

ACOUSTIC ANALYSIS OF LONG TRANSDUCER ARRAYS

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1. INTRODUCTION

Many structures occurring in underwater acoustics are constructed with an axially repeating section. Examples of this are a periodically rib-stiffened cylindrical hull, fig 1, or a line array of transducers, fig 2. Such a system is often excited, either by an incident plane wave or directly on the structure, in a manner with constant phase difference between periodic units. If the number of repeating sections is large and also many acoustic wavelengths in the extent, then it is often assumed that the system is infinite. Bloch's theorem can then be used to state periodic relations for displacements and pressures. In this paper a finite element based method for analysing such problems is proposed. A combined structural/acoustic FE mesh is constructed for a cylindrical region of fluid containing one periodic unit. Periodic constraints are applied between the ends. The curved surface of the acoustic FE faces is coupled to a series which automatically satisfies the Bloch-Floquet relations.

2. PRESSURE EXPANSION IN FLUID

Consider an axially periodic structure immersed in an acoustic medium. Assume that the x direction is axial, and let the periodic length be $2d$. If the system is vibrating in a steady state, excited by harmonic loading at circular frequency ω , and has a constant phase difference $-2qd$ between neighbouring periodic units, then from Bloch's theorem it follows that all response quantities (displacement, pressure, force,...) must satisfy the relationship

$$F(x + 2d, r, \theta) = F(x, r, \theta)e^{-2iqd} \quad (1)$$

For the case of an incident plane wave travelling in direction $\underline{n}$ the phase is determined by the angle of incidence, and

$$q = kn_x \quad (2)$$

where k is the wavenumber. By expanding $F(x, r, \theta)e^{-iqx}$ in the Fourier series it follows that

$$F(x, r, \theta) = e^{-iqx} \sum_{s=-\infty}^{+\infty} \bar{F}_s(r, \theta) e^{i \frac{\pi x}{d} s} \quad (3)$$

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where

$$\overline{F}_s(r, \theta) = \frac{1}{2d} \int_{-d}^{+d} e^{-ik_s x} F(x, r, \theta) dx \quad (4)$$

and

$$k_s = \frac{\pi S}{c} - q \quad (5)$$

In the acoustic medium the scattered component of the pressure field satisfies the Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \quad (6)$$

Taking a cylindrical axis set and assuming an $e^{im\theta}$ circumferential variation, this reduces to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) + \frac{\partial^2 p}{\partial x^2} + \left(k^2 - \frac{m^2}{r^2} \right) p = 0 \quad (7)$$

Assuming further an $e^{ik_s x}$ axial dependency this becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) - \left(\gamma_s^2 + \frac{m^2}{r^2} \right) p = 0 \quad (8)$$

where

$$\gamma_s^2 = k_s^2 - k^2 \quad (9)$$

The solutions to equation (8) are Bessel functions. Choosing the functions to satisfy the Sommerfeld radiation condition for travelling wave cases, and choosing radially bounded solutions otherwise, results in

$$p(x, r, \theta, t) = f_m(r; \gamma_s^2) e^{ik_s x} e^{im\theta} e^{i\omega t} \quad (10)$$

where

$$f_m(r; \gamma_s^2) = \begin{cases} H_m^{(2)}(\beta_s r) & \text{if } \gamma_s^2 < 0 \\ K_m(\gamma_s r) & \text{if } \gamma_s^2 > 0 \\ r^{-m} & \text{if } \gamma_s^2 = 0 \end{cases} \quad (11)$$

$$\text{and } \beta_s = \sqrt{-\gamma_s^2}$$

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3 COUPLING TO FINITE ELEMENTS

Assume that the pressure field in the region $\{(x, r, \theta) : -d \leq x \leq +d, r \geq a\}$ is modelled as a superposition of a finite number of terms of the type in equation (10). The system in the finite cylindrical region can be modelled with a mesh of acoustic/structural FE as in figure 3.

Assume that a is taken large enough such that the interface S consists of acoustic FE faces. The discretized equations are of the form:

$$\begin{bmatrix} [S] - \omega^2 [M] & [T]^T \\ [T] & \frac{1}{\omega^2} [S_a] - [M_a] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{F_a\} \end{Bmatrix} \quad (12)$$

where $[S]$ and $[M]$ are the structural stiffness and mass matrices, $[T]$ is a coupling matrix, $[S_a]$ and $[M_a]$ are acoustic "stiffness" and "mass" matrices, $\{u\}$ is a vector of displacements on the structural mesh, $\{p\}$ is a vector of pressures on the fluid mesh, $\{F\}$ is a vector of forces applied directly to the structural mesh and $\{F_a\}$ are the 'fluid forces' caused by surface vibration on S .

$$\{F_a\} = \int_S [N]^T u_r dS \quad (13)$$

where $[N]$ is the interpolation function row vector for pressures on the fluid mesh. The $\{F_a\}$ vector must be constructed so as to correctly couple the FE mesh to the series solution. Furthermore the Bloch-Floquet relations must be applied as constraints to the FE matrices before equation (12) is solved.

Using only a finite number of terms from the series, the total pressure in the external region can be represented as:

$$P(x, r, \theta) = P_l(x, r, \theta) + \sum_{s=-N_s}^{+N_s} \sum_{m=-N_m}^{+N_m} f_m(r; \gamma_s^2) e^{ik_s x} A_{ms} e^{im\theta} \quad (14)$$

The coefficients can be computed as:

$$A_{ms} f_m(a; \gamma_s^2) = \frac{1}{4\pi d} \int_{x=-d}^{+d} \int_{\theta=-\pi}^{+\pi} (P(x, a, \theta) - P_l(x, a, \theta)) e^{-ik_s x} e^{-im\theta} d\theta dx \quad (15)$$

Differentiating (14) with respect to r gives

$$\frac{\partial p}{\partial r} = \frac{\partial P_l}{\partial r} + \sum_{s=-N_s}^{+N_s} \sum_{m=-N_m}^{+N_m} \frac{\partial f_m(r; \gamma_s^2)}{\partial r} e^{ik_s x} A_{ms} e^{im\theta} \quad (16)$$

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Substituting in for A_{ms} in equation (16) from (15) and then substituting into (13) results in an equation of the form:

$$\{F_a\} = [\bar{A}]^T [D] [A] \{P\} + \{F_{p_l}\} \quad (17)$$

where $[A]$ has $(2N_s + 1)(2N_m + 1)$ rows, and the $(s, m)th$ row has the form

$$\{A\}_{sm}^T = \int_{x=-d}^{+d} \int_{\theta=-\pi}^{+\pi} [N(x, a, \theta)] e^{-ik_s x} e^{-im\theta} d\theta dx \quad (18)$$

$[D]$ is a diagonal matrix and $\{F_{p_l}\}$ is a term due to the incident wave loading

$$\begin{aligned} \{F_{p_l}\} = & -\frac{1}{4\pi\omega^2\rho d} \sum_{s=-N_s}^{+N_s} \sum_{m=-N_m}^{+N_m} \frac{\partial f_m(a; \gamma_s^2)}{\partial r} \cdot \int_{\theta=-\pi}^{+\pi} \int_{x=-d}^{+d} [N(x, a, \theta)]^T e^{ik_s x} e^{im\theta} \\ & \int_{\theta'=-\pi}^{+\pi} \int_{x'=-d}^{+d} P_l(x', a, \theta') e^{-ik_s x'} e^{im\theta'} d\theta' dx' + \frac{1}{\omega^2\rho} \int_{\theta=-\pi}^{+\pi} \int_{x=-d}^{+d} [N]^T \frac{\partial P_l}{\partial r} d\theta dx \end{aligned} \quad (19)$$

using the expression

$$P_l(x, r, \theta) = P_l e^{-ikn_x x} e^{-ikn_r r \cos(\theta - \theta_a)} \quad (20)$$

it can be shown that only $s = 0$ contributes to the first term of equation (19).

Thus the equations to be solved are:

$$\begin{bmatrix} [S] - \omega^2 [M] & [T]^T \\ [T] & \frac{1}{\omega^2} [S_a] - [M_a] - [\bar{A}]^T [D] [A] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{F_{p_l}\} \end{Bmatrix} \quad (21)$$

The periodic constraints must be applied before solution.

The methodology outlined above is similar to the doubly periodic case, which has been described by Hladky-Hennion and Decarpigny [1].

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4. ANALYSIS RESULTS

The method described above has been implemented in the PAFEC VibroAcoustics code for both 3D and axisymmetric models. This functionality has been tested on 3 problems, scattering of a normally incident plane wave by a rigid cylindrical surface, an infinite line array of point sources and the sound field generated by an infinite linear array of coaxial ring transducers. In all these examples, the properties of water were assumed to be density = 1000kgm^{-3} and speed of sound = 1500ms^{-1} .

A 3D mesh composed of 12 quadratic 20 noded brick elements, illustrated in figure 4, was used to compute scattering by the rigid cylinder, the radius of the cylinder was taken as 1m. A comparison of pressure at the 'far point in the shadow zone' against frequency computed by the FE method and using a closed form series solution is made in figure 5.

For the array of point sources, the n th source was assumed to lie at $(n,0,0)$. The phase of the n th source was taken to be at $0.296192\pi n$. Each source was assumed to produce an $\frac{e^{-ikr}}{r}$ distribution. Results were computed using the axisymmetric mesh of figure 6. Table 1 gives a comparison with a series method using 2000001 terms for the pressure at $(0, 0.5, 0)$.

Table 1

Frequency	Pressure (series)	Pressure (FE)
100	4.14317-3.12439i	4.14330-3.12437i
200	2.08882-3.02216i	2.07786-3.02215i

The ring projectors were taken to have inner radius 0.0508m, outer radius 0.05715m, axial length 0.028m and be made of PZT4, polarized radially. The axial separation was taken as 0.052m. The analysis was performed using the axisymmetric mesh of figure 7. The conductance/ring graph computed is shown in figure 8. Results from finite arrays calculated using BE have been published previously by Gallaher [2] but these are not directly comparable with the current work.

5. CONCLUSIONS - FUTURE WORK

The proposed method has been shown to work well on the first two test problems. The results from the analysis of the transducer array are plausible. It is hoped to compute results for large finite arrays for comparison with the infinite results. The technique seems a promising method for analysing very long linear arrays of regularly spaced transducers.

6. REFERENCES

- [1] A Hladky-Hennion and J Decarpigny
 "Analysis of scattering of a plane acoustic wave by a doubly periodic structure using the finite element method; Application to Alberich Anerchoic Coatings"
 Jou. Acoust. Soc. Am. Vol 90 No 6 1991 pp3356-3367

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- [2] A B Gallaher
"Performance prediction of an array of free flooding ring transducers"
Proc. IOA Vol 17 Pt 3 1995 pp34-43

Fig 1

PERIODICALLY RIB-STIFFENED CYLINDER

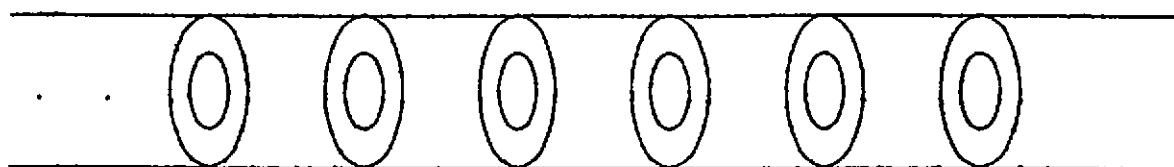
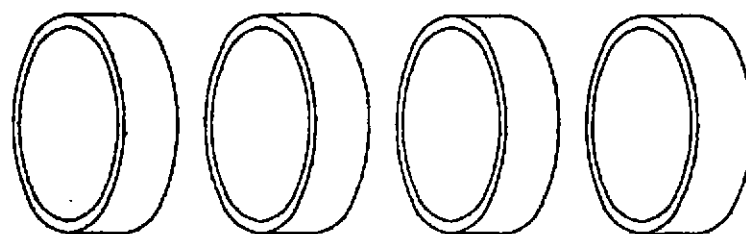


Fig 2

COAXIAL ARRAY OF RING TRANSDUCERS



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Fig 3

FINITE ELEMENT MESH FOR AXIALLY PERIODIC ANALYSIS

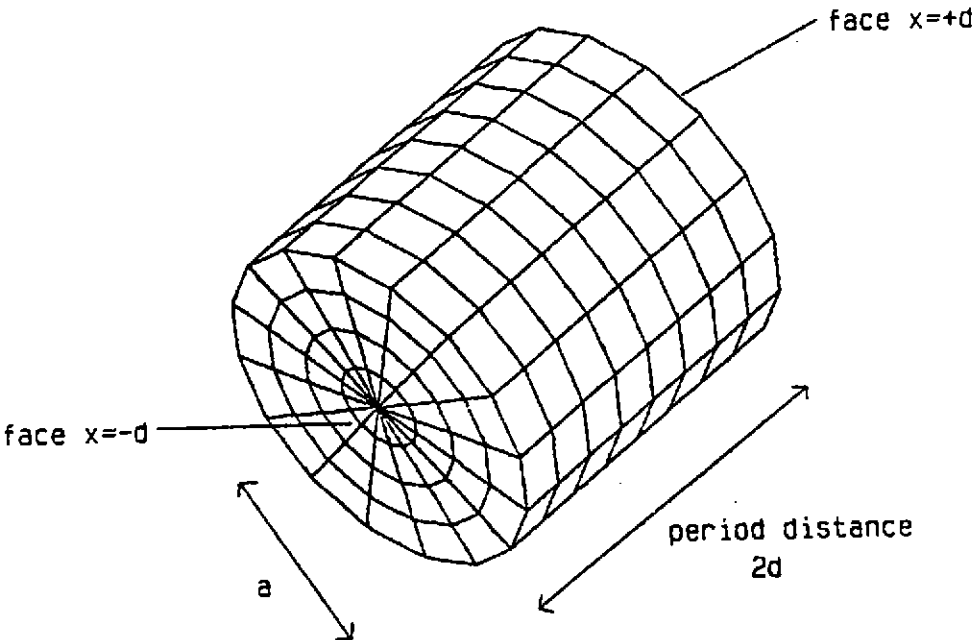


Fig 4

MESH USED TO ANALYSE SCATTERING BY RIGID CYLINDER

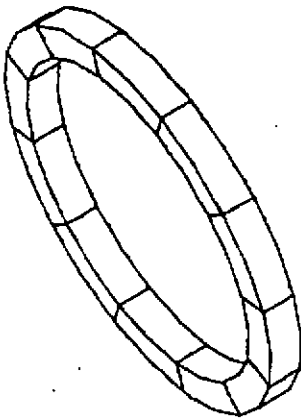
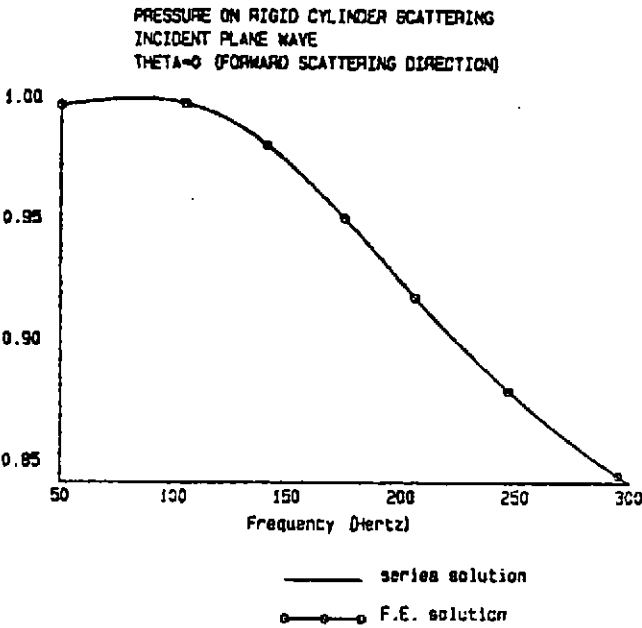


Fig 5



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Fig 6 MESH USED TO ANALYSE INFINITE ARRAY OF POINT SOURCES

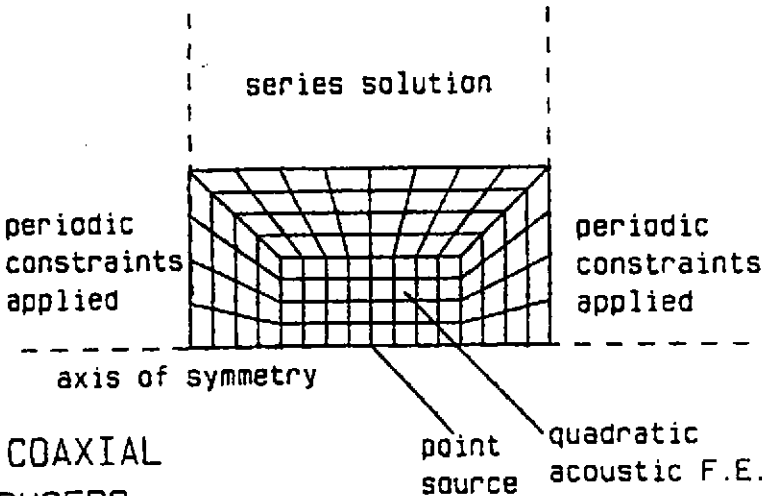
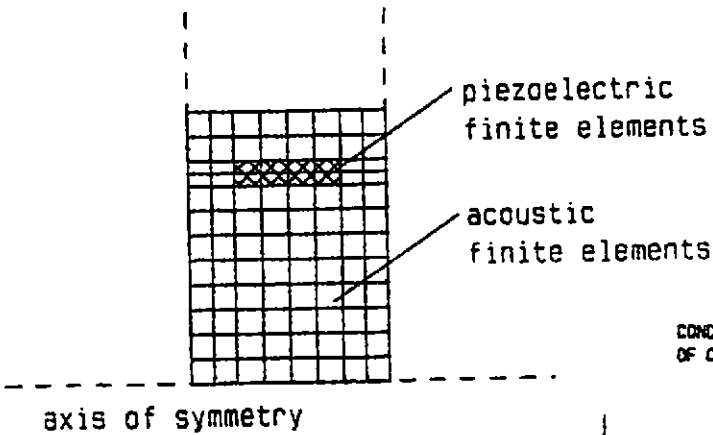


Fig 7 MESH USED TO ANALYSE COAXIAL ARRAY OF RING TRANSDUCERS



CONDUCTANCE/RING OF INFINITE ARRAY OF COAXIAL RING TRANSDUCERS

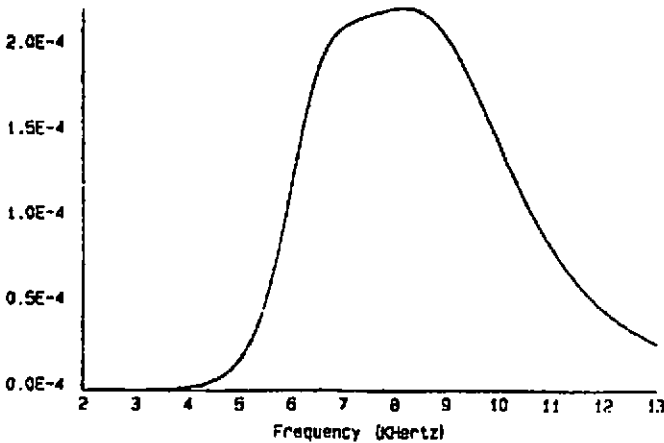


Fig 8