FINITE ELEMENT ANALYSIS OF SURFACE SCATTERING PROPERTIES

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1. INTRODUCTION

The scattering properties of rough surfaces can be investigated by studying the idealised situation of scattering by a sinusoidal surface. These types of problem have been traditionally investigated by series solutions methods, e.g. Heaps [1], which are usually accurate only for h/d << 1, where h is the amplitude of the spatial oscillation and 2d is the wavelength, and possibly only for small angles of incidence. In this paper a hybrid finite element/series solution is proposed. This FE based solution does not suffer from either of the above limitations. The FE solution can be used to analyse scattering by more general periodic surfaces. Comparison is made with scattering by a periodic square wave surface.

2. SEPARATION OF VARIABLES

Consider the situation in figure 1, where a plane monochromatic wave is incident on a periodic surface.

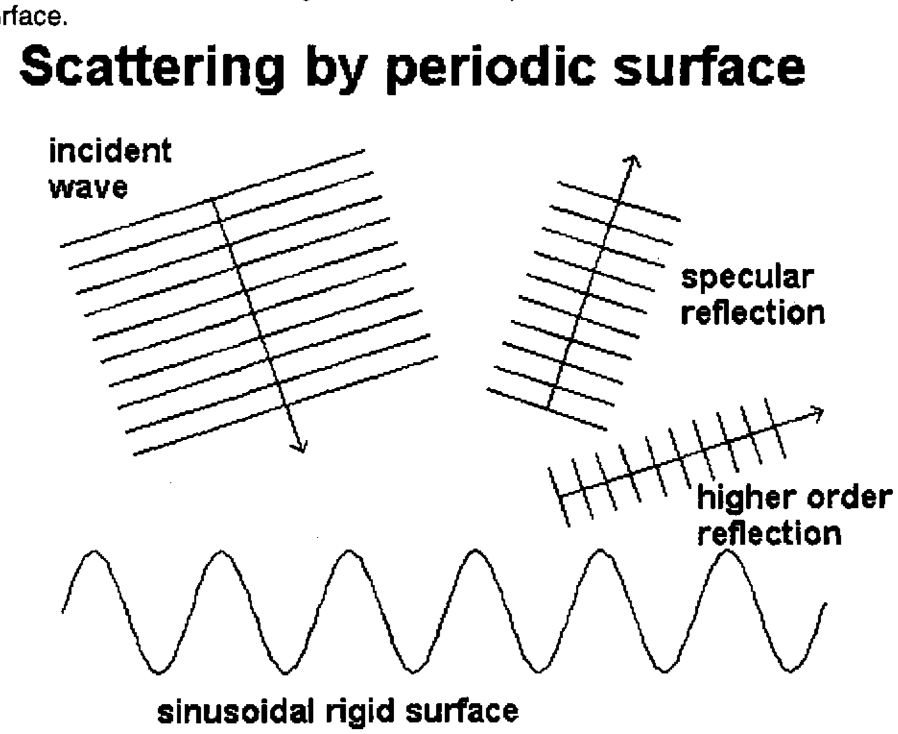

Figure 1

Let $\underline{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ be the direction of propagation of the incident wave. Assuming an $e^{i\omega t}$ time variation then the incident wave will have the form

$$P_I(x, y, t) = P_I \, e^{i(\omega t - k \underline{n} \cdot \underline{x})} \tag{1}$$

From Bloch's theorem the pressure must satisfy the condition

$$P(x + 2d, y) = P(x, y) e^{-2 i n_1 d k} \tag{2}$$

Hence it follows that $P(x, y) e^{ikn_1 x}$ is periodic with period 2d and can be expanded in a Fourier series, resulting in

$$P(x, y) = e^{-ikn_1 x} \sum_{r=-\infty}^{r=\infty} \overline{P}(y) e^{i\frac{r\pi}{d}} \tag{3}$$

Each term in the above sum must satisfy the Helmholtz formula

$$\nabla^2 P + k^2 P = 0 \tag{4}$$

Hence $\overline{P}(y)$ is determined and equation (3) becomes

$$P(x, y) = P_I + \sum_{r=-\infty}^{+\infty} R_r e^{i k_r y} e^{i \alpha_r x} \tag{5}$$

where

$$k_r^{\,2} = k^2 - \alpha_r^{\,2} \quad , \quad \alpha_r = \frac{r\pi}{d} - k\, n_1 \tag{6}$$

If $k_r^{\,2} \geq 0$ then the negative square root must be taken to ensure a reflected wave. If $k_r^{\,2} < 0$ then the square root must be taken to have a positive imaginary part, and the term is an evanescent wave, which decays exponentially in the y direction, and is a travelling wave in a plane of constant y.

3. HYBRID FE/SERIES SOLUTION

For the hybrid finite element/series solution, a method similar to Hladky-Hennion et al [2] is followed. A length 2d of surface is modelled with pressure based acoustic elements up to a plane of constant y, as in figure 2.

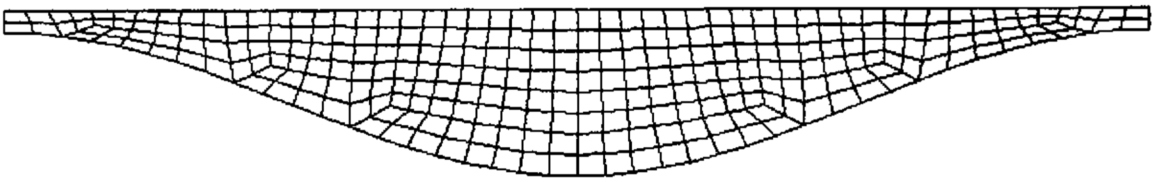

Figure 2

The Neuman boundary condition, required for the lower surface of the mesh, is the natural boundary condition for pressure-based acoustic finite elements. The condition of equation (2) is applied as a constraint linking corresponding nodes on the left and right hand sides of the mesh. The acoustic F.E. equations take the form, as in ref [3]

$$\left([S_a]-\omega^2[M_a]\right)\{P\}=\omega^2\int_\Gamma [N]^T u_y d\Gamma \qquad (7)$$

where u_y is the normal displacement on the upper surface Γ. The pressure in the region above Γ is represented by a truncation of the series solution in equation (5), from r=-N to r=+N.

$$P(x,y)=P_I+\sum_{r=-N}^{r=+N} R_r e^{ik_r y} e^{i\alpha_r x} \qquad (8)$$

The reflection coefficients can be computed as

$$R_r e^{ik_r y}=\frac{1}{2d}\int_\Gamma (P(x,y)-P_I)e^{-ik_r x} d\Gamma \qquad (9)$$

By differentiating equation (8) an expression for u_y can be obtained.

$$\omega^2 \rho u_y=\frac{\partial P_I}{\partial y}+\sum_{r=-N}^{r=+N} ik_r R_r e^{-ik_r y} e^{i\alpha_r x} \qquad (10)$$

Substituting from equations (8), (9) and (10) into (7) a set of equations for the hybrid FE/series solution is obtained. This can be solved to determine {P} the nodal pressures on the FE mesh. Equation (9) can then be used to determine the reflection coefficients.

The method described in this section has been included in the PAFEC VibroAcoustics finite element program.

4. SCATTERING BY DIFFERENT SURFACES

The FE mesh of figure 2 was used to analyse the effect of scattering by a sinusoidal rigid surface. N=20 was taken in equation (8). The speed of sound was taken to be 340 ms^{-1}. The spatial period 2d was taken as 2m. The spatial amplitude was taken to be 0.125m. Results were computed for the frequency range 100 Hertz to 1000 Hertz in steps of 5 Hertz. It is of course possible to scale results up/down in frequency and factor down/up the length scales involved. Normal incidence scattering was analysed for the sinusoidal surface case and for a square wave surface with the same root mean square variation from the mean surface level, i.e. with a distance D=0.17678m between the upper and lower surfaces. A comparison between the specular scattering coefficients over the frequency range is shown in figure 3.

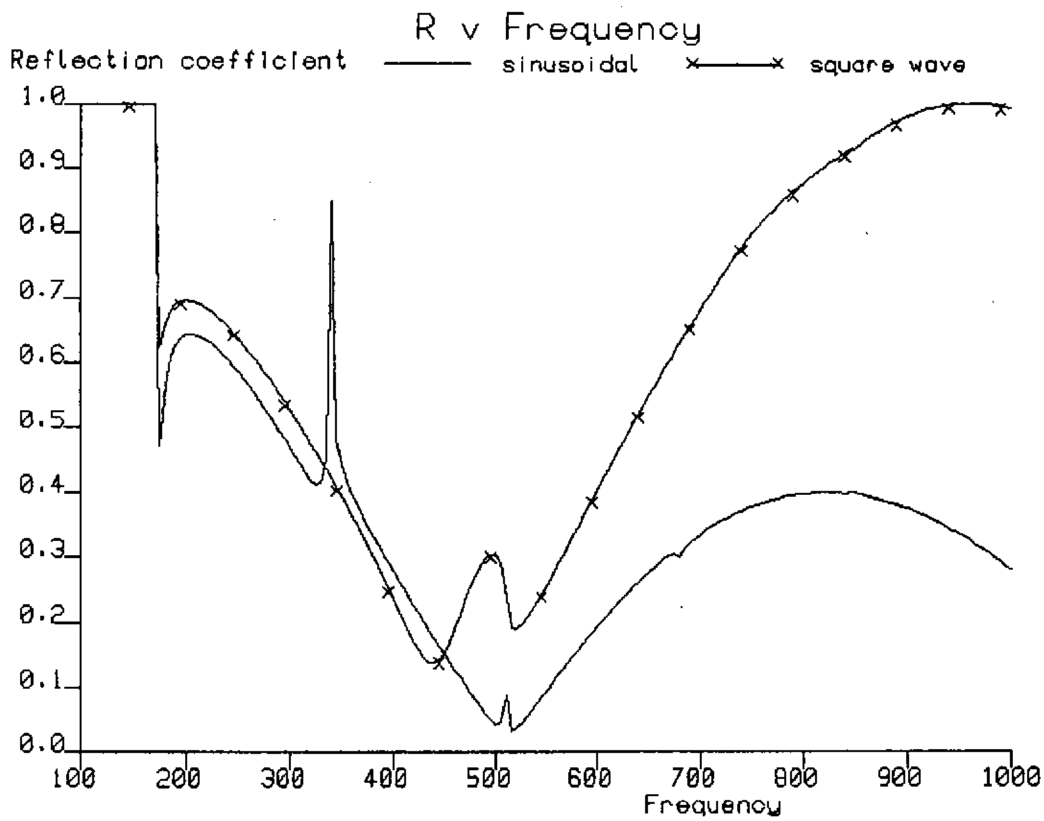

Figure 3

Below 170 Hertz there is only one travelling wave in the summation in equation (8) and the specular reflection coefficient has unit magnitude. At 170 Hertz the first higher order travelling waves occur. These take up some of the energy in the reflected field and there is a sharp drop in the specular reflection coefficient. Other terms change from evanescent to travelling waves at 340, 510, 680 and 850 Hertz. Each new travelling wave has a resonant type of effect on the specular reflection coefficient, but this diminishes with the order r. The higher order travelling waves have much less influence on the square wave case.

In figure 4 results for the reflection coefficient by a square wave surface with D=0.707107, i.e. four times the amplitude. Comparison is made with a simple ray tracing model. This assumes that 50% of the contribution to the specular reflection is by a 'perfect reflection' from the upper surface and that 50% arises from a 'perfect reflection' from the lower surface. The latter contribution has an additional path length of 2D. This results in a magnitude for the reflection coefficient of $|\cos(kD)|$.

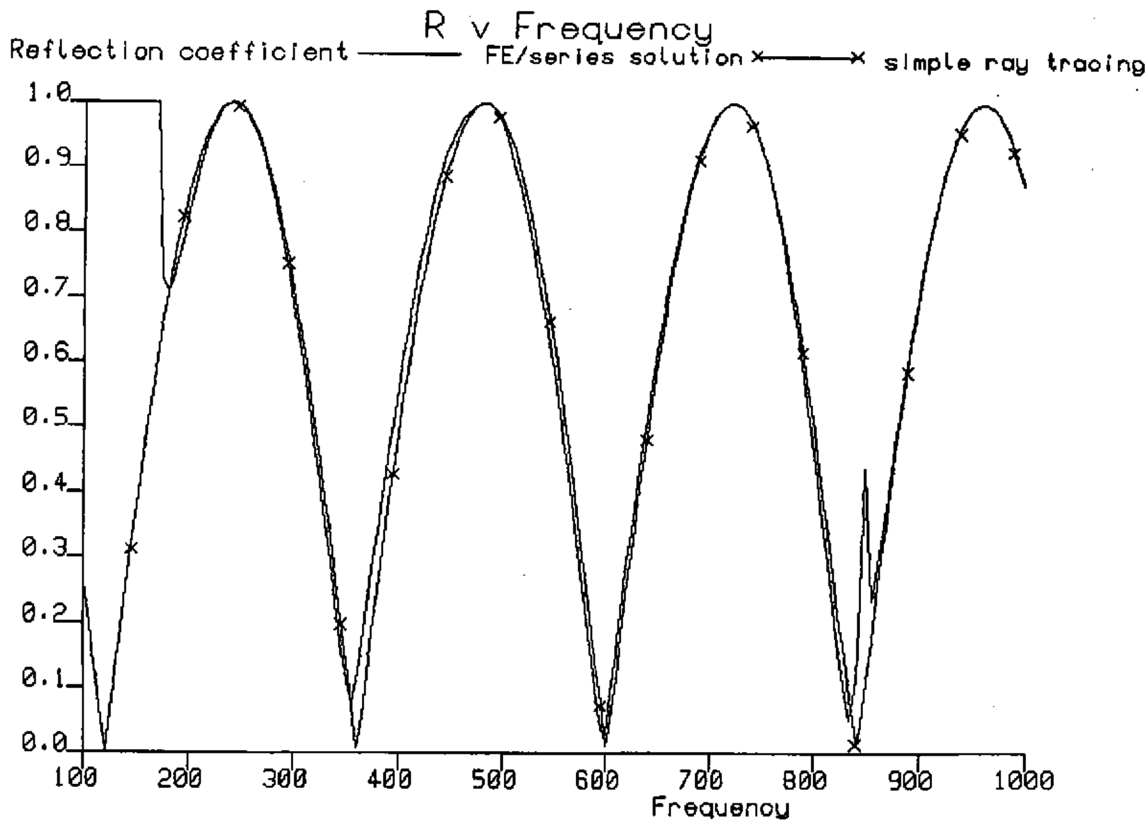

Figure 4

The hybrid FE/series solution is believed to be more accurate. The element side length is very small compared with an acoustic wave length and hence the FE part of the solution should be very accurate. The series solution also includes 41 terms and should be very accurate. The simple ray tracing model does not include diffraction effects. The hybrid and ray tracing solutions disagree at low frequency. The level of agreement generally increases with frequency, apart from a difference at 850 Hertz, corresponding to the onset of a particular higher order travelling wave. This increase in accuracy of the ray tracing with frequency would be expected because diffraction effects decrease.

5. CONCLUSIONS

A combined finite element/series soution has been shown to be suitable for investigating the reflective properties of periodic surfaces. It should be valid for arbitrary values of h/d and any angle of incidence. Results shown here are only for normal incidence. Further results for off-normal incidence will be computed. The results shown here are only for reflection by a rigid surface. This can easily be generalised to include an impedance boundary condition on the surface or a layer of damped acoustic medium on the surface.

The methodology proposed may be of benefit in design of diffusers. It may perhaps be a reasonable assumption to make, for a finite length of a surface profile with a few 'imperfections', that the specular component can be predicted by the above method and that the remainder of the energy in the higher order travelling waves enters the diffuse field.

6. REFERENCES

[1] Heaps H.S. "Non-specular Reflection of Sound from a Sinusoidal Surface"
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