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ADVANCED DESIGN METHODS FOR LOUDSPEAKERS USING VIBROACOUSTIC FINITE ELEMENT AND BOUNDARY ELEMENT MODELS

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1. INTRODUCTION

Design of loudspeakers is a challenging task. Many aspects must be considered, electrical, magnetic, mechanical vibration and acoustics. Some loudspeakers are based on models which have evolved over many decades and been extensively tested, without the use of analytical techniques. However the consumer market is very competitive and new designs have to evolve more rapidly nowadays. Analytical methods offer a tool which can be used to assist in the design process and which can potentially permit the consideration of very radical changes using non-standard components which would be expensive to construct on a one-off basis, thus accelerating the design cycle. They also have a role to play in helping the engineer to comprehend the phenomena and hence make more efficient design changes.

The finite element method is a numerical technique which can be applied to many fields of engineering, including statics, dynamics, magnetic and acoustics. It has been around since the 1950's where it was initially used in the aerospace industry. The increasing power and decreasing cost of computer hardware together with the refinement of numerical techniques and the evolution of commercial software based on these techniques has encouraged the spread of finite elements (FE) into many diverse applications. The boundary element method is a more recent technique. Boundary elements (BE) are partly competitive with FE and partly complimentary. It is available in some commercial computer codes and its use is also increasing.

In this paper the FE and BE methods are described and their application to loudspeaker design is considered. Attention is mainly confined to acoustics with a little consideration of structural vibration.

2. ACOUSTIC EQUATIONS

For small amplitude oscillations in an inviscid, irrotational, compressible fluid, with no mean flow, the pressure distribution satisfies the wave equation

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \qquad (1)$$

where c is the acoustic wavespeed. For steady state oscillations at circular frequency ω, this reduces to the Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \qquad (2)$$

where $k = \frac{\omega}{c}$ is the acoustic wavenumber. On the interface between the fluid and some structure there is continuity of normal velocity

$$\frac{\partial p}{\partial n} = \omega^2 \rho u_n \tag{3}$$

and pressure with negative normal stress

$$p = -\sigma_{nn} \tag{4}$$

where n is the normal direction, ρ is the density, u is the displacement and σ the stress. If there is a point source or incident plane wave in the fluid domain, then the total pressure field p can be decomposed as

$$p = p_I + p_S \tag{5}$$

where p_I is the incident free field pressure and p_s is the scattered pressure. For external problems the scattered pressure field must satisfy the Sommerfeld radiation condition

$$\lim_{r\to\infty} r\left|\frac{\partial p_s}{\partial r} - ikp_s\right| = 0 \tag{6}$$

which ensures that it consists only of outgoing waves.

3. ACOUSTIC FINITE ELEMENTS

The FE method requires the volume of the region modelled to be subdivided into elements. Within each element the pressure is interpolated from the nodal reference points, which define its shape, using polynomial functions. In simple terms, the wave equation can be solved exactly, subject to these constraints on the pressure distribution. In a rigorous manner, the equations can be derived using either variational or Galerkin weighted residual methods. The general transient form of the equations is

$$[M_a]\{\ddot{p}\} + [A_a]\{\dot{p}\} + [S_a]\{p\} = \int_\Gamma [N]^T \ddot{u}_n d\Gamma \tag{7}$$

where {p} is a vector of the nodal pressures, u_n is the normal displacement on the boundary Γ. $[S_a]$, $[A_a]$ and $[M_a]$ are sparse, square symmetric matrices and [N] is a row vector of interpolation functions. A more complete theoretical description is given by Petyt [1].

If steady state response at circular frequency ω is to be considered then the equation becomes

$$\left([S_a] + i\omega[A_a] - \omega^2[M_a]\right)\{p\} = -\omega^2 \int_\Gamma [N]^T u_n d\Gamma \tag{8}$$

The matrix $[A_a]$ arises from surface absorbency. If volume absorbency is to be modelled, as in a foam, then the matrices $[S_a]$ and $[M_a]$ become complex.

The natural frequency calculation for a cavity can be done by solving the eigenvalue problem

$$\left([S_a]-\omega^2[M_a]\right)\{p\}=\{0\} \tag{9}$$

4. WAVE ENVELOPE ELEMENTS

Acoustic FE are not the best method for solving exterior, infinite domain, problems. It would be possible to model up to a truncation surface with acoustic FE and then apply some impedance boundary condition, e.g.

$$p=\rho cV \tag{10}$$

which would be an exact non-reflective condition for a plane wave impinging normally on the surface. To ensure that this is satisfied adequately would require modelling to a large distance from the structure, but this would then be computationally expensive. Furthermore at low frequency large elements would be most efficient, whereas at high frequency small elements would be required.

Wave envelope elements are a more accurate technique. They explicitly model a zone extending to infinity and assume that the wave travels normally across the boundary and that it is composed of a linear combination of poles of various orders coming from a "source". The distance of the "source" can vary along the boundary. This technique requires the (infinite) boundaries to follow the ray paths of the acoustic problem but can be used much nearer to the structure than a simpler impedance condition. The equations for these elements are of a similar form to (7) and they can be merged into a FE solution procedure. A more formal description is given by Cremers [2].

5. ACOUSTIC BOUNDARY ELEMENTS

The boundary element method requires only the surface of the region to be modelled with boundary element patches. Within each patch the pressure and its normal gradient are interpolated using polynomials. The surface Helmholtz formula can be used, together with collocation at the surface nodal points to derive a set of equations of the form

$$[H]\{p\}=[G]\left\{\frac{\partial p}{\partial n}\right\}+\{p_I\} \tag{11}$$

where [H] and [G] are complex dense matrices and {p} and $\left\{\frac{\partial p}{\partial n}\right\}$ are vectors of pressures and pressure normal gradients at the surface mesh points. The surface Helmholtz formulation, described above, works well apart from at a set of characteristic frequencies, which become dense in the higher frequency range. A number of improved boundary element methods are available, which overcome these difficulties, e.g. CHIEF, due to Schenck [3].

The acoustic BE method does not require the direction of wave propagation to be normal to the boundary and it is thus possible to use this technique right on the surface of a vibrating structure.

6. COUPLING TO STRUCTURAL MODELS

An acoustic FE or BE mesh can be coupled to a structural FE mesh. A fully coupled analysis can be performed, satisfying equations (3) and (4) on the fluid-structure interface and solving simultaneously for both displacements on the structure and pressures in the fluid. The usual FE equations become modified to

$$\left([S]+i\omega[C]-\omega^2[M]\right)\{u\}+[T]^T\{p\}=\{F\} \tag{12}$$

where [T] is a coupling matrix such that $-[T]^T\{p\}$ gives the loading on the structure due to the pressure distribution in the fluid. Evaluating the right hand side of equation (8) using the structural normal velocities results in

$$\left([S_a]+i\omega[A_a]-\omega^2[M_a]\right)\{p\}=\omega^2[T]\{u\}=\{0\} \tag{13}$$

Coupling between structural FE and acoustic BE requires the construction of a matrix [E] such that $[E]^T\{u\}$ gives the normal displacements at the degrees of freedom on the BE mesh. Equation (11) becomes

$$[H]\{p\}=\omega^2\rho[G][E]^T\{u\}+\{p_I\} \tag{14}$$

It is possible to couple acoustic FE directly to acoustic BE, as described by Macey [4], which is advantageous in some circumstances.

For structures vibrating in light fluids the pressure loading on the structure is often assumed to be insignificant, thus permitting an uncoupled solution. The structural vibration calculation is performed without including the fluid loading, and then the acoustic analysis is performed, using the structural surface motion to compute the right hand side of equation (8) or (14). This is computationally more efficient. However in loudspeaker analysis the cone is very light also and hence it is probably advisable to perform fully coupled analyses.

7. COMPARISON OF METHODS

Acoustic analysis techniques are still evolving and being refined. Thus the best method for solving a particular problem may not still be so in a few years time. It is certainly easier to create a surface mesh than a volume mesh, which gives BE some advantage, but then automatic 3D meshing schemes are another area of current research. FE are definitely better if the fluid medium is has inhomogeneous properties. BE do not deal as well with natural frequency calculations. For steady state exterior problems BE is more robust than FE combined with wave envelope elements, but less efficient. However a technique for interpolating acoustic BE matrices between frequencies can be used to speed up BE analysis, at the cost of a slight loss in accuracy. For interior problems either FE or BE could be used; the author's opinion is that FE has the edge. The relative computational advantage of FE will increase with the ratio of boundary to volume, as the geometry of the problem changes.

8. APPLICATION TO THE LOUDSPEAKER INDUSTRY

A commonly used performance indicator for a loudspeaker box is the pressure against frequency response at some point on axis in front of it. Ideally this should be fairly flat for a constant electrical input against frequency. However there are many phenomena which affect the final result. The cone may be vibrating rigidly if it is stiff or at low frequency. Under other conditions bending may occur, causing a different radiation efficiency. The loading on the cone due to the pressure field inside the box will generally be small compared with the forces driving the system, but under some conditions there may be an effect at the cavity resonance of the box. The loading of the air in front of the cone may have an added mass effect at low frequencies, particularly if the cone is light. The sound received at the measurement point will be affected by diffraction from the edges of the speaker box and reflections off the walls, ceiling, floor and furniture. If the designer does not understand why a particular system is producing its response then new models can only evolve at a slow evolutionary pace. To acquire this comprehension it is useful to isolate different phenomena. This can be done experimentally, e.g. by using an anechoic chamber to remove reflection effects, measuring cone vibration in air at low pressure to study the added mass contribution etc. ... However these will be time consuming. Alternatively each of these phenomena can be studied individually using FE/BE models.

The natural frequencies and mode shapes of the voice coil/cone/surround can be determined using structural eigenvalue analysis. Results of this type are given by Fackrell [5]. The natural frequencies of the interior cavity can be determined by eigenvalue calculation on an acoustic FE mesh, e.g. Fackrell [5]. The effects of added mass of the external air will be incorporated in a fully coupled acoustic analysis, which will also include the radiation damping associated with the external air. This has been done by modelling the external fluid as a combination of acoustic FE and BE as in Fackrell [5], Wright [6] and Bank [7]. These three studies consider the case of radiation into an infinite half space, i.e. ignoring diffraction/reflection effects. An example of this type using wave envelope elements is contained in this paper. Radiation into a full 3D space can be treated by an acoustic BE coupled directly to a structural FE model of the cone/dust cover/box. The placement of speakers in a room has been considered by Wright [8] where a low frequency representation of the speaker as a point source was used in an acoustic BE model of the room. In principle it would be possible to have an analytical model of a speaker within a computational model of a room. This would be computationally expensive on today's processors, but should become a possibility in the future.

The drive unit is axisymmetric, thus it is often sufficient to perform a computationally cheaper axisymmetric analysis, where FE/BE methods require only a mesh of the generator plane to be created. The main difficulties are determining the correct material properties for cone, surround and foams and the modelling of ports where fluid flow and/or viscosity effects may need to be taken into account. Another area which needs to be addressed is that steady state FE/BE analysis assumes that materials are operating in the linear region whereas in practice a loudspeaker may be driven in the non-linear region, producing distortion, which is not taken into account in the analytical models.

9. EXAMPLE RESULTS

The example of a concave piston mounted in an infinite rigid baffle studied by Wright [6] has been analysed more recently using the wave envelope element method. The air inside the cone and extending out up to a hemispherical surface are modelled using acoustic FE. The rest of the infinite half space is modelled with wave envelope elements. The mesh is shown in figure 1. The computed impedance is normalised by ρcA where A is the projected area of the cone. In figure 2 the graphs of resistance and reactance against normalised frequency ka, where a is the radius of the piston, are in close agreement with the results published in [6]. The current results computed with the wave envelope method took much less time than those of [6] which used the BE method.

10. POSSIBLE FUTURE TRENDS

In the future computer hardware will be more powerful and software will use more sophisticated and robust numerical techniques and be more user friendly. Application specific semi-intelligent programs will be much more common, e.g. prompting the designer for the dimensions of the box, the position of the drive unit, the materials, the frequency range,... and automatically creating, running and displaying results from an appropriate numerical model. Possibly future systems will have optimisation functionality, so that some of the redesign process can be more automatic. Some research in this field has already been done by Geaves [9] where shape optimisation of a cone is considered. Perhaps future design systems for loudspeakers will integrate the calculation of the magnetic field with the vibroacoustic analysis. This would permit computation of the effect that changing components in the electrical matching network will have on the sound field. Thus it seems certain that analytic methods will play a greater role in loudspeaker design in the coming years.

11. ACKNOWLEDGEMENTS

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Figure 1

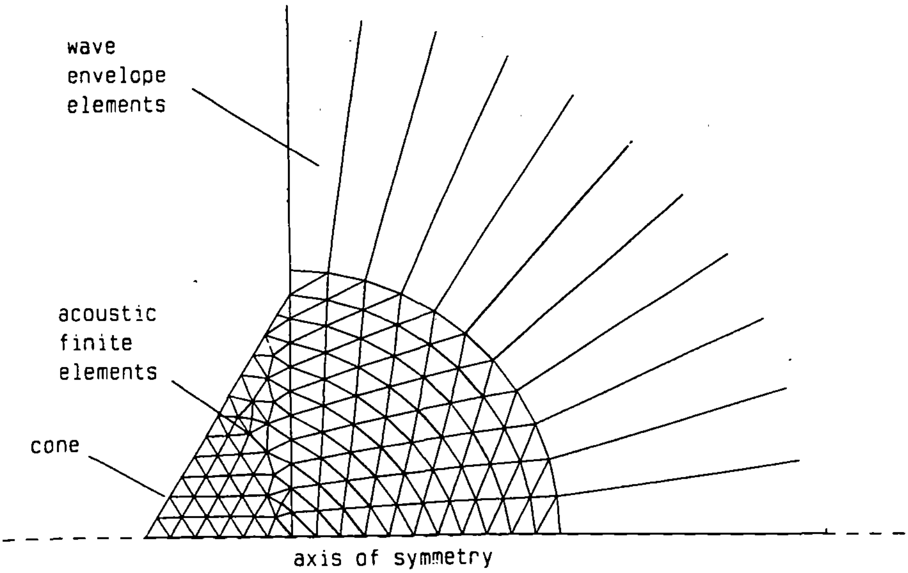

Figure 2