ANALYSIS OF LINEAR ARRAYS OF RING TRANSDUCERS USING WAVE ENVELOPE ELEMENTS

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1. INTRODUCTION

Piezoelectric transducers have many applications in underwater acoustics. In active mode a device is driven electrically by an alternating voltage applied across the electrodes. The piezoelectric coupling effect causes mechanical vibration. The surface motion in contact with the water causes an acoustic field to be radiated. The pressure distribution in the surrounding fluid will generally apply significant loading to the system, due to the high density of water. In passive mode the reverse processes causes acoustic energy to be converted to mechanical and then electrical energy, permitting the detection of an incoming wave. Transducers are often used in arrays, increasing power and improving directivity. There is then a coupling though the fluid between transducers, particularly when the spacing between elements is less than half an acoustic wavelength.

Even the ring transducer, perhaps the geometrically most simple design, has no analytical solution describing its operation. Numerical techniques are needed to predict performance before production. The finite element method, as proposed by Allik and Hughes[1], can be used to model the electromechanical behaviour of the transducer. The boundary element method (BE), as described by Schenck[2], can be used to model the surrounding fluid. This requires only a surface mesh and automatically satisfies the Sommerfeld radiation condition. Alternatively acoustic finite elements (FE) can be used in the immediate neighbourhood of the structure combined with either a series solution for the far field, also proposed by Hunt et al[3], or with wave envelope elements (WEE), described by Cremers and Fyfe[4].

WEE are potentially faster but not as robust as BE. This paper considers the accuracy of various modelling techniques using WEE for modelling linear arrays of transducers, which are long and thin in a structural sense. A safe but possibly inefficient method is to model to a spherical surface with acoustic FE. A more efficient but risky method is to follow more closely the natural geometry of the system.

2. ACOUSTIC ANALYSIS TECHNIQUES

The BE method requires only the boundary of the domain, ie, the wet surface of the structure, to be modelled. A small but dense set of equations is produced. The time to set up the BE matrices is of the form:

$$T_1 = C_1 b^2 \tag{1}$$

where C_1 is a constant and b is the number of nodes on the boundary element. The time to solve the dense set of equations is

$$T_2 = C_2 b^3 \tag{2}$$

ANALYSIS OF ARRAYS OF RING TRANSDUCERS

There is also computational time to eliminate the structural freedoms. The surface mesh required will usually need to be constructed of surface patches with side lengths proportional to the acoustic wavelength. Thus for a 2D or axisymmetric model, b increases linearly with frequency and for a 3D model, it increases with the square of frequency. Thus analysis using BE increases rapidly in computational requirements with frequency.

The acoustic FE method can be derived using standard Galerkin weighted residual methods. The volume of the region being modelled is broken down into a mesh of elements over each of which the pressure is interpolated with simple polynomial functions. The set of equations generated is large but sparse. The time to solve the equations using a direct, frontal solution approach, is of the form:

$$T_3 = C_3 w^2 n \tag{3}$$

where w is the wavefront and n is the total number of freedoms.

Infinite elements are similar, except that they model a region extending to infinity. The shape functions are modified to include radial decay and oscillation. The element matrices, which are now complex, can be merged in with acoustic FE. The integration to compute the infinite element matrices is difficult due to the oscillatory part of the shape functions. This problem is overcome in the WEE technique where the weighting function is the complex conjugate of the shape function. The equations produced are still sparse, but not now symmetric. Further details are given in ref [4].

Forming the WEE matrices is very quick. Solving the equation is again governed by (3). Thus very efficient solutions are possible if the wavefront can be kept low, which would be achieved if the acoustic FE mesh can follow the 'natural geometry' for a linear array of transducers. It is not clear whether this will always be possible because there may be near field irregularities in the pressure field which are not adequately modelled by the assumed interpolation functions of the WEE. It can be shown however that the pressure field outside a spherical surface containing all the sources and vibrating structure is sufficiently simple to be modelled well by WEE. However modelling with acoustic FE up to a sphere will lead to a relatively high wavefront. It is one of the objectives of the current study to determine whether it is possible to work with more efficient meshes which follow the natural geometry of long thin structures. It is likely that the accuracy of results can be improved in such cases by adjusting the WEE such that the infinite edges point in the direction of the acoustic particle velocities determined from acoustic FE mesh results in an initial analysis. Unfortunately, it has not been possible to investigate this corrective method yet.

A generalisation of the infinite element method described by Burnett[5] has already been shown to be efficient at analysing long thin structures.

3. TRANSDUCER ARRAY DESCRIPTION

An array of four coaxial ring transducers was used to compare the analysis methods. Each ring was constructed from radially polarized PZT4, with an inner radius of 0.0508m, an outer radius of 0.05715m and an axial length of 0.028m. The axial spacing between rings was 0.052m. Arrays of these transducers, but with different spacing, have been analysed previously by Gallaher[6] using the boundary element method. The actual transducers are often coated with a 0.005m layer of polybutadiene, a viscoelastic polymer, to prevent a short circuit through the water. This is almost acoustically transparent, but does support shear waves and also introduce significant damping. This layer can be modelled explicitly with finite elements and given frequency dependent properties, as described by Macey[7]. For the purposes of the current work, investigating the modelling of the fluid, the simplified model of [6], including the effect of the coating as structural damping within the piezoelectric ring, was adopted. Hysteretic damping with $\tan\delta=0.13$ was used. The properties assumed for water were $\rho = 1000\text{kgm}^{-3}$ and $C = 1500\text{ms}^{-1}$.

4. MODELS USED FOR ANALYSIS

The results shown in this paper were produced using the PAFEC VibroAcoustics program. All models used to analyse the array were axisymmetric. Within the scope of the current work, it was only possible to consider the case where all transducers were driven in phase with identical voltage excitation. It was thus possible to exploit the plane of symmetry and model only two transducers explicitly. Both piezoelectric and acoustic finite elements used were quadratic and isoparametric. The boundary element used was composed of 3 noded line patches, with a quadratic variation of both pressure and normal velocity. The wave envelope elements used had a quadratic tangential variation and were 6th or 8th order in the radial direction.

Figure 1 illustrates a simple wave envelope element mesh following the natural geometry of the system. Figure 2 shows another wave envelope element mesh where the acoustic finite element region has been extended out to a spherical surface of radius 0.15m. These will be henceforth referred to as mesh 1 and mesh 2 respectively. To test the accuracy of results from these two wave envelope element models a boundary element analysis, using the mesh of figure 3, was used as the benchmark. This used acoustic finite elements immediately surrounding the structure, coupled to a boundary element for the remainder of the exterior region, using the method proposed by Macey[8]. This removed any source of error due to the boundary element method being inaccurate at modelling thin regions, which would have been a possible concern if the boundary element had been coupled directly to the piezoelectric finite elements.

5. RESULTS

Figures 4 and 5 compare the radial and axial pressure sensitivities of mesh 1 with the benchmark solution. Figures 6 and 7 give a similar comparison for mesh 2. Both meshes predict the radial sensitivity well; mesh 2 is extremely accurate. The axial sensitivity is poorly predicted by mesh 1. There is a definite improvement with mesh 2, but still errors of several dB at some frequencies. Figure 8 shows the axial sensitivity for mesh 2 when the radial order is increased to 8. The results are more accurate, but have become slightly erratic. Other users of wave envelope elements [4] have also found that very high order radial variation causes the system equations to become ill-conditioned. Increasing the radial order to 10 for the current problem was found to make the results even more erratic.

6. CONCLUSIONS

The more prudent strategy of modelling up to a spherical surface with acoustic finite elements (mesh 2) seems worthwhile if a wide range of results is required. For the case considered the faster method was sufficient for predicting radial sensitivites. It may be possible to improve on the more economical method (mesh 1) by computing pressure gradients in an initial solution and using this to adjust the infinite edges of the wave envelope elements for a subsequent resolution. This idea has not been tried yet.

7. REFERENCES

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Fig 1

FINITE/WAVE ENVELOPE ELEMENT
MESH NUMBER 1

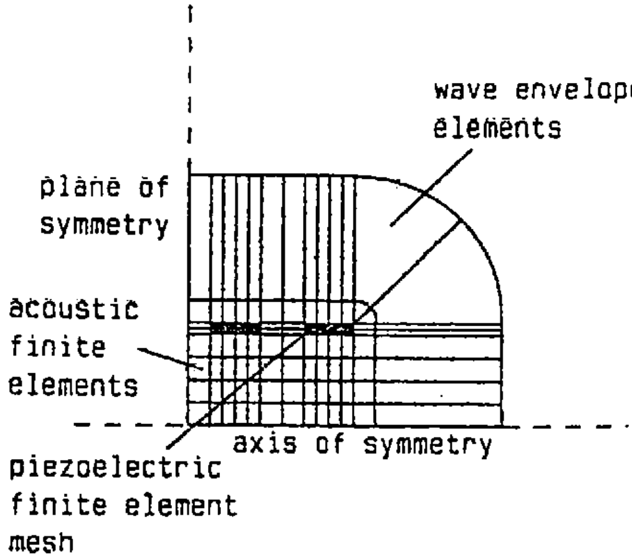
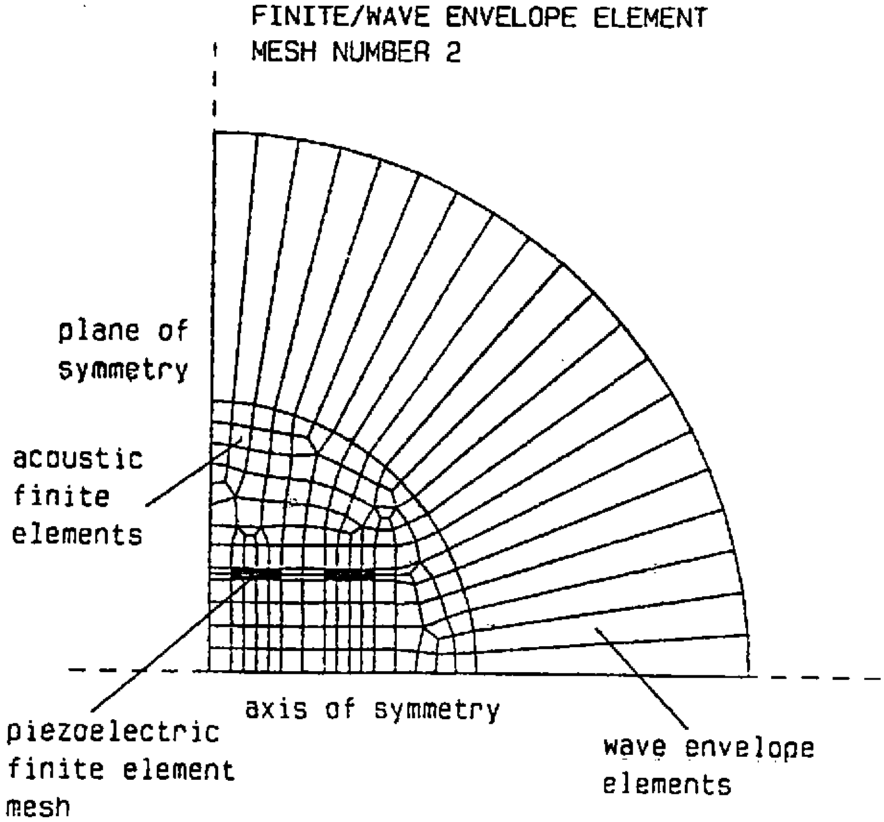


Fig 2

FINITE/WAVE ENVELOPE ELEMENT
MESH NUMBER 2



ANALYSIS OF ARRAYS OF RING TRANSDUCERS

Fig 3

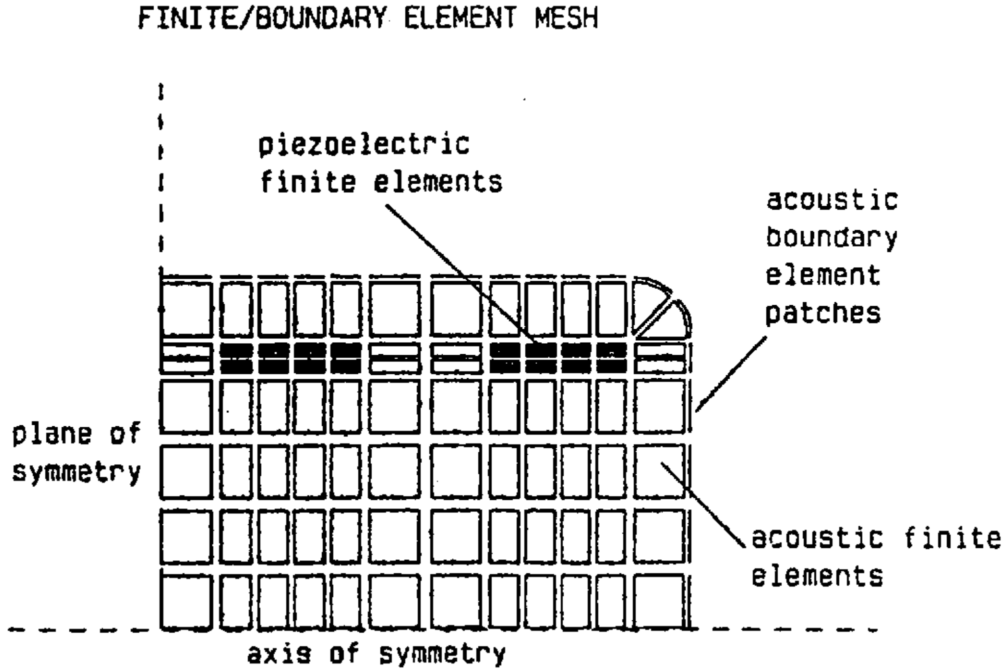


Fig 4

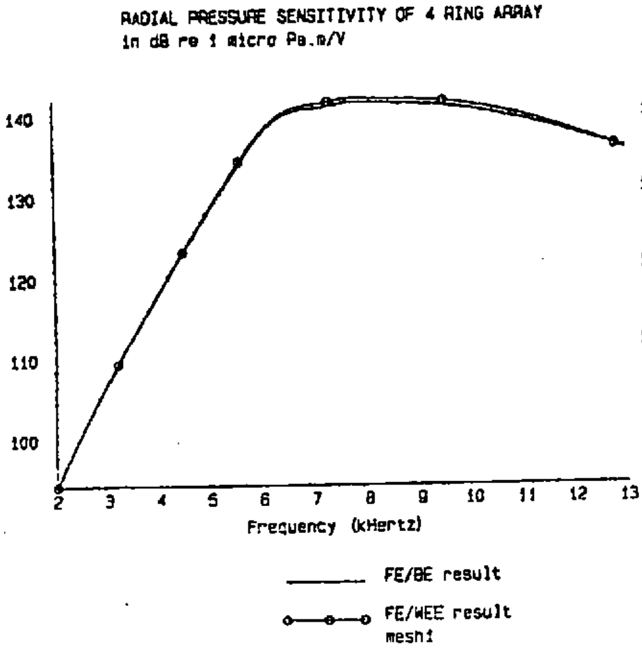
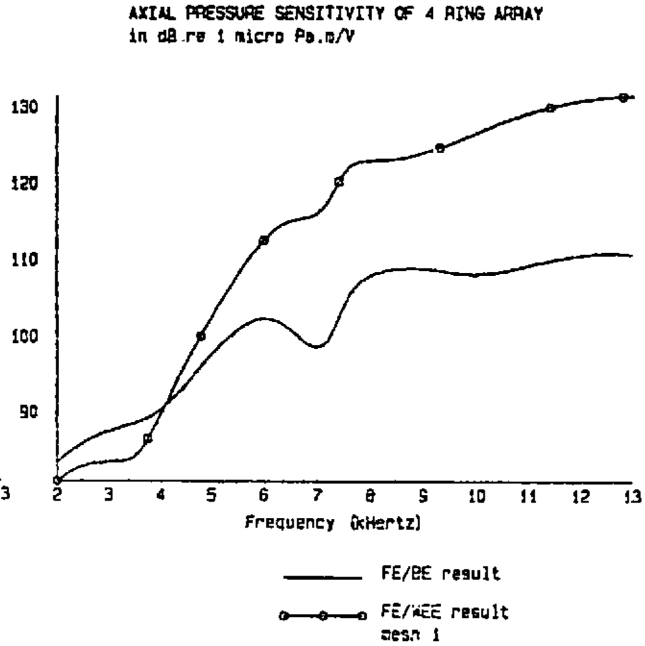


Fig 5



ANALYSIS OF ARRAYS OF RING TRANSDUCERS

Fig 6

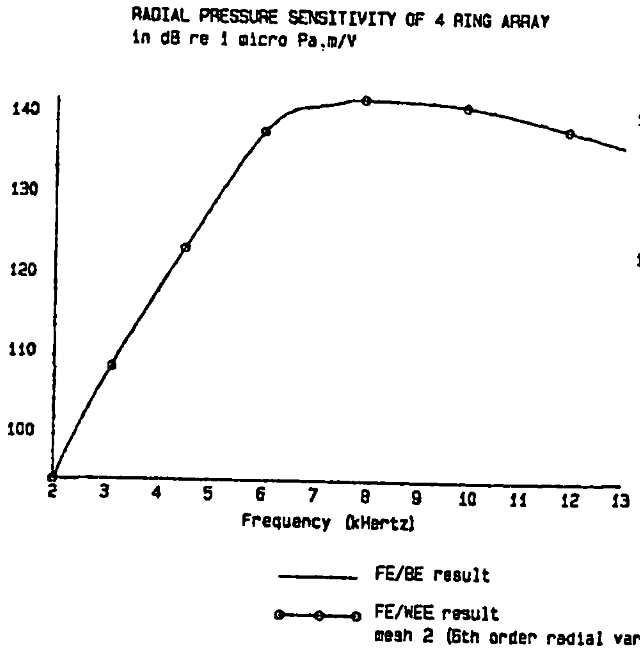


Fig 7

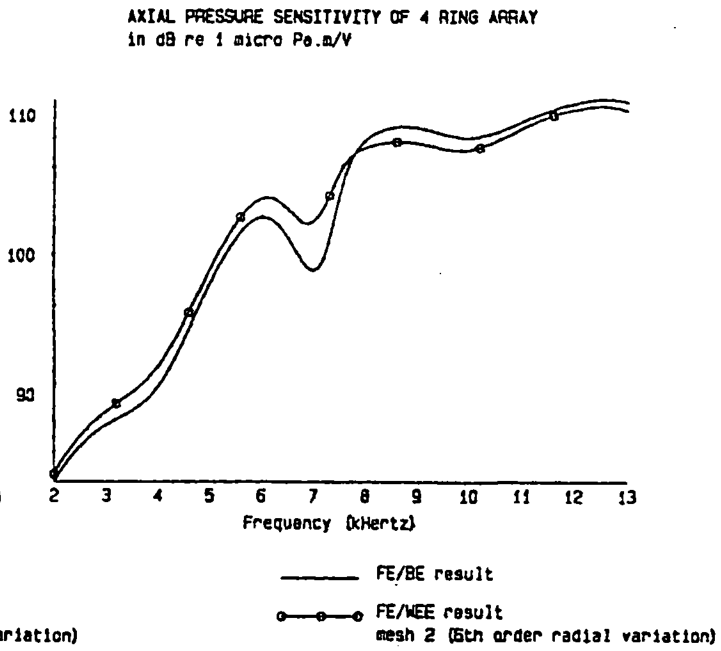


Fig 8