FAST ACOUSTIC ANALYSIS FOR ROOMS OF CONSTANT HEIGHT

PC Macey PACSYS Limited, Strelley Hall, Nottingham, UK

1 INTRODUCTION

For rooms of simple shape, acoustic analysis can use closed form solutions. This is common for cuboid-shaped rooms, but could be applied more generally to rooms bounded by surfaces of constant coordinate, for a coordinate system in which the wave equation is separable. For irregularly shaped rooms the finite element method (FEM)[1] or the boundary element method (BEM)[1] are applicable at low frequency. These techniques have the merit of solving the wave equation "exactly", in the sense that as the mesh is refined, the solution will converge to the true solution, if exact arithmetic is used. Thus diffractive effects, which are important at low frequency, will be included, as this phenomenon is predicted by the wave equation. However as frequency increases the required element size decreases, resulting in a rapid increase in memory, CPU time and storage requirements. Ray tracing, an alternative simulation technique, does not solve the wave equation exactly, and is not good at modeling diffraction, and hence not applicable for low frequency. However it is useful for high frequency where diffraction is negligible. There is clear benefit in extending the frequency range over which FEM can be applied. This paper considers analysis of rooms of constant height using hybrid part FE / part analytical approaches. Firstly a modal approach is considered, then a direct solution of the cross section equations obtained from separation of variables is discussed.

2 MODES OF A ROOM OF CONSTANT HEIGHT

Using the separability of the wave equation in Cartesian coordinates, it is easily shown that modes of a room of constant height h can be expressed in terms of the cross section modes as

$$\Phi_{nm} = \sqrt{\frac{\varepsilon_m}{h}}\varphi_n(x,y)\cos\left(\frac{m\pi z}{h}\right) \tag{1}$$

where $\varphi_1, \varphi_2, \varphi_3, \ldots$ are the cross section modes, orthonormal with respect to integration over the area of the cross section, and

$$\varepsilon_n = \begin{cases} 1 \text{ if } & n = 0 \\ 2 \text{ if } & n > 0 \end{cases} \tag{2}$$

The associated frequencies are

$$\omega_{nm} = \sqrt{\omega_n{}^2 + \frac{m^2\pi^2c^2}{h^2}} \tag{3}$$

where $\omega_1, \omega_2, \omega_3, ...$ are the radian frequencies of the cross section modes. n refers to the cross section mode, and m the number of half wavelengths through the height. The scaling in equation (1), ensures that the 3D modes are orthonormal with respect to integration over the volume.

Computing the 2D cross section modes is a significantly smaller computational task than computing the 3D modes. Hence the modal computation of (1) and (3) can be used for a fast analysis of a 3D room of constant height, as described in the next section.

3 MODAL COMPUTATION FOR SOURCE EXCITATION

3.1 Steady State Sinusoidal

In a cavity with rigid boundaries, and a single ideal point source at $\underline{x_s}$, the pressure distribution satisfies

$$\nabla^2 p + k^2 p = -\delta\left(\underline{x} - \underline{x_s}\right) \tag{4}$$

within the domain, and

$$\frac{\partial p}{\partial n} = 0 \tag{5}$$

at the boundaries. Using the divergence theorem and equation (4), it can be shown that the pressure field can be represented as a sum of modal contributions as

$$p\left(\underline{x}, \underline{x_s}, \omega\right) = \sum_{n,m} \alpha_{nm} \Phi_{nm}\left(\underline{x_s}\right) \tag{6}$$

where the modal contribution factors are given by

$$\alpha_{nm} = \frac{c^2}{\omega_{nm}{}^2 - \omega^2} \Phi_{nm}\left(\underline{x_s}\right) \tag{7}$$

In equation (4), a single source is assumed, but this is trivially generalized to include multiple sources.

This approach, based on modes with the Neumann condition, includes no damping. Modal damping is easily included, assuming that the (m,n)th mode decays as $e^{-\gamma_{nm}\omega_{nm}t}$, where γ_{nm} is the proportion of critical damping.

$$\alpha_{nm} = \frac{c^2}{\omega_{nm}{}^2 + 2i\gamma_{nm}\omega_{nm}\omega - \omega^2} \Phi_{nm}\left(\underline{x_s}\right) \tag{8}$$

This assumption is not “close to the physics”, because in practice the damping occurs at the boundaries rather than an overall modal decay. The approach described above can be generalized to include this by means impedance conditions on the walls which are included in a complex eigenvalue calculation for the cross section modes, but even then this does not permit a general variation of impedance as a function of frequency.

3.2 Modal Transient

In the transient case, the wave equation, with source included is

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -S(\underline{x},t) \tag{9}$$

where $S(\underline{x},t)$ is a source density term. In the equations below a single point source is considered

$$S(\underline{x},t) = \delta(\underline{x} - \underline{x_s}) f(t) \tag{10}$$

but the extension to a distribution of sources is straightforward. As in the sinusoidal case, an arbitrary pressure distribution, satisfying the boundary conditions can be represented as a sum of modal contributions as

$$p(\underline{x}) = \sum_{n,m} \alpha_{nm}(t)\Phi_{nm}(\underline{x}) \tag{11}$$

Substituting this into the wave equation, multiplying by Φ_{nm} , integrating over the volume and using the orthonormal properties results in a differential equation for the modal contribution factor α_{nm}.

$$\ddot{\alpha}_{nm} + {\omega_{nm}}^2 \alpha_{nm} = c^2 \Phi_{nm}(\underline{x_s}) f(t) \tag{12}$$

In the results below, the source term will be assumed to be a pulse of duration τ of the form

$$f(t) = \begin{cases} \dfrac{12\pi t^2(\tau - t)^2(\tau - 2t)}{(\tau/2)^6} & \text{for t} \le \tau \\ 0 & \text{for } t > \tau \end{cases} \tag{13}$$

Standard techniques can be used to solve the differential equation (12), resulting in

$$\alpha_{nm} = \begin{cases} A_{nm}\cos(\omega_{nm}t) + B_{nm}\sin(\omega_{nm}t) + g_{nm}(t) & \text{for } t \le \tau \\ A'_{nm}\cos(\omega_{nm}t) + B'_{nm}\sin(\omega_{\text{nm}}t) & \text{for } t > \tau \end{cases} \tag{14}$$

The particular integral, $g_{nm}(t)$, is a 5th degree polynomial. The trigonometric terms are the compilementary function and the constants are chosen to satisfy the initial conditions and continuity at time τ .

4 MODIFIED CROSS SECTION EQUATIONS

4.1 General Remarks

An alternative approach, which can have different boundary conditions included more easily, is to decompose the pressure field by a Fourier series through the height,

$$p(x, y, z, t \text{ or } \omega) = \sum_{m=0}^{\infty} p_m(x, y, t \text{ or } \omega)\cos\left(\frac{m\pi z}{h}\right) \tag{15}$$

which can be done because of the rigid condition at z=0 and z=h.

The coefficients are time or frequency dependent cross section distributions. They satisfy a modified form of the 2-dimensional wave equation. This approach is not as rapid as the modal approaches described above, but nevertheless orders of magnitude faster than a full 3-dimensional analysis. Furthermore this approach can be quite naturally parallelized.

4.2 Sinusoidal Case

Substituting from equation (14) into the Helmholtz equation (4), and using the orthogonality of the Fourier terms, results in

$$\nabla_{xy}{}^2 p_m + \left(k^2 - \frac{m^2\pi^2}{h^2}\right)p_m = -\frac{\varepsilon_m}{h}\cos\left(\frac{m\pi z_s}{h}\right)\delta(x - x_s, y - y_s) \tag{16}$$

Thus a 2D FE solution can be used to solve for the cross section distribution $p_m(x, y, \omega)$. Absorbent regions, such as porous media at the walls can be included by having complex acoustic properties for part of the domain[2]. Note that when

$$m > \frac{2fh}{c} \tag{17}$$

The effective wave number for the cross section in the air region is imaginary, giving rise to an evanescent wave field, decaying exponentially away from the source point. As m increases further, the exponential decay becomes more rapid. Hence the convergence of the series in equation (15) would be expected to become more rapid as the separation between the source and receiver increases.

4.3 Transient Case

The source distribution can be decomposed into Fourier components

$$S(\underline{x}, t) = \sum_{n=0}^{\infty} S_n(x, y, t)\cos\left(\frac{n\pi z}{h}\right) \tag{18}$$

After substituting (14) and (17) into (8) a modified wave equation can be derived

$$\nabla_{xy}^2 p_n(x, y, t) - \frac{n^2\pi^2}{h^2} p_n(x, y, t) - \frac{1}{c^2}\frac{\partial^2 p_n(x, y, t)}{\partial t^2} = -S_n(x, y, t) \tag{19}$$

Thus it is possible to solve for the $p_n(x, y, t)$ distributions using 2D FEM and combine to obtain the full 3D pressure distribution using equation (15).

5 RESULTS

5.1 Description of Room

Results were computed for a room of height 2.94m and cross section as shown in figure 1.

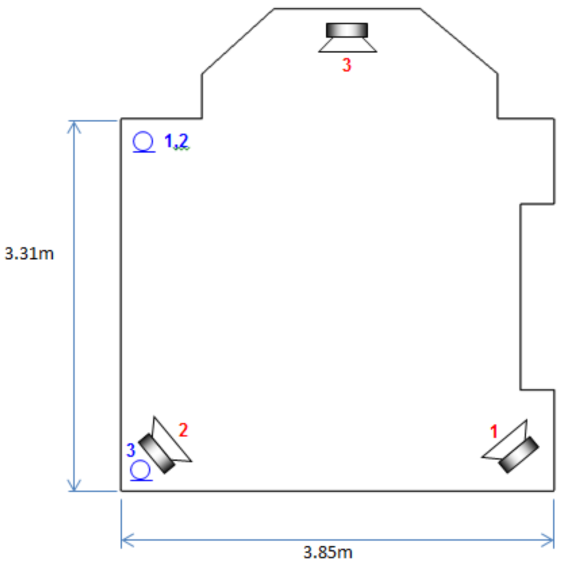

Figure 1, cross section of room

The speed of sound in air was assumed as 340 ms^{-1}.

The cross section mesh used for the section mode calculation had 313 quadratic elements and 918 degrees of freedom. 40 modes were computed. Figure 2 shows some of the modes.

Three pairs of source / receiver positions were considered, in the sinusoidal results of section 5.2. In each case the ideal sources were assumed to be 0.2m from walls and floor and the receivers to be 0.2m from walls and ceiling.

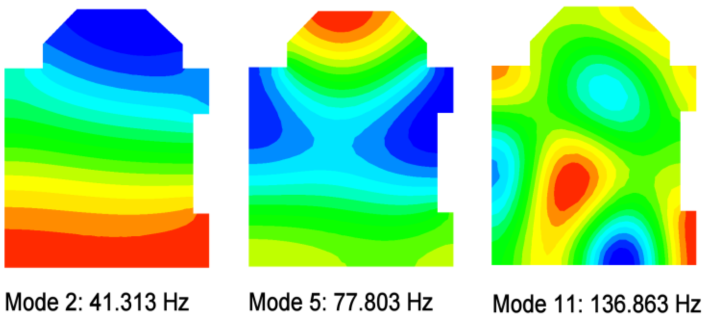

Figure 2, some cross section modes

5.2 Sinusoidal Results

Two 3D models were used to compute results for comparison, a BEM model with 1276 quadratic patches and 4192 degrees of freedom, shown in figure 3, and a FEM model with twice the surface mesh density, consisting of 23232 element and 98317 degrees of freedom.

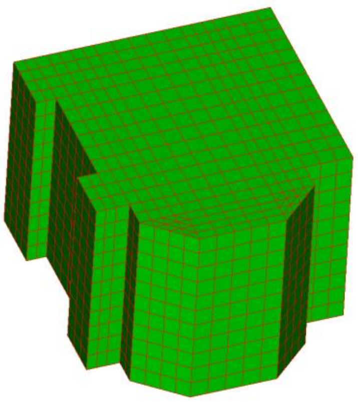

Figure 3 , BEM comparison model

Figures 4, 5 and 6 compare the results from the hybrid modal method with the 3D models. Agreement is good at low frequency, but small discrepancies arise around resonance / antiresonances in the upper frequency range.

The hybrid 2.5D approach was several orders of magnitude faster.

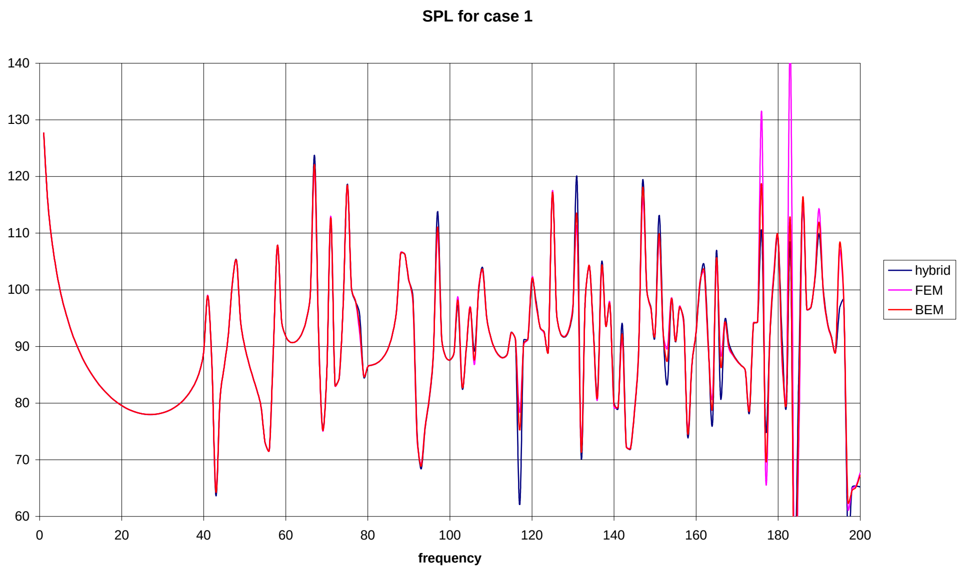

Figure 4, comparison case 1

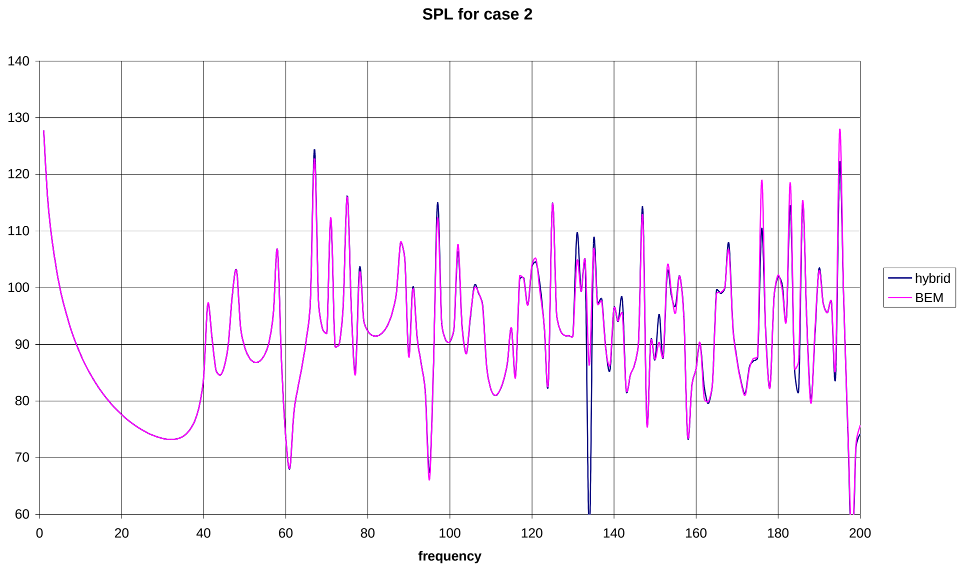

Figure 5, comparison case 2

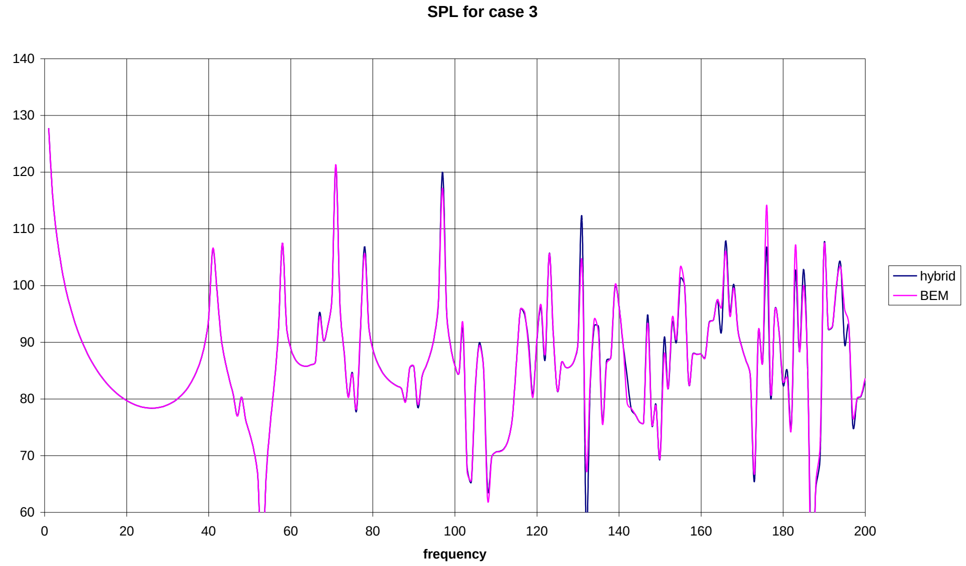

Figure 6, comparison case 3

5.3 Transient Results

A transient acoustic analysis was performed for the room. The source and receiver were as in figure 7. The pulse duration was 0.006 seconds. The receiver point was taken at ceiling height in the corner. Comparison is made with a 3D finite element model, using a direct time marching scheme. The finite element model, shown in figure 8 comprises of 21252 quadratic elements and 90275 nodes. 2.5D results, using both Fourier and modal approaches, were computed using the cross section mesh in figure 9, comprising of 5901 linear triangle and 3067 nodes, and taking a maximum of 30 half wavelengths through the height. The three sets of results are shown in figure 10. The agreement is good, particularly at early times. The two 2.5D approaches agree more closely with each other than with the 3D model. This is to be expected since they use the same mesh.

The 2.5D approaches were several orders of magnitude faster than the 3D computation.

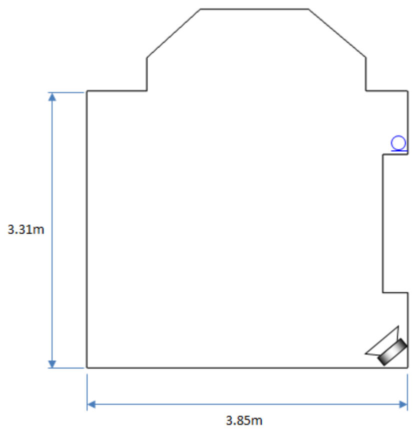

Figure 7 source and receiver (transient case)

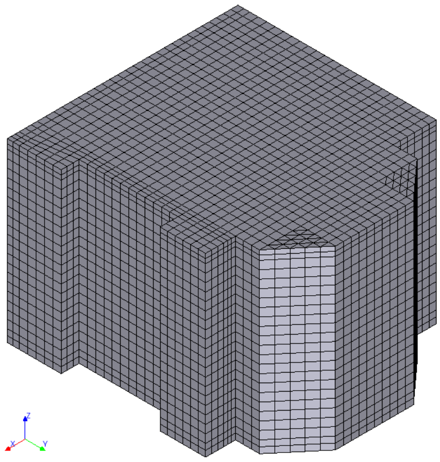

Figure 8, 3D model used for transient analysis

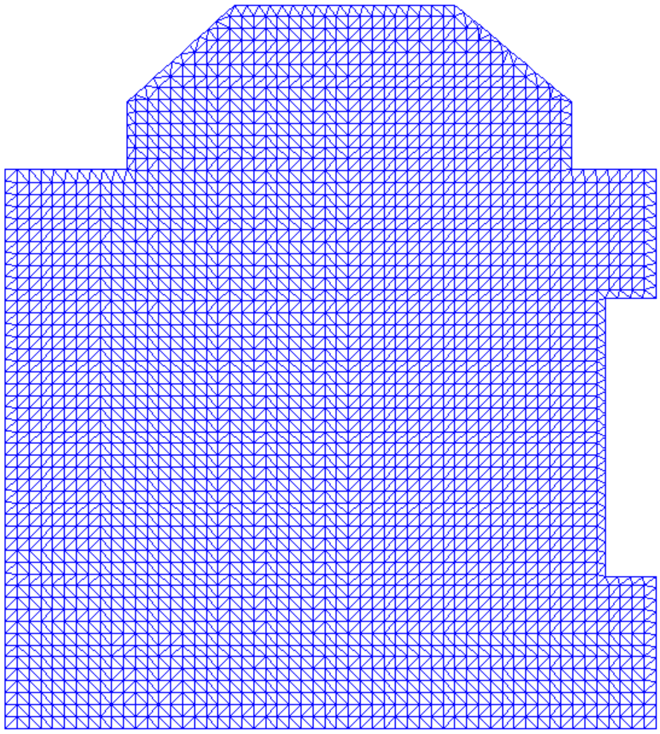

Figure 9, cross section mesh used for tyransient analysis

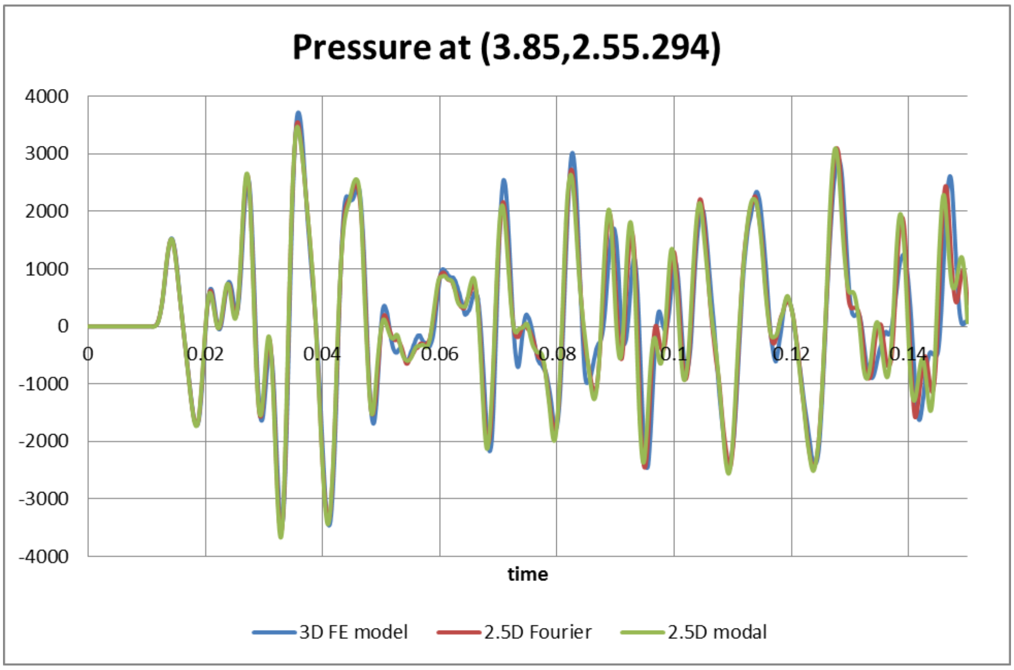

Figure 10, comparison of transient analysis results

6 ROOMS OF ALMOST CONSTANT HEIGHT

In practice some rooms are “approximately constant height”. To investigate the effect of minor deviations from the constant height assumption the 3D model was reanalysed, but with a reduction of 0.15m at the front, bay window section. The comparison of results is in figure 11. At early times the responses are identical. There is a tendency for the discrepancy to increase as time progresses, but there are some well matched peaks at quite late time. These are believed to results from multiple reflections off surfaces in the section containing the source, where the height is constant.

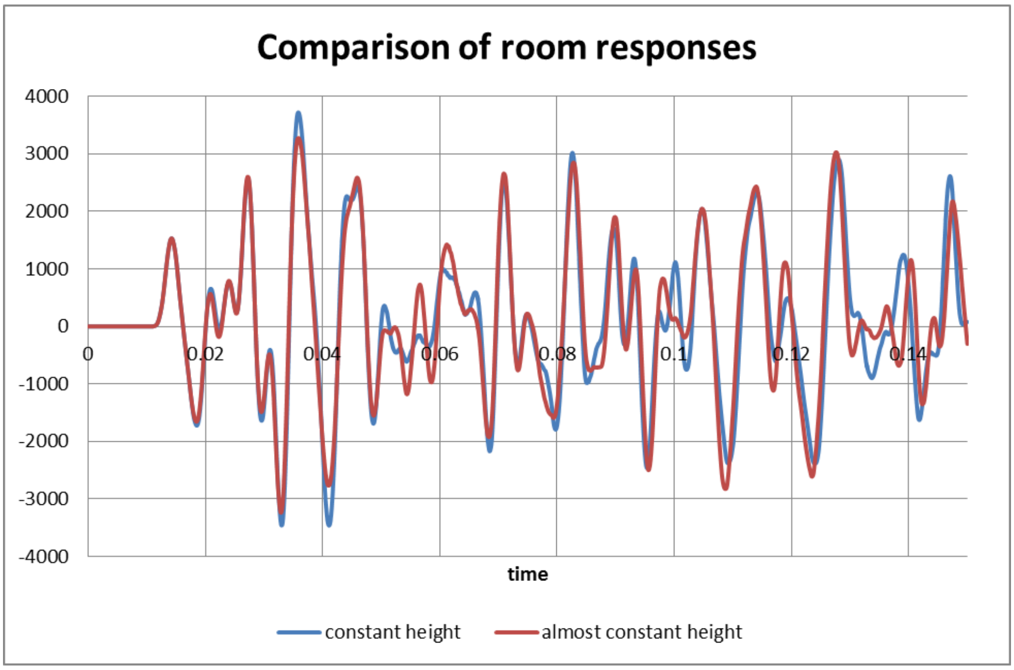

Figure 11, minpor deviation from constant height

7 FURTHER WORK/CONCLUSIONS

The 2.5D approaches have been shown to be accurate and computationally efficient in both time and frequency domain analysis. for suitable rooms.

Further work is underway to investigate the extent to which the method can be used for rooms which are only approximate constant in height.

8 REFERENCES

1. S. Marburg and B. Nolte, Computational Acoustics of Noise Porpagation in Fluids, Springer (2008) ISBN 978-3-540-77447-1.
2. P.C.Macey and K.F.Griffiths., ‘Room Acoustic Analysis using a 2.5 Dimkensional Approach’, Proc IoA Vol 33, Pt 6 (Brighton 2011).