inter-noise 96
25th Anniversary Congress – Liverpool

INCE: 37.7

OPTIMAL PLACEMENT OF SECONDARY SOURCES AND ERROR MICROPHONES FOR ACTIVE NOISE CONTROL IN A STRAIGHT WAVEGUIDE

P Sergent & D Duhamel

Ecole Nationale des Ponts et Chaussées, CERAM, 1 avenue Montaigne, 93167 Noisy Le Grand Cedex, France

1. INTRODUCTION

The locations of secondary sources and error microphones greatly affect the performance of an active noise control system. For a single frequency excitation, it has been shown that a combined strategy of *selection* and *gradient minimization* can enhance their placement [1]. In this paper, an original strategy [2] of *selection* of secondary sources and error microphones for active control is briefly reviewed. The main feature of this strategy is the use of a new objective function: the largest modulus of the sound pressure at a large number of points. Results in an infinite straight duct are then presented and compared with a placement that was proposed in a previous paper [3].

2. STRATEGY OF PLACEMENT

Let us note $\underline{y}_i$ and $\underline{x}^j$ the points whose respective coordinates are (u_i, v_i, w_i) and (u^j, v^j, w^j). Let us consider N secondary point sources located at $\underline{Y} = (\underline{y}_1, .., \underline{y}_i, .., \underline{y}_N)$ with volume velocities $\underline{Q} = (q_1, .., q_i, .., q_N)$ and M_0 error sensors located at $\underline{X} = (\underline{x}^1, .., \underline{x}^j, .., \underline{x}^{M_0})$. Let us write $p_p(\underline{x}^j)$ the primary noise at the location $\underline{x}^j$ and $G(\underline{y}_i|\underline{x}^j)$ the transfer function between the source i and sensor j. The sound pressure $p(\underline{Q}, \underline{Y}, \underline{x}^j)$ at sensor j can now be written as:

$$p(\underline{Q}, \underline{Y}, \underline{x}^j) = p_p(\underline{x}^j) + \sum_{i=1}^{N} q_i G(\underline{y}_i|\underline{x}^j) \quad (1)$$

In order to take advantage of the features of linear programming, the maximum modulus of the sound pressure at the M_0 error sensors is chosen as objective function J:

$$J(\underline{Q}, \underline{Y}) = \max_{j \in \{1,..,M_0\}} |p(\underline{Q}, \underline{Y}, \underline{x}^j)| \quad (2)$$

After minimization of function J with respect to the volume velocities $\underline{Q}$, a solution $\underline{Q}_0$ is determined. The minimum value of J is called J_0. A method of selection of error sensors can then be presented. Let us select the error sensors with index j such as:

$$|p(\underline{Q}_0, \underline{Y}, \underline{x}^j)| = J_0 \tag{3}$$

Let us call G_1 the subset of error sensors verifying the equation (3). The cardinal number of this subset is M. Let us introduce another objective function J_1 retaining only the error sensors belonging to subset G_1.

$$J_1(\underline{Q}, \underline{Y}) = \max_{j \in G_1} |p(\underline{Q}, \underline{Y}, \underline{x}^j)| \tag{4}$$

It can be shown (see [2]) that the minimization of J_1 with respect to the volume velocities $\underline{Q}$ leads to the same solution as the minimization of J. The notion of *sufficient* number of error microphones can then be introduced. These M error sensors provide the same acoustic attenuation as M_0 error sensors. Their number and locations are found simultaneously by solving a linear programming problem. The locations are found at the maximums of the modulus of the residual pressure. Generalization to the placement of secondary sources is also possible (see [2]). Among N_0 candidate locations for N secondary sources, the optimal placement for sources is determined. We have to deal with an integer programming problem, more difficult than linear programming to solve. Different algorithms are however available in the litterature.

3. APPLICATION TO A STRAIGHT WAVEGUIDE

The model used in the following analysis is a hardwalled infinite waveguide. The cross-section is rectangular with side lengths L_y and L_z. The transfer functions are given by

$$G(\underline{y}_i|\underline{x}^j) = \frac{\rho_0 \omega}{2} \sum_{I=0}^{\infty} \frac{1}{k_I} \psi_I(v_i, w_i) \psi_I(v^j, w^j) e^{jk_I|u_i - u^j|} \tag{5}$$

j is the imaginary unit, ω the angular frequency, ρ_0 the air density and C_0 is the speed of sound. The mode shapes $\psi_I(y, z)$ are:

$$\psi_I(y, z) = \sqrt{\frac{2 - \delta_{0m_I}}{L_y}} \sqrt{\frac{2 - \delta_{0n_I}}{L_z}} cos(\frac{m_I \pi y}{L_y}) cos(\frac{n_I \pi z}{L_z}) \tag{6}$$

where m_I and n_I are the modal integers. The quantity k_I is the wavenumber associated with the I^{th} mode and is given by

$$k_I = \sqrt{\frac{\omega^2}{C_0^2} - \frac{m_I^2 \pi^2}{L_y^2} - \frac{n_I^2 \pi^2}{L_z^2}} \tag{7}$$

Our example is similar to the problem studied by Stell and Bernhard [3]. The dimensions L_y and L_z are equal to 1.0 m and 0.1 m respectively. The zone of silence (see figure 1) is limited by the sections u_c and u_d equal to 5.0 m

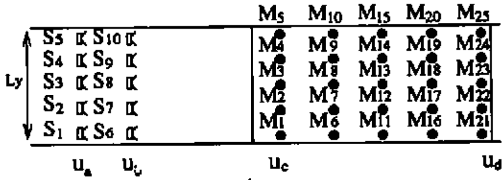

Fig. 1: Candidate locations for sources and sensors.

and 7.0 m. This zone of silence is occupied by 25 sensors. Their locations are defined by the coordinates $(5.0 + 0.5m, 0.25n, 0.05)$ with m and n belonging to $\{0, 1, 2, 3, 4\}$.

The location of a unit point primary source is $\underline{y}_p = (0, 0, 0)$ so that all the modes are excited. The zone of control, occupied by secondary sources, is limited by the sections u_a and u_b equal to 3.0 m and 3.5 m. The number of candidate locations N_0 is equal to 10. Their locations are defined by the coordinates $(3.0 + 0.5m, 0.25n, 0.05)$ with m and n belonging to $\{0, 1\}$ and $\{0, 1, 2, 3, 4\}$ respectively. The summation of 10 modes is found to be sufficient for the calculations. The frequency f in the rest of the text is an adimensional number equal to $\frac{L_y \omega}{\pi C_0}$. The results presented in the next section concern the case of N equal to 2 secondary sources.

4. RESULTS

frequencies f	1.7	1.8	1.9
sources	S_2-S_4	S_2-S_4	S_7-S_9
sensors	M_1-M_2-M_4	M_1-M_3-M_5	M_1-M_3-M_5

Table 1: Optimal locations of secondary sources and error sensors.

Let us first consider frequencies f below 2 so that the number of secondary sources N is at least equal to the number of propagative modes. Results are summed up in table 1. We can compare these results with considerations of Stell and Bernhard [3]. They propose to move error sensors as far downstream from secondary sources as it is practical. These error sensors and the secondary sources should be located on the nodal lines of the most significant evanescent mode in the waveguide. In short, these considerations advise us the locations (S_2-S_4) for sources and (M_{22}-M_{24}) for error sensors. Table 1 shows that our method mostly determines on the one hand the same location for sources, on the other hand location for sensors at the maximums of the most significant evanescent mode and close to the sources. The *sufficient* number of sensors is found equal to 3.

The value of the objective function J is then computed for a control with the optimal placement of sources and three different placements of sensors: (a) the optimal placement for sensors determined by linear programming, (b) locations on the nodal lines of the most significant evanescent mode, first close to sources

frequencies f	1.7	1.9
sensors M_2-M_4	5.2	0.8
sensors M_{22}-M_{24}	1.3	0.4

Table 2: Gap in dB between the optimal solution and placements (b) and (c).

(locations M_2-M_4), (c) then far downstream from the sources (locations M_{22}-M_{24}). Table 2 shows the gap in decibels between the optimal locations and the two other placements. Results confirm first that error sensors should be located as far downstream from the sources as it is possible if the sensors are on the nodal lines of the first evanescent mode. It is noticed also that the gap between the three placements is reduced when the frequency f approaches the cut-off frequency 2. It can be shown actually that these three placements would give exactly the same control if there were a single evanescent mode. The control would consist then in cancelling the two propagative modes. The results show also that the placement proposed by Stell is interesting because it uses only two sensors and it provides a control close to the optimal solution that we have determined for our own objective function.

When the frequency f exceeds 2, there are more propagative modes than secondary sources. The control can not cancel the complete set of propagative modes. Since the optimal locations for sensors is found at the maximums of the modulus of the residual sound pressure, let us focus on the shape of the residual sound pressure. It contains mainly propagative components. The maximums of the modulus are now spread out along the duct, so are the optimal locations for sensors.

5. CONCLUSION

This paper is devoted to the optimal placement of secondary sources and error sensors in a straight infinite waveguide. An original method of placement determines simultaneously a *sufficient* number and locations of error sensors. This method is also adapted to the placement of secondary sources. When the number of sources is equal to the number of propagative modes, the locations of sources should be on the nodal lines of the most significant evanescent mode and mostly far upstream from the zone of silence. The optimal location for error sensors is found at the maximums of the most significant evanescent mode and close to the zone of control. The locations of error sensors on the nodal lines of this evanescent mode seems a *good* placement if these sensors are far enough from the zone of control. When the number of propagative modes exceeds the number of secondary sources, sensors should be spread out along the axis of the waveguide.

References

[1] **Benzaria E. and Martin V.**, J. Sound Vib., 173(1):137-144, 1994.
[2] **Sergent P.**, Active 95, 661-671, 1995.
[3] **Stell J.D. and Bernhard R.J.**, J. Sound Vib., 173(2):197-215, 1994.