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ACOUSTIC ANALYSIS OF AXIALLY PERIODIC STRUCTURES

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1. INTRODUCTION

Many structures occurring in underwater acoustics are constructed with an axially repeating section. Examples of this are a periodically rib-stiffened cylindrical hull or a line array of transducers. Such a system is often excited, either by an incident plane wave or directly on the structure, in a manner with constant phase difference between periodic units. If the number of repeating sections is large and also many acoustic wavelengths in extent, then it is often assumed that the system is infinite in extent. Bloch's theorem can then be used to state periodic relations for displacements and pressures. In this paper a finite element based method for analysing such problems is proposed. A combined structural/acoustic FE mesh is constructed for a cylindrical region of fluid containing one periodic unit. Periodic constraints are applied between the ends. The curved surface of the acoustic FE faces is coupled to a series solution.

2. PRESSURE EXPANSION IN FLUID

Consider an axially periodic structure immersed in an acoustic medium. Assume that the x direction is axial, and let the periodic length be 2d. If the system is vibrating in a steady state, excited by harmonic loading at circular frequency ω, which has a constant phase difference $-2qd$ between neighbouring periodic units, then from Bloch's theorem it follows that all response quantities (displacement, pressure, force, ...) must satisfy the same periodic relationship:

$$F(x+2d,r,\theta)=F(x,r,\theta)e^{-2iqd} \qquad (1)$$

For the case of an incident plane wave, travelling in direction $\underline{n}$ this relation is $q=kn_x$, where k is the wavenumber.

By expanding $F(x,r\theta)e^{iqx}$ in Fourier series it follows that:

$$F(x,r,\theta)=e^{-iqx}\sum_{s=-\infty}^{+\infty}\overline{F}_s(r,\theta)e^{i\frac{\pi x}{d}} \tag{2}$$

$$\overline{F}_s(r,\theta)=\frac{1}{2d}\int_{-d}^{+d}e^{-ik_s x}F(x,r,\theta)dx, k_s=\frac{\pi s}{d}-q \tag{3}$$

In the acoustic medium the scattered component of the pressure field satisfies the Helmholtz equation. Using separation of variable techniques in a cylindrical axis set, assuming a $\cos(m\theta)$ circumferential variation and an $e^{-ik_s x}$ axial dependency results in solutions of the form:

$$p(x,r,t)=f_m(r;\gamma_s^{\,2})e^{-ik_s x}\cos(m\theta)e^{j\omega t} \tag{4}$$

where

$$f_m(r;\gamma_s^2)=H_m^{(2)}(\beta_s r)\text{ or }K_m(\gamma_s r)\text{ and }\gamma_s^2=k_s^2-k^2=-\beta_s^2 \tag{5}$$

3. COUPLING TO FINITE ELEMENTS

Assume that the pressure field in the region $\{(x,r,\theta):-d\le x\le +d, r\ge a\}$ is modelled as a superposition of a finite number of terms of the type in equation (5). The system in the finite cylindrical region can be modelled with a mesh of acoustic/structural FE.

Assume that a is taken large enough such that the interface S consists of acoustic FE faces. The discretized equations are of the form:

$$\begin{bmatrix}[S]-\omega^2[M] & [T]^T\\ [T] & \frac{1}{\omega^2}[S_a]-[M_a]\end{bmatrix}\begin{Bmatrix}\{u\}\\ \{p\}\end{Bmatrix}=\begin{Bmatrix}\{F\}\\ \{F_a\}\end{Bmatrix} \tag{6}$$

$\{F\}$ is a vector of directly applied structural forces and $\{F_a\}$ are 'fluid forces', due to the surface vibration on S. $\{F_a\}$ can be split into incident and scattered contributions. The latter can be evaluated using the radial derivative of the Fourier sum for the pressure field, whose coefficients can be evaluated by the integrating over the pressure field on S. $\{F_a\}$ can then be expressed in the form:

$$\{F_a\}=\{F_{p_i}\}+[\overline{A}]^T[D][A]\{P\} \tag{7}$$

Before solution, the periodic constraints of equation (1) must be imposed.

The derivation of the above equations is similar to the doubly periodic case, which is covered in more detail in Ref [1].

4. ANALYSIS RESULTS

The method described above has been implemented in the PAFEC VibroAcoustics code. The code has been tested on two problems, an infinite line array of point sources, and the sound field generated by an infinite linear array of coaxial ring transducers. In all the calculations the properties of water were assumed to be density = $1000kgm^{-3}$ and speed of sound = $1500ms^{-1}$.

The nth point source is assumed to lie at (n,o,o). The phase of the nth source was taken to be 0.296192xn. Results were computed using the mesh in figure 1. Table 1 gives a comparison with a series method using 2000001 terms, for the pressure at (0, 0.5, 0).

Table 1

Frequency	Pressure (series)	Pressure (FE)
100	4.14317 - 3.12439i	4.14330 - 3.12437i
200	2.07772 - 3.02216i	2.07786 - 3.02215i

The ring projectors were taken to have inner radius 0.0508m, outer radius 0.05715m, axial length 0.028m and to be made from PZT4, polarized radially. The axial separation was taken as 0.052m. The analysis was performed using the mesh of figure 2. The conductance graph computed is shown in figure 3.

5. CONCLUSIONS - FURTHER WORK

The method has been shown to work well for an infinite array of point sources. Finite arrays of the above ring projectors have been analysed using the BE method [2]. However these results are not directly comparable with the present work. Further work will be performed to compute results for large, finite arrays and compare with the infinite results.

References

[1] A. Hladky-Hennion and J. Decarpigny
'Analysis of the scattering of a plane acoustic wave by a doubly periodic structure using the finite element method: Application to Alberich Anerchoic Coatings'. J. Acoust. Soc. Am. 3356 (1991)

[2] A. B. Gallaher
'Performance prediction of an array of free flooding ring transducers'
Proc I.O.A. Vol 17, part 3 p34

Fig 1

MESH USED TO ANALYSE
INFINITE ARRAY OF POINT SOURCES

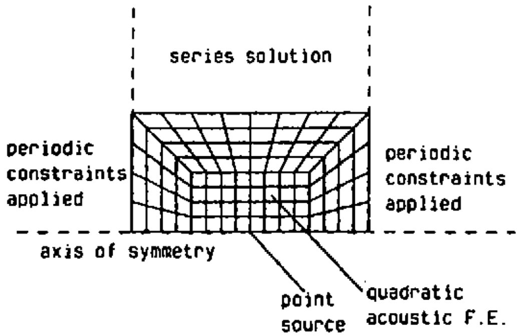

Fig 2

MESH USED TO ANALYSE COAXIAL
ARRAY OF RING TRANSDUCERS

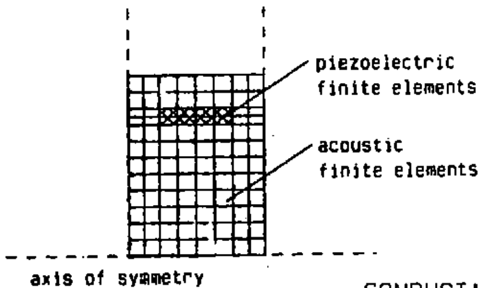

Fig 3

CONDUCTANCE OF INFINITE
ARRAY OF RING TRANSDUCERS