ON THE USE OF ARRAY SNAKING FOR RESOLVING THE LEFT/RIGHT AMBIGUITY OF LINEAR ANTENNAS IN TOWED ARRAY SONARS

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1. INTRODUCTION

Straight linear antennas do not allow to determin whether a plane wave impinges on the array from the right or left side. Their directivity patterns have a rotational symmetry with the axis of rotation being the antenna axis. Changing the course of the towing vessel while having several target contacts, the right/left ambiguity can be resolved by observing the change of the direction of arrival with respect to the own heading. This method fails when target signals can only be received intermittendly or occasionally. In towed array sonar systems the array tends to oscillate slightly around the mean course (array snaking). This effect can be exploited for resolving the right/left ambiguity. This may be done on the beamforming level when performing the bearing measurements. The steering vectors in the beamformer have to be modified to take into account the array deviation. The paper describes a simple procedure assuming an array shape of a circular arc. It will be shown how the probability of a correct decision depends upon the signal to noise ratio (SNR) and the curvature. Proven by simulations as well as by at-sea experiments, the results show that a precise knowledge of the radius of curvature is not necessary and that already very slight curvatures lead to reasonable results at reasonable SNR.

2. MODIFICATION OF THE BEAMFORMER

Assuming a plane wave impinging a set of sensors from an angle β_j with a phasevelocity v_m, the signal arrives a certain sensor with its polarcoordinates (a_i, α_i) with a time-shift τ_{ij} of

$$\tau_{ij} = \frac{a_i}{v_m}\cos(\alpha_i - \beta_j) \tag{1}$$

relative to the choosen origin. Now, summing up the sensor outputs with respect to the time-shift for various angles β_j leads to the beampattern, which show a maximum at the angle $\beta_j = \beta_q$, if there is a target at the angle β_q.

With knowledge of the sensorspacing, efficient computation of (1) for an towed array requires assumptions about the shape. For a straight linear array and an origin upon the array axis, all a_i are known and all $\alpha_i = const.$ For a given array system of N sensors with spacings of a half wavelength, a computationally attractive model for the array's shape is that of a circular arc. For the origin being the first sensor and 180 degrees being defined by the direction to the last sensor, the time-delays can be calculated from (1) with

$$\alpha_i = \frac{N-i}{4R_a} \text{ and} \tag{2}$$

$$a_i = 2R_a \sin\left(\frac{i-1}{4R_a}\right), \tag{3}$$

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with R_a being the radius of the array's shape. The deviation h from the straight linear array can be calculated with

$$h \approx \frac{l^2}{8R_a}, \tag{4}$$

for an array length l being short compared with the radius of curvature R_a. For resolving the right/left ambiguity, the variation of the beamformer output on the right and left side as a function of the direction of arrival β_q and radius of curvature R_a has to be investigated. In practice, obviously there is an additional detail that effects the results: the radius of curvature R_e used in calculating the time shifts is only a result of measurement, or estimation, and therefore may differ from R_a.

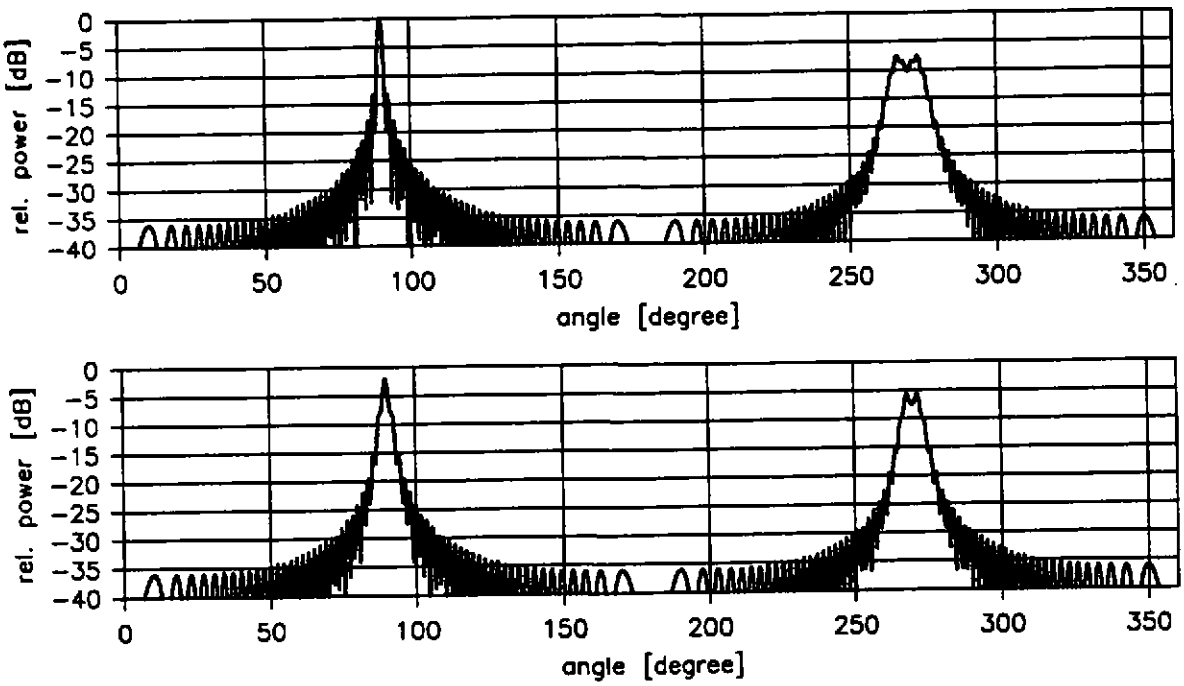

Figure 1: Beampattern obtained with an array of 64 hydrofones in spacings of half a wavelength. The targets direction is $\beta_q = 90$ degrees, the deviation of the array is half a wavelength. Top: beampattern calculated for known deviation ($R_e = R_a$), bottom: beampattern calculated under the assumption of a smaller deviation ($R_e = 3R_a$). The use of a much too large R_e leads to a decrease of beam power at the bearing angle $\beta_q = 90$ degrees, but the power obtained there is still larger than that at the corresponding wrong direction at 270 degrees.

With an array shape of a circular arc of radius R_a and a source located at an angle β_q, the beamformer output for the direction β_q as a function of the number of hydrofones, their spacings, R_a, β_q and the radius R_e is larger than that for the corresponding direction $360 - \beta_q$ degrees, though, under reasonable large R_a, their difference is very small. Due to the slight differences of the corresponding time shifts, the modified beamformer will, as well as for the target signal, present nearly the same amount of background or ambient noise for the corresponding wrong angles. By chance, the remaining differences caused of noise may lead to a larger output in the opposite direction, so a wrong decision about the side will occur in a statistical sense.

3. PROBABILITY OF A WRONG DECISION

With no target present, the sensor amplitudes as well as the beamformer output amplitudes are Rayleigh-distributed. With a target in direction β_q, the beamformer amplitudes for the angles β_q (denoted by 'x') and $360 - \beta_q$ degrees (denoted by 'y') are approximatively normal distributed, the higher the signal to noise ratio, the better the approximation. While the variances σ^2 being identically, their means μ are determined by the way of beamforming as shown in the first chapter. Taking into account the correlation ϱ of noise, their joint distribution is:

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\varrho^2}} \exp \frac{-1}{2(1-\varrho^2)} \left(\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\varrho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right) \quad (5)$$

Using 'x' for the correct side involving $\mu_x > \mu_y$, a random variable $Z = X - Y$ describes the decision rule: the target is on that side with the higher output. The distribution of Z can be evaluated analytically:

$$f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma_x\sqrt{2(1-\varrho)}} \exp - \frac{1}{4\sigma_x^2(1-\varrho)}(z - (\mu_x - \mu_y))^2 \quad (6)$$

that means

$$Z \sim N(\mu_x - \mu_y, 2\sigma_x^2(1-\varrho)) \,. \quad (7)$$

Observing the behaviour of mean and variance of Z with respect to their dependence on geometry shows that the probability of a wrong decision strongly depends on the radius of curvature R_a and the bearing angle β_q (and signal to noise ratio), but is nearly independent of the used radius R_e within a wide range of variations. So, for the decision, on which side of the towing ship a detected target is located, one only has to know, on which side of the towing ship the center of the curvature of the array is.

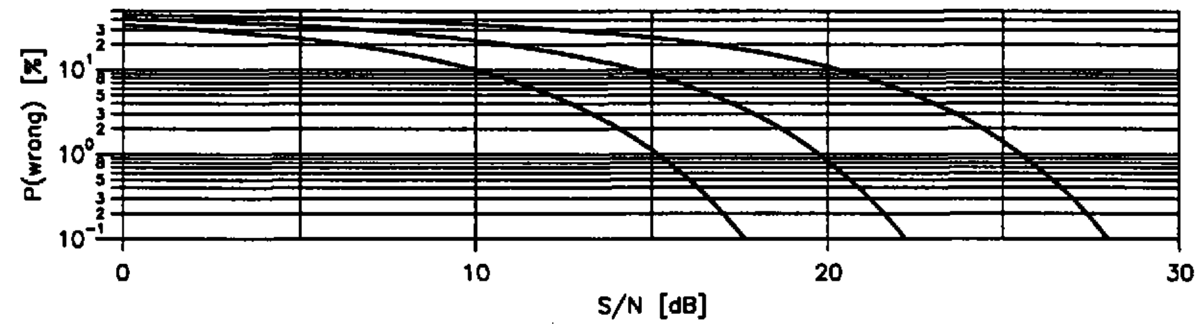

Figure 2: Probability of a wrong decision denoted in percent as a function of the signal to noise ratio, calculated for targets on broadside. Top line: array deviation 0.06 wavelength, medium line: deviation 0.12 wavelength, bottom line: deviation 0.25 wavelength. The array has 64 hydrofones with spacings of a half wavelength.

In addition to Monte-Carlo-simulations the algorithm has also been tested with experimental at-sea-data.

4. AT-SEA EXPERIMENTS

The signals used were short pulses radiated from the target at 60 sec intervalls. In both experiments presented here a 64-element array with sensor spacings of half a wavelength is used. The target is far away and the signal to noise ratio is high. The experiments should give an answer to the following questions:

- is the assumption of a circular array shape valid for useful results (both experiments),
- is it possible to reconstruct the target's right/left position over 270 degrees, when the deviation h is relatively high (first experiment),
- is it possible to reconstruct the target's right/left position on broadside, when the deviation h is relatively small (second experiment).

FIRST EXPERIMENT: The towing vessel proceeds on a 75 per cent circle, i. e. 270 degrees. The mean deviation h is around 0.08 wavelength. The effect of array snaking in this experiment is small compared with the deviation caused by the ship's maneouvre. Out of 39 measurements the target was detected on the correct side while the bearing angle changed for 270 degrees. Only 1 wrong decision occured at a bearing angle of 25 degrees (with 90 degree denoting broadside).

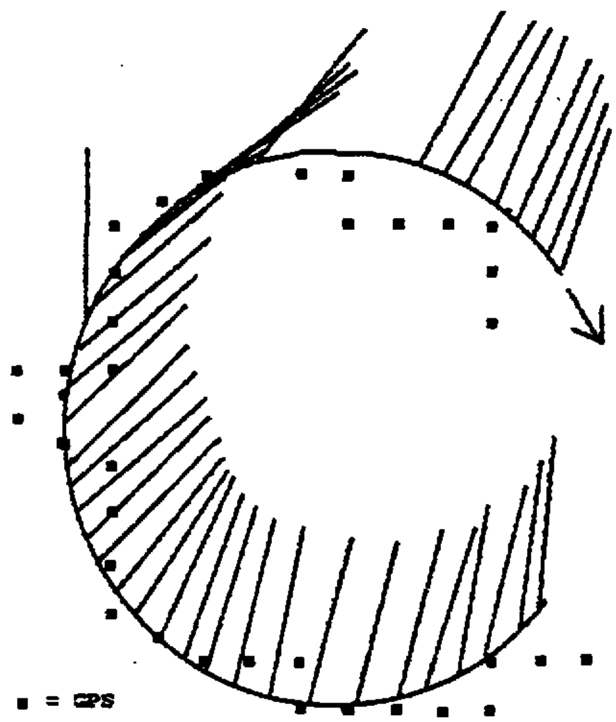

Figure 3: Outline of the first experiment. The tow vessel's course is derived from GPS-meaurements and the estimated target bearings are outlined as lines. The target's position is 18 kilometers on the top.

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SECOND EXPERIMENT: In this SONAR-experiment the towing vessel and the target went on parallel courses with the target echoes being detected in the broadside beam. At this bearing angle, the optimal results for deciding the correct side are expected, but the mean deviation h of the array is only around 0.025 wavelength. Consecutive decisions for one side can be observed, followed by consecutive decisions on the other side. This is a consequence of the array snaking, for the array curvature changes its orientation slowly from one to the other side. So the approximation of the shape as a part of a circle may work fine even for small deviations. Unfortunately, the heading sensors for determing the array orientation give no instantly reliable outputs for such small deviations. Averaging the heading measurements, however, showed similar curvature parameters as those estimated with the modified beamformer.

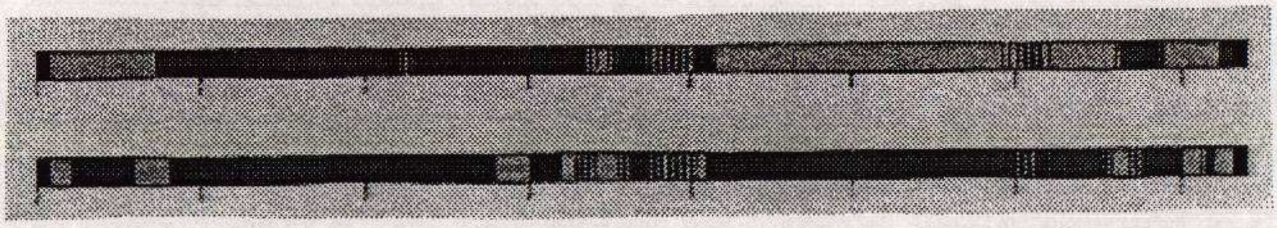

Figure 4: Top: estimated curvature from heading sensors: red: center of curvature on port, green: center on starboard. Bottom: output of detector: red: target on port, green: target on starboard. Target position: port.

RESULTS: There are too few measurements to find out, whether the predicted probabilities of a wrong decision do correspond to the simulation results. Depending on the bearing and estimated deviation, most decisions were correct. As expected, relatively strong deviations enable correct decisions even for targets far off broadside. For targets at broadside, even very slight deviations often lead to correct decisions. Comparing the measured outputs for both sides with the predicted behaviour proves, that the approximation of the array shape by a circular bending of the array is sufficient.

5. CONCLUSION

For resolving the right/left ambiguity in estimating the location of a source with towed array sonar systems, the small deviations of the array shape were exploited. Approximating the shape by a circular arc works well and is computationally attractive. Due to the only small corrections for both sides, the beamformer outputs for both sides will only slightly differ for the signal component as well as for the noise component. For the outputs differing as a function of array geometry, a calculation of the probability of a wrong decision is enabled. These investigations are described with more extent in [1]. Simulations as well as at-sea experiments proved that a precise knowledge of the radius of curvature is not necessary and that already very slight curvatures lead to reasonable results at reasonable SNR.

6. REFERENCES

[1] P. Voßen, 'Rechts-/Linksunterscheidung mit gekrümmten Antennen', FWG-TB 1994-4, Kiel 1994