressures are eliminated then an equation for $\{\alpha_1\}$ is obtained.

$$[D']\{\alpha_1\}=\{F'\}$$
(13)

where

$$[D']=[D_{11}]+\omega^2\rho[U_1]^T[T_1]^T[H]^{-1}[G][E_1]^T[U_1] \quad (14)$$

and

$$\{F'\}=[U_1]^T\{F_1\}-([S_{12}]-\omega^2[M_{12}])\{u_2\}-[U_1]^T[T_1]^T[H]^{-1}(\{p_I\}+\omega^2\rho[G][E_2]^T\{u_2\}) \quad (15)$$

Equation (13) is a fairly compact set of equations. If there are t transducers, each with m modal degrees of freedom then the $[D']$ matrix is of dimension mt by mt.

4. SENSITIVITY COMPUTATION

If the material properties or transducer dimensions differ slightly from those intended then this can be approximated by assuming that the $[D']$ matrix changes, but that $[U_1]$ is unchanged. It would be expected that $[D']$ would be more sensitive than $[U_1]$ to structural changes. (If the ceramic in a ring transducer were a little stiffer then the breathing mode would occur, with identical mode shape, but at a higher frequency. In a flextensional transducer if the ceramic were stiffer then there would be a second order change in mode shape, because the structural change is not uniform throughout.)

The matrix $[D_{11}]$ is symmetric. Assume that the (jk)th/(kj)th terms vary slightly. Differentiating equation (13) with respect to D'_{jk} results in

$$[D']\frac{\partial\{\alpha_1\}}{\partial D_{jk}}+\{\beta_{jk}\}=\{0\} \tag{16}$$

where $\{\beta_{jk}\}$ has α_j in the (k)th row, α_k in the (j)th column and zeroes elsewhere. Hence, once equation (13) has been solved to determine $\{\alpha_1\}$ then the sensitivities of the modal contribution factors with respect to the $[D']$ matrix can be determined by solving the equations

$$\frac{\partial\{\alpha_1\}}{\partial D_{jk}}=-[D']^{-1}\{\beta_{jk}\} \tag{17}$$

Given that the modal variables within one transducer only couple to modal variables within the same transducer, the number of (j,k) pairs which need to be considered is m(m+1)t/2. It is possible to solve equation (17) with multiple right hand side vectors reusing the factorisation used in solving equation (13).

The complex directivity, in a particular direction, can be computed once the surface pressures and pressures gradients are known, using an equation of the form

$$d(\theta)=[H_\theta]\{p\}+[G_\theta]\left\{\frac{\partial p}{\partial n}\right\} \tag{18}$$

where $[H_\theta]$ and $[G_\theta]$ are row vectors. Using the standard boundary element equations

$$[H]\{p\}+[G]\left\{\frac{\partial p}{\partial n}\right\}=\{0\} \tag{19}$$

this can be rewritten in the form

$$d(\theta)=\omega^2\rho\left([H_\theta][H]^{-1}[G]+[G_\theta]\right)[E_1]^T[U_1]\{\alpha_1\} \tag{20}$$

Using the derivative of this equation with respect to D_{jk}, the derivatives of the directivities with respect to the $[D']$ matrix can be determined.

In a similar manner, the variation of other quantities, such as the admittance, can also be studied.

5. RELATION TO VARIATIONS IN NATURAL FREQUENCY

If the transducers coming off the production line show deviations in the (j)th natural frequency of Δf_j. Then this would cause variations in the D'_{jj} term given by

$$\Delta D'_{jj} = 4\pi(\omega_j + i\gamma_j\omega)\Delta f_j \qquad (21)$$

Using this together with the derivative of equation (20), estimates of the variation of directivity of the transducer array, due to manufacturing tolerances, can be made.

6. CONCLUDING REMARKS

A methodology for estimating the variation in performance of an array of transducers due to variations in manufacturing tolerances has been outlined. This could be used to decide whether more expensive manufacturing, to tighter tolerances in the transducer design, is necessary to achieve the required array performance.

7. REFERENCES

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