ON THE PERFORMANCE OF PHASE BASED ARRAY PROCESSING SCHEMES.

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1. INTRODUCTION

Passive sonar systems present data to operators in a variety of forms. One of the most useful of which are Time-Bearing Displays (TBDs), these aim to represent the energy arriving at a sensor array from each bearing at a particular time. This allows the operator to detect, localise and track an acoustic source as it moves through the environment. Each line in a TBD represents a spatial spectrum based on a block of data of fixed length. The most common method used to compute such spatial spectra is the conventional beamformer (CBF). There are a variety of methods that can be used in place of CBF. This paper is particularly concerned with phased based schemes that have been tested and found to produce TBD that appear superior to those produced by CBF, see Figure 1 for an example.

The goal of this paper is to assess the performance of phase based TBD algorithms. This will provide quantitative performance comparisons with CBF and in addition provide (empirical) measures that can be used by the system designer to predict sonar performance.

1.1 Conventional Beamforming (CBF)

CBF has several properties that make it attractive: its performance is good (under limited circumstances), it is robust (to noise and calibration errors) and is computationally efficient. To be more precise the CBF algorithm is the basis of optimal directional of arrival (DOA) estimation schemes and of optimal detectors [1]; assuming that there is a single source present in Gaussian, spatial white, noise. These conditions are unrealistic in most passive sonar applications, but they do provide and indication of the fact that CBF can be a powerful tool.

The major shortcomings of the CBF algorithm occur in the presence of multiple targets. If two sources are present at similar bearings then the ability of the CBF algorithm to resolve them is dictated by the array length (an optimal algorithm should have a performance that is signal to noise ratio (SNR) dependent [2]). Secondly if two sources are present, and one is relatively weak, then sidelobes from the stronger source may mask the presence of the second source. This latter effect can be mitigated by the use of array shading, but the introduction of shading further reduces the ability of the array to resolve close sources and reduces the accuracy of bearing estimates.

The CBF algorithm can be expressed in a variety of forms and implemented in numerous ways. Traditionally one tends to conceive of the algorithm as forming beams, through a delay and sum beamformer, and computing the energy occurring in each beam (the energy computation maybe performed after application of a band-pass filter, to focus the processing on the frequency band of interest). Choosing the number of beams presents a compromise; the greater the number of beams the greater the computational load and the easier it is to generate accurate bearing estimates. Note to generate highly accurate DOA estimates one can use interpolation to achieve accuracy greater than that implied by simply considering the number of beams.

An alternative flexible and efficient scheme for implementing CBF is to realise it in the frequency domain. In such implementations one performs a Fast Fourier Transform (FFT) on the incoming data stream, then each of the frequency bins is processed using CBF and an output can then be reconstructed using an inverse FFT. Due to the fact that the data in each frequency bin is narrow-band, one need only implement a simpler, narrow-band, form of CBF. For a uniform line array (ULA) narrow-band CBF can be realised using a Fourier transform, wherein additional beams are formed by zero padding [3]. An additional advantage of this structure is that one has the option of employing one of the gamut of high-resolution narrow-band spatial spectral estimation algorithms, for example, Capon's method [4], MUSIC [5], linear prediction [6], in place of CBF.

1.2 Phase-Based Processing

An alternative to CBF that has gained some favour in recent years is to adopt a phase-based processing scheme. The basis of these techniques is that in a narrow-band configuration with a ULA geometry there is a simple relationship between the phases of the signals on the sensors. Specifically the phase should increase linearly as one progresses along the array, implying that pairs of adjacent sensors exhibit identical phase differences. This phase difference is related through simple geometry to the DOA. Such schemes generate a DOA estimate for each frequency bin in the FFT. The DOA estimates can then be combined in some manner to yield a distribution of DOA estimates across frequency. In the absence of a source this distribution resembles a uniform distribution, but in the presence of a broadband source a strong peak at the bearing to that source occurs.

The presence of noise will clearly disturb this idealisation causing the measured phases and phase differences to depart from their predicted forms. Averaging or regression can be used to reduce the influence of this noise. A more troublesome problem is due to phase wrapping, a consequence of the measured phase being constrained to lie in a 2π interval. This results in the measured phase containing discontinuities. In the presence of noise and/or rapid phase variations determining when phase wrapping has occurred is a non-trivial task.

There are a variety of methods that can be defined based solely on the phase measurements from an array. These methods can look for linear relationships in the raw phase data or alternatively seek to identify constant behaviour in the phase differences. Processing the phase differences has the advantage of generating simpler algorithms, but is less robust to noise. This is because the noise on the phase differences has a variance that is twice that on the individual phase measurements. All phase based methods yield a value reflecting the confidence one has in the DOA estimate. This measure is based on the regression error computed when fitting the phase data to either a linear or constant model (depending upon whether one is processing phase or phase difference data).

1.3 Circular Precision Algorithm

In order to focus our discussion on phase based processing, we shall consider one particular scheme that is typical of the phase difference based methods. It neatly avoids the need to perform phase unwrapping. Define $\delta\phi_n$ as the phase difference between the n^{th} pair of sensors then the DOA, θ. is given by

$$\overline{\delta\phi}_r = \frac{1}{L-1}\sum_{n=1}^{L-1} \cos(\delta\phi_n) \qquad \overline{\delta\phi}_i = \frac{1}{L-1}\sum_{n=1}^{L-1} \sin(\delta\phi_n)$$

$$\theta = \frac{1}{kd}\sin^{-1}(\beta) \qquad \beta = \tan^{-1}\left(\frac{\overline{\delta\phi_i}}{\overline{\delta\phi_r}}\right)$$

where k is the wavenumber, L is the number of sensors in the array and d is the sensor spacing. The quality metric, ν, for this scheme is given by

$$\nu = 1 - \sqrt{\overline{\delta\phi_r}^{\,2} + \overline{\delta\phi_i}^{\,2}}$$

For strong signals then ν takes a value close to zero, whereas if the signal is weak then ν is close to unity. This information can be used to weight data in the construction of the DOA's distribution.

a) b)

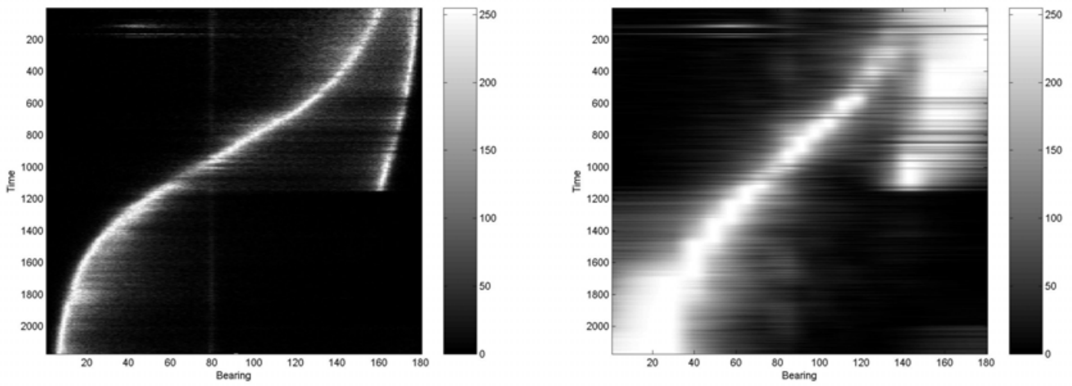


Figure 1: Example TBDs for a) Phase Based Processing and b) CBF (courtesy of QinetiQ)

Figure 1 shows a typical example of TBDs constructed using this algorithm and presents the equivalent CBF TBD. The experimental data in this case came from a 14 element line array and two sources where known to be present. One source passed from 160° to 10° during the observation interval, whilst the second source started close to end-fire (180°) and move to roughly 160° before becoming quiet. This figure illustrates the benefits of phase based processing over CBF based schemes, showing the tightness of the tracks that can be obtained, allied with the ability to track sources that are closely spaced.

2. PERFORMANCE COMPARISIONS

In order to compare the performance of the phase based method and CBF we conduct simulations of two scenarios. The first considers the bearing accuracy, i.e. accuracy with which the methods can identify the DOA of single source in noise (in this case Gaussian white noise). The second set of simulations evaluates the detection performance of the two approaches. Both sets of scenarios assess the performance of the narrow-band schemes, measuring the effectiveness of the algorithms within a single frequency bin. This separates the core processing algorithms that create information within a frequency bin, from the higher level processing that accumulate information across frequency to yield a final output "spectrum" and ultimate constructs the TBD.

2.1 Bearing Accuracy

The absolute limit on the bearing accuracy is provided by the Cramer-Rao Lower Bound (CRLB). This predicts the smallest possible mean squared error that can be achieved by any unbiased DOA estimation algorithm. For the case of a single source is Gaussian white noise the maximum likelihood estimator of bearing is the CBF processor. In addition, since maximum likelihood schemes are asymptotically efficient [7] we know that the performance of the CBF should be close to the CRLB. The CRLB for a single source is Gaussian white noise is given by

$$CRLB(\theta) \approx \frac{500\lambda^2}{d^2 L^3 \cos(\theta)^2} \frac{1}{snr}$$

where *snr* is the SNR measured on a linear scale and θ is the true source bearing. To test the performance of CBF and phase based schemes 1000 realisations of data sets were crated at various SNRs and the mean squared error (MSE) of the estimates computed. The results are shown in Figure 2 for a source at a bearing of 45°.

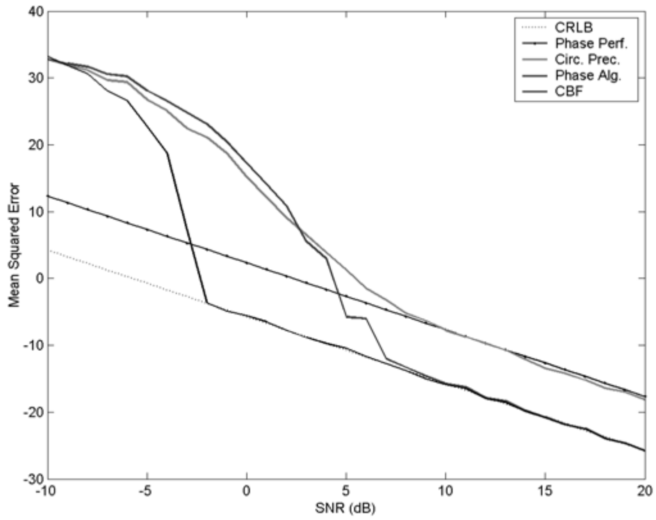


Figure 2: MSE for Several Algorithms: 16 Element ULA d=0.5λ, θ=45°

From Figure 2 one can see that the CBF does indeed achieve the CRLB for high SNRs. This performance is matched by the phase-based schemes up to an SNR of around 6 dB, whereupon the MSE of the phase based scheme exceeds the CRLB because of errors that begin to occasionally appear in the regression scheme. The CBF algorithm continues to achieve the CRLB down to a SNR of rough –2 dB. The phase-difference based algorithm (circular precision) offers poorer performance than either of the two alternative approaches because of the additional noise induced by the differencing of the phases. In Figure 2 a line is plotted that represents an empirical prediction of the performance of the circular precision algorithm obtained by analysing data across a range of simulation studies examining the effect of various parameters. This empirical prediction MSE for circular precision is

$$MSE = \frac{100\lambda^2}{d^2 L^2 \cos(\theta)^2} \frac{1}{snr}$$

Note this expression differs from the CRLB in that it is scaled by a different constant term and the power law for the filter length (*L*) is a square rather than a cube.

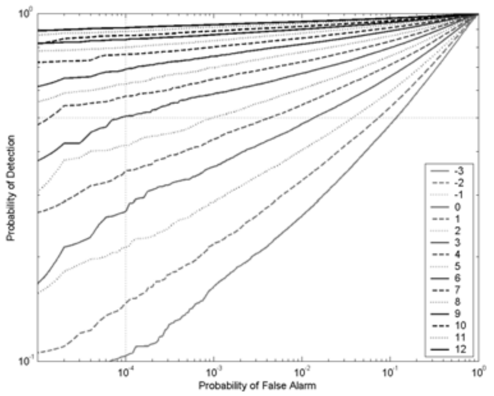
2.2 Detection

The performance of the phase-difference based schemes (circular precision) and CBF are assessed for detecting single sources in Gaussian white noise. In this case the detection statistics used are the quality metric ν for circular precision and the peak height for CBF. With appropriate scaling, the peak height of the CBF spectrum can be regarded as a measure of the size of errors in a regression analysis and so is directly analogous to ν.

Simulations are conducted using 100,000 realisations of the process and receiver operating characteristic (ROC) curves are computed. From these ROC curves the SNR at which one can achieve a 0.5 probability of detection for a false alarm rate of 10^{-4} is identified. This SNR is traditionally viewed as being the sum of two terms [8]: the detection threshold (DT), which is the SNR required by a system with no directivity to make a detection, and the directivity index (DI), which defines the performance improvement resulting from the increase in directionality obtained by using multiple sensors. Hence DI depends on the number of sensors whilst DT is a constant. The form we shall assume for DI is given below, adopting his model allows one to separate the effects of DI and DT.

$$DI = -\alpha \log_{10}(L)$$

a)



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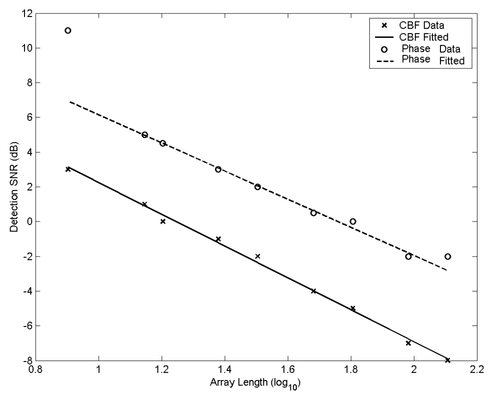

Figure 3: a) CBF ROC Curve and b) Detection SNR Computed for an 8 Element ULA

	CBF Sinusoidal Signal	Circular Precision Sinusoidal Signal	CBF Gaussian Signal	Circular Precision Gaussian Signal
Slope (α)	-9.2	-8.0	-9.2	-8.1
Intercept (DT)	9.8	12.7	11.4	14.3

Table 1: DI Values Obtained from Regression Analysis

Figure 3 shows examples of ROC curve and the detection SNRs computed for various array lengths. The results from curves of the form Figure 3b) are summarised in Table 1. Two different source types are modelled. A sinusoidal signal, which generates a constant amplitude signal in each frequency bin (with random phase), and a Gaussian signal, which is simulated in each frequency bin by drawing a complex random variable.

The phase-based schemes exhibit poorer detection performance. The DT value associated with them is roughly 3 dB less than that required by CBF and their DI reduces more slowly with array length. The phase based schemes suffer from poorer detection performance under all modelled conditions.

3. CONCLUSIONS

Phase-based methods have poorer performance than CBF both in terms of accuracy and detection. Expressions for the bearing MSE, DT and DI have been developed. The reduced performance of these methods is at odds with the enhancements observed in the TBDs. The key to understanding this dilemma is to understand that the two approaches also employ different methods for accumulating data from multiple frequency bins. The results in this paper are only concerned with performance within a single frequency bin. Consequently it is clear that the apparently enhanced TBD created phase based method is not related to the use of a phase-based DOA estimation scheme but is a result of the manner of constructing a display from the information in frequency bins.

REFERENCES

[1] Rife DC and Boorstyn RR, Multiple-Tone Parameter Estimation from Discrete Time Observations, Bell Sys. Tech. J., 1389-1410, 1976.

[2] Stoica P, Nehorai A, MUSIC, Maximum Likelihood, and Cramer-Rao Bound, IEEE Trans. on Acoustics Speech and Signal Processing, Vol. 37, 720-741, 1989.

[3] Nielson RO Sonar Signal Processing, Artech House, 1991.

[4] Capon J, High Resolution Frequency-Wavenumber Spectrum Analysis, Proc. IEEE, Vol. 57, 1408-1418, 1969.

[5] Schmidt RO, A Signal Subspace Approach to Multiple Emitter Location and Spectral Estimation, Ph.D. Thesis, Stanford University, 1982.

[6] Johnson D, The Application of Spectral Estimation Methods to Bearing Estimation Problems, Proc. IEEE, Vol. 70, No. 9, 1018-1028, 1982.

[7] Van Trees HL, Detection, Estimation and Modulation Theory Part I, John Wiley, 1968.

[8] Urick, RJ Principles of Underwater Sound (3rd Ed.), McGraw-Hill, 1983

ACKNOWLEDGEMENTS

The author would like to acknowledge the valuable technical input of Peter Glynn and Patrick Tindell, QinetiQ (Winfirth) and Jenny Johnson (DSTL Farnborough). In addition he would like to acknowledge DSTL for the financial support of DSTL.