FINITE ELEMENT ANALYSIS OF SURFACE SCATTERING

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1. INTRODUCTION

The scattering properties of rough surfaces can be investigated by studying the idealised situation of scattering by a sinusoidal surface. These types of problem have been traditionally investigated by series solutions methods which are usually accurate only for h/d << 1, where h is the amplitude of the spatial oscillation and 2d is the wavelength, and possibly only for small angles of incidence. In this paper some results from a series solution method due to Heaps [1] are compared to a hybrid finite element/series solution. The FE based solution does not suffer from either limitation. The FE solution can be used to analyse scattering by more general periodic surfaces. Comparison is made with scattering by a periodic sawtooth and square wave pressure release surfaces.

2. SEPARATION OF VARIABLES

Consider the situation in figure 1, where a plane monochromatic wave is incident on a periodic surface.

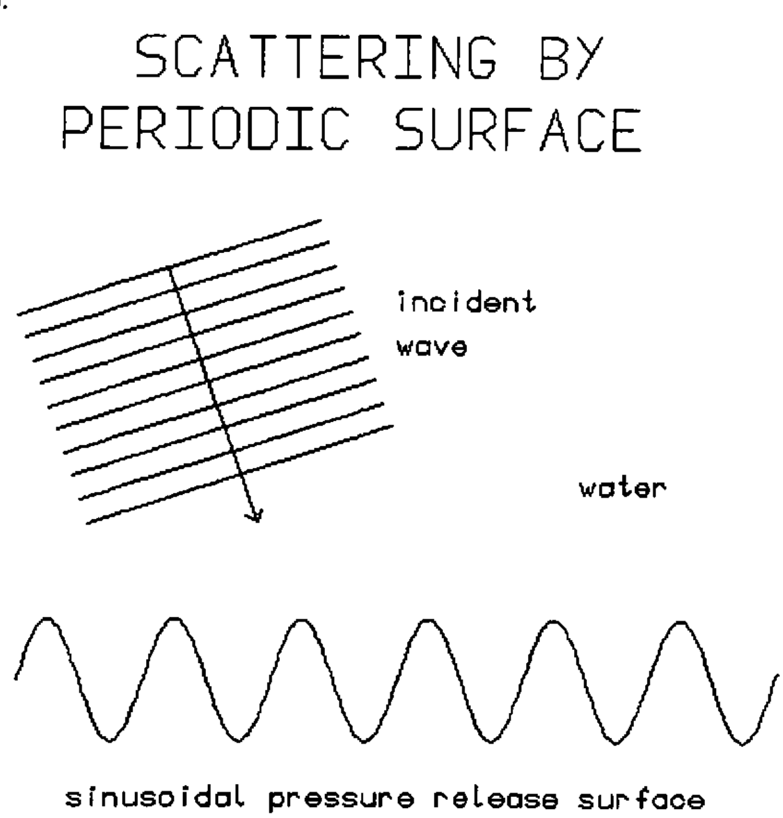

Figure 1

Let $\underline{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ be the direction of propagation of the incident wave. Assuming an $e^{i\omega t}$ time variation then the incident wave will have the form

$$P_I(x,y,t) = P_I\, e^{i(\omega t - k\,\underline{n}.\underline{x})} \tag{1}$$

From Bloch's theorem the pressure must satisfy the condition

$$P(x+2d,y) = P(x,y)e^{-2in_1dk} \tag{2}$$

Hence it follows that $P(x,y)e^{ikn_1x}$ is periodic with period 2d and can be expanded in a Fourier series, resulting in

$$P(x,y) = e^{-ikn_1x}\sum_{r=-\infty}^{r=\infty}\overline{P}(y)e^{i\frac{r\pi x}{d}} \tag{3}$$

Each term in the above sum must satisfy the Helmholtz formula

$$\nabla^2 P + k^2 P = 0 \tag{4}$$

Hence $\overline{P}(y)$ is determined and equation (3) becomes

$$P(x,y) = P_I + \sum_{r=-\infty}^{+\infty} R_r e^{ik_r y} e^{i\alpha_r x} \tag{5}$$

where

$$k_r^{\,2} = k^2 - \alpha_r^{\,2} \quad , \quad \alpha_r = \frac{r\pi}{d} - k\,n_1 \tag{6}$$

If $k_r^{\,2} \geq 0$ then the negative square root must be taken to ensure a reflected wave. If $k_r^{\,2} < 0$ then the square root must be taken to have a positive imaginary part, and the term is an evanescent wave, which decays exponentially in the y direction, and is a travelling wave in a plane of constant y.

The series solution due to Heaps [1] can be used to determine the reflection coefficients R_r. His derivations for the reflection coefficients are given via a recurrence relation. Expressions are in the form of power series exansion of a small parameter, $\varepsilon = kh$, and are therefore valid at low frequencies. Heaps ignores terms of fourth order and higher. Heaps' theory is also limited to cases where the profile amplitude is small compared with the spatial wavelength.

3. HYBRID FE/SERIES SOLUTION

For the hybrid finite element/series solution, a method similar to Hladky-Hennion et al [2] is followed. A length 2d of surface is modelled with pressure based acoustic elements up to a plane of constant y, as in figure 2.

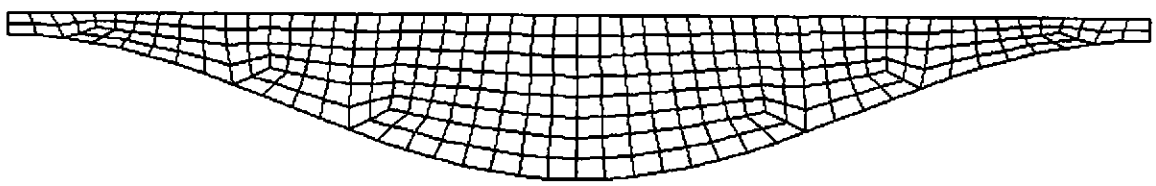

Figure 2

The Dirichlet boundary condition is appled to the lower surface of the mesh by restricting the trial functions in the usual way. The condition of equation (2) is applied as a constraint linking corresponding nodes on the left and right hand sides of the mesh. The acoustic F.E. equations take the form, as in ref [3]

$$\left([S_a]-\omega^2[M_a]\right)\{P\}=\omega^2\int_\Gamma [N]^T u_y \, d\Gamma \qquad (7)$$

where u_y is the normal displacement on the upper surface Γ. The pressure in the region above Γ is represented by a truncation of the series solution in equation (5), from r=-N to r=+N.

$$P(x,y)=P_I+\sum_{r=-N}^{r=+N} R_r e^{ik_r y} e^{i\alpha_r x} \qquad (8)$$

The reflection coefficients can be computed as

$$R_r e^{ik_r y}=\frac{1}{2d}\int_\Gamma (P(x,y)-P_I)e^{-ik_r x} \, d\Gamma \qquad (9)$$

By differentiating equation (8) an expression for u_y can be obtained.

$$\omega^2 \rho u_y=\frac{\partial P_I}{\partial y}+\sum_{r=-N}^{r=+N} ik_r R_r e^{-ik_r y} e^{i\alpha_r x} \qquad (10)$$

Substituting from equations (8), (9) and (10) into (7) a set of equations for the hybrid FE/series solution is obtained. This can be solved to determine {P} the nodal pressures on the FE mesh. Equation (9) can then be used to determine the reflection coefficients.

The method described in this section has been included in the PAFEC VibroAcoustics finite element program.

4. COMPARISON OF SERIES/HYBRID SOLUTION RESULTS

The fluid for the test problem was taken to be water, with properties wavespeed=1500 ms^{-1} and density=1000 kgm^{-3}. The profile had h=0.5m and d=4m. The finite element mesh in figure 2, with 280 elements and 856 degrees of freedom was used, taking N=8 in equation (8). Two separate sets of calculations are presented in tables 1 and 2. The first has a fixed angle of incidence (10 degrees off normal) and varying the frequency range from 100 Hertz to 400 Hertz. The second takes a fixed frequency of 200 Hertz and varies the angle of incidence from 0 degrees, normal incidence, to 80 degrees. Only travelling wave componants are shown; a null entry corresponds to an evanescent wave.

Freq	$\|A_0\|$	$\|A_0^*\|$	$\|A_{-1}\|$	$\|A_{-1}^*\|$	$\|A_{+1}\|$	$\|A_{+1}^*\|$	$\|A_{-2}\|$	$\|A_{-2}^*\|$
100 Hz	1.0021	1.0000						
150 Hz	1.0024	1.0002						
200 Hz	0.9443	0.9453	0.3976	0.4041				
250 Hz	0.8354	0.8431	0.4626	0.4902	0.4499	0.4826		
300 Hz	0.7051	0.7214	0.5146	0.5669	0.5075	0.5566		
350 Hz	0.5602	0.6012	0.5698	0.5991	0.5400	0.6217	0.2466	0.2276
400 Hz	0.3958	0.4722	0.6115	0.6152	0.5805	0.6758	0.3302	0.2903

Table 1 : reflection coefficients computed using Heaps series solution and PAFEC VibroAcoustics (*) for a fixed angle of incidence of 10 degrees

θ	$\|A_0\|$	$\|A_0^*\|$	$\|A_{-1}\|$	$\|A_{-1}^*\|$	$\|A_{-2}\|$	$\|A_{-2}^*\|$
0	0.9389	0.9424	0.4032	0.4009		
10	0.9443	0.9453	0.3946	0.4041		
20	0.9322	0.9354	0.3664	0.3828		
30	0.9256	0.9343	0.3267	0.3498		
40	0.9224	0.9368	0.2782	0.3136		
50	0.9215	0.9453	0.2232	0.2638		
60	0.9219	0.9578	0.1630	0.2036		
70	0.9225	0.9717	0.1084	0.1371	0.0300	0.0284
80	0.9237	0.9856	0.0544	0.0697	0.0152	0.0145

Table 2 : reflection coefficients computed using Heaps series solution and PAFEC VibroAcoustics (*) for a fixed frequency of 200 Hertz

For the fixed angle of incidence the agreement between the two solutions is within a fraction of 1% up to 200 Hertz. In this region Heaps' solution should be accurate, although an amplitude greater than unity is predicted. At 400 Hertz the results are about 16% different. At this frequency the convergence of Heaps series solution is in doubt with the smallness parameter ε^4 being about 0.5. The finite element mesh should still be accurate here as the element size is a small fraction of a wavelength.

In table 2 there is generally good agreement. At low angles of incidence the discrepancy is less than 1%. The onset of the second order wave is well reproduced. However there is some disparity in the magnitude of the specular amplitude $|A_0|$ particularly at high angles of incidence. The Heaps' prediction decreases monotonically between 10 degrees and 50 degrees, whereas the FE result rises after 30 degrees getting close to unity at 80 degrees. Given that Heaps' formulae are approximate and the FE mesh is very fine it is expected that the FE results are more accurate.

5. SCATTERING BY DIFFERENT SURFACES

The scattering effects of different periodic pressure release surfaces was investigated using the hybrid FE/series solution. Sawtooth and a square wave periodic surfaces with the same root mean square deviation from the mean level were considered. A comparison of the specular reflection coefficients over the frequency range 100 Hertz to 800 Hertz is given in figure 3. The angle of incidence was taken to be 10 degrees in all cases.

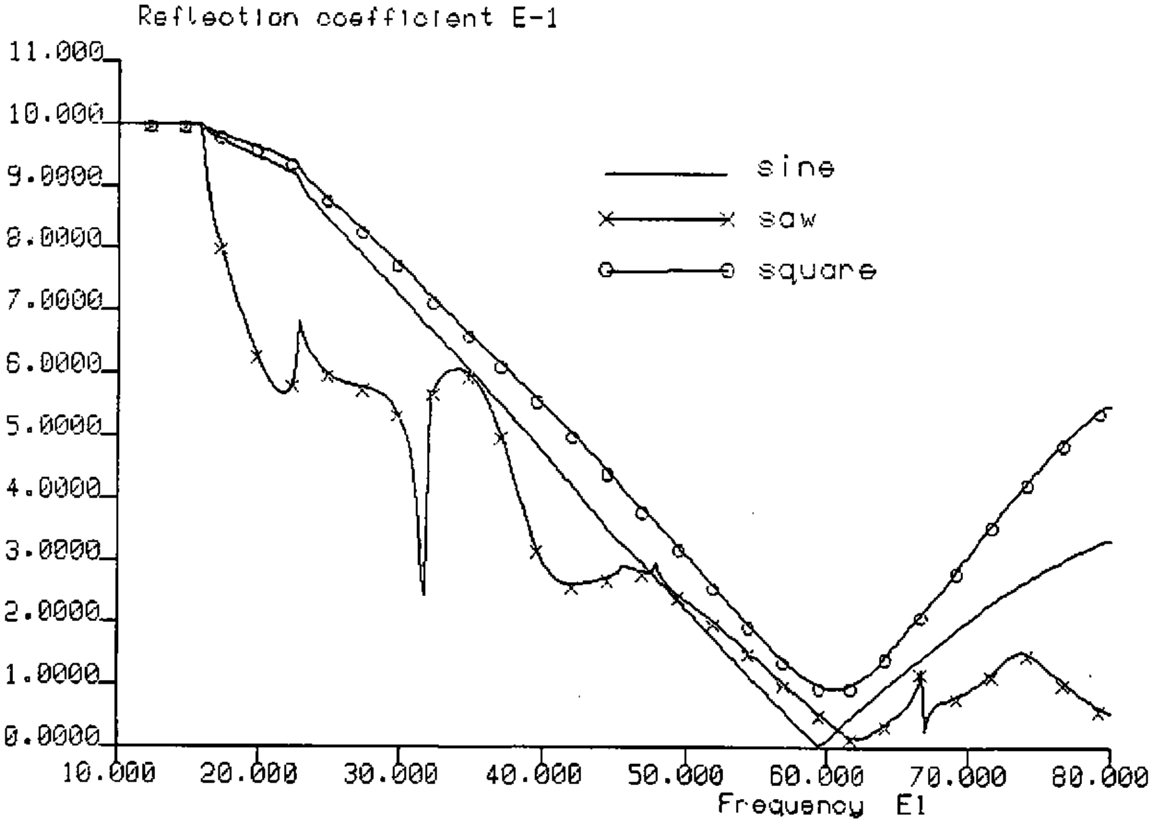

Figure 3

The sine wave and square surfaces are very similar, showing the same generally smooth variation with frequency. The reflection coefficients gradually diverge with increasing frequency, as higher order terms become more important. The sawtooth case does follow the same broad trends, but is very much more irratic and generally lower than the sine and square wave cases. This latter factor must result in greater values on average for the other travelling wave reflection coefficients. This is perhaps because half the surface is angled to reflect directly back in a direction close to one of the travelling wave directions of equation (5), which are of course independent of the profile of the scattering surface. The extent to which a 'direct reflection' is close to travelling wave direction will clearly be frequency dependent. This may perhaps explain the 'peaky' behaviour of the sawtooth specular reflection coefficient. This matter needs further investigation.

6. CONCLUSIONS

A combined finite element/series soution has been shown to be suitable for investigating the reflective properties of periodic surfaces. It should be valid for arbitrary values of h/d and any angle of incidence. The method has been tested on periodic pressure release surfaces. However it can easily be extended to consider more general surface impedance conditions. Whilst the FE/series solution can never truly simulate a realistic sea bottom surface it could be used to provide validation of more general, albeit approximate, theories of sea bed scattering.

7. REFERENCES

[1] Heaps H.S. "Non-specular Reflection of Sound from a Sinusoidal Surface"
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