FINITE ELEMENT METHODS FOR TRANSIENT ACOUSTIC ANALYSIS OF AUDIO PROBLEMS

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1 INTRODUCTION

Numerical modeling techniques are becoming common in design of loudspeakers, horns and other devices used in the audio industry. Most commonly steady state sinusoidal response results are computed. However many phenomena are much better understood or more naturally studied in the time domain. The diffraction from cabinet edges is one example. Reflections from walls of a small room are similarly most easily identified in the time domain. The current work compares implicit and explicit transient finite element schemes for some audio applications.

2 TRANSIENT RESPONSE WITH ACOUSTIC FE

2.1 Implicit scheme

For a model including displacement-based structural finite elements and pressure-based acoustic finite elements, the coupled set of equations is

$$\begin{bmatrix} [S] & [T]^T \\ [0] & [S_a] \end{bmatrix} \begin{Bmatrix} (u) \\ \{p\} \end{Bmatrix} + \begin{bmatrix} [C] & [0] \\ [0] & [C_a] \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \{\dot{p}\} \end{Bmatrix} + \begin{bmatrix} [M] & [0] \\ -[T] & [M_a] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{p}\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{F_a\} \end{Bmatrix} \qquad (1)$$

using the notation of reference [1], where {u} and {p} are vectors of the nodal displacements and pressures on the two meshes, {F} and {F_a} are vectors of nodal forces and acoustic volume accelerations, [S], [C] and [M] are the structural stiffness, damping and mass matrices respectively, [S_a], [C_a] and [M_a] are matrices for the acoustic mesh and [T] is a coupling matrix. These equations can be marched forward in time, using the Newmark Beta method[2], assuming typically that the acceleration during a time step δt is constantly equal to the mean value. This results in a set of sparse linear equations to be solved at each time step to march forward. However the system matrix only needs to be factorized once, unless the time step δt is changed. If there is an infinite external fluid, then wave envelope elements[3] can be used in the model.

2.2 Explicit scheme

The costly solution of linear equations required by the implicit solution is avoided in an explicit solution where the mass matrix is approximated by a lumped diagonal matrix. The accelerations can be easily computed once the forces are evaluated, which only requires element-level matrix x vector operations. In order to ensure stability it is usually necessary to work with velocities at half time step points. The basic structural scheme is given in ref 4. The scheme below is a generalization to include structural and acoustic finite elements.

Assume that results have been evaluated up to time n x δt. Let the vectors of structural displacements and acoustic pressures at this time be $\{u_n\}$ and $\{p_n\}$ respectively, and the time derivatives half a time step previously be $\{\dot{u}_{n-\frac{1}{2}}\}$ and $\{\dot{p}_{n-\frac{1}{2}}\}$. Firstly the accelerations are computed as

$$\{\ddot{u}_n\} = [M]^{-1}\left(\{F_n\} - [S]\{u_n\} - [C]\{\dot{u}_{n-\frac{1}{2}}\} - [T]^T\{p_n\}\right) \tag{2}$$

Next the second derivatives are pressure are computed as

$$\{\ddot{p}_n\} = [M_a]^{-1}\left(\{F_{an}\} - [S_a]\{p\} - [C_a]\{\dot{p}_{n-\frac{1}{2}}\} + [T]\{\ddot{u}_n\}\right) \tag{3}$$

The vectors are then updated as

$$\{\dot{u}_{n+\frac{1}{2}}\} = \{\dot{u}_{n-\frac{1}{2}}\} + \{\ddot{u}_n\}\delta t \tag{4}$$

$$\{\dot{p}_{n+\frac{1}{2}}\} = \{\dot{p}_{n-\frac{1}{2}}\} + \{\ddot{p}_n\}\delta t \tag{5}$$

$$\{u_{n+1}\} = \{u_n\} + \{\dot{u}_{n+\frac{1}{2}}\}\delta t \tag{6}$$

$$\{p_{n+1}\} = \{p_n\} + \{\dot{p}_{n+\frac{1}{2}}\}\delta t \tag{7}$$

Explicit schemes require a small time step size for stability. Strictly this should be related to the maximum eigenvalue of the system, but in practice this can be effectively taken into account by

$$\delta t \le f\frac{L}{c} \tag{8}$$

where L is the minimum element side length and c is the speed of sound in the acoustic medium or compressional wavespeed in a solid. f is a scaling factor, typically 0.85 or 0.9.

3 ROOM EXAMPLE

The sound field set up by a point source in a room was used to compare methods. The room was taken to have dimensions 4m x 3m x 2m. The source position was taken as (1,0.6,0.2) and a listening position as (2,0.6,0.2), see figure 1. The free-field source was taken as $\frac{S\left(t-\frac{r}{c}\right)}{r}$, where the function S is defined as

$$S(t) = \begin{cases} 0 & \text{for } t < 0 \\ \dfrac{t^2(t-t_0)^2(t-2t_0)}{t_0{}^5} & \text{for } 0 \le t \le t_0 \\ 0 & \text{for } t > t_0 \end{cases} \tag{9}$$

where t_0=0.005, as shown in figure 2. This is similar to a single cycle, but smooth, to ensure the high frequency content is reduced. The properties of air used were c=340ms^{-1} and ρ=1.2 kgm^{-3}. The case of rigid walls was considered, since this has a closed form solution using mirror images of the source in the walls.

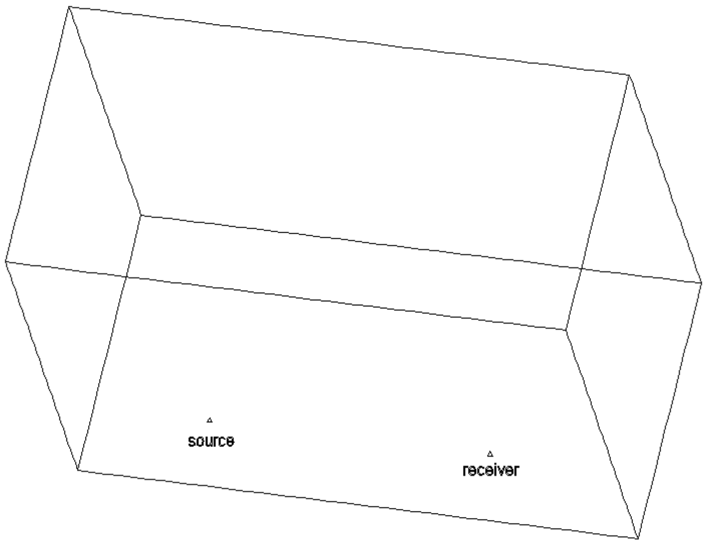

Figure 1
source and receiver point
in room

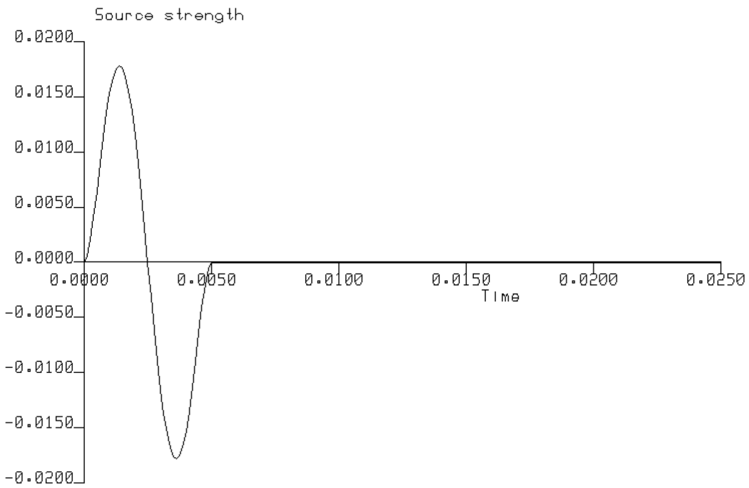

Figure 2
source strength

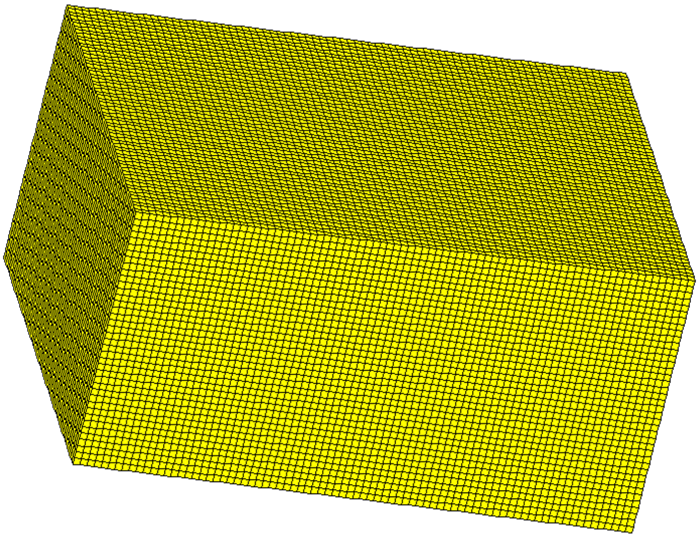

Figure 3
Finite element
mesh for explicit
solution

An explicit finite element analysis was run with a 80 x 60 x 40 mesh of 8-noded linear acoustic brick elements, shown in figure 3. This had 192000 elements and 202581 degrees of freedom. A time step δt=0.000125 secs was used. An implicit solution was run using a 20 x 15 x 10 mesh of 20-noded quadratic brick elements, with 3000 elements and 14041 degrees of freedom and using a time step δt=0.00025 secs. A frontal solution method was used; the maximum front size was 642. A comparison of the pressures computed, with the analytic solution is given in figure 4.

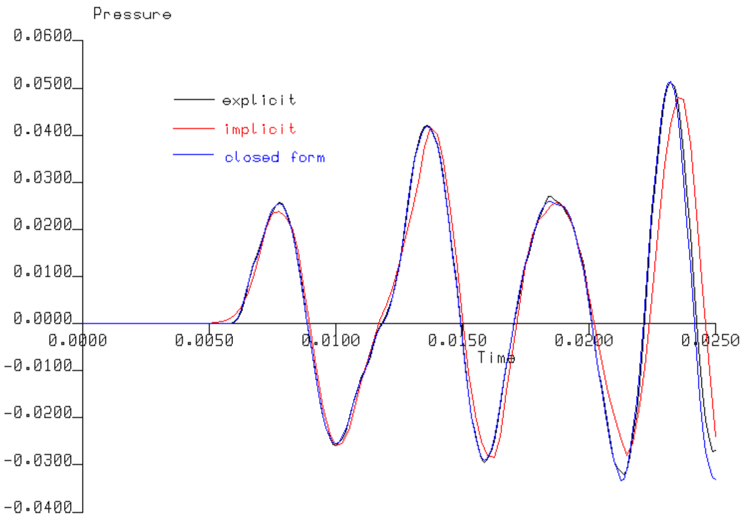

Figure 4
Comparison of results

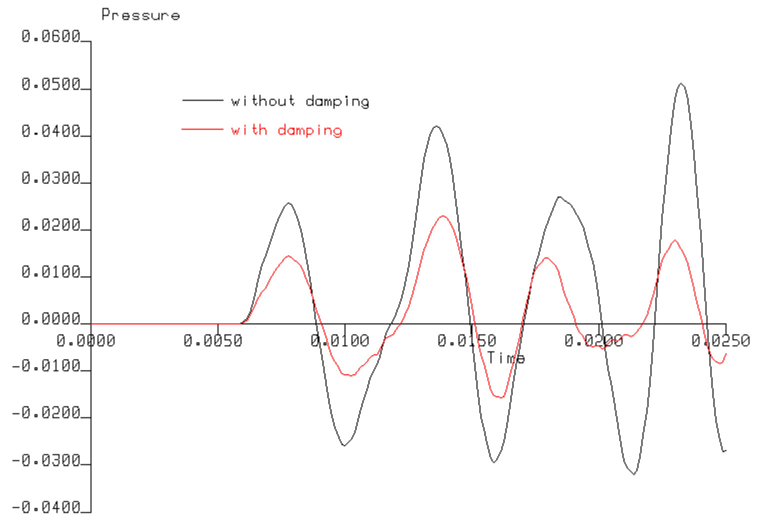

Figure 5
Explicit solution results with/without damping

The explicit solution results are more accurate here, but the run time was about an order of magnitude longer. However as the problem size is increased the relative computation times are likely to change. If the element size, and time step, are halved, then the number of elements and degrees of freedom in the models will increase by a factor of 8. Thus the time for an explicit solution time step will increase by a factor of 8. Also of course twice as many time steps are likely to be needed, but that will also be a requirement of the implicit solution. For the implicit solution, the system matrix needs to be factorized at the start of the analysis. The time for this is proportional to the number of degrees of freedom and the square of the front size, which increase by 8 and 4 respectively. Thus the matrix factorization increases by a factor 128. After this for each time step an equation solution, using this factorization is needed. The time for this is proportional to the product of the number of degrees of freedom and the maximum front size. Thus the time to march forward one time step increases by a factor 32. It is clear that for large problems the explicit approach will be more efficient. However for large implicit solutions, the solution time can be reduced by replacing the frontal solution with generalized sparse solution techniques, based on graph partitioning theory.

Work to investigate this is in progress. However both the frontal solution and generalized sparse solution techniques have a rapidly increasing memory requirement as the problem size increases, whereas the memory required for the explicit solution is proportional to the number of degrees of freedom.

Both methods above can include the effect of impedance conditions, although analytic comparisons are harder. Figure 5 shows two sets of results from the explicit solution, with/without damping. The floor impedance Z=3ρc and the walls Z=1.5ρc.

4 HORN EXAMPLE

Work is currently in progress to compare transient radiation through a horn, using implicit and explicit approaches. A horn with a throat radius of 0.0125m, a mouth radius of 0.075m an axial length of 0.18m. mounted in an infinite rigid baffle is being analysed. A light rigid piston is excited by a finite duration force of similar form to figure 2 The finite element model for the explicit solution is shown in figure 6.

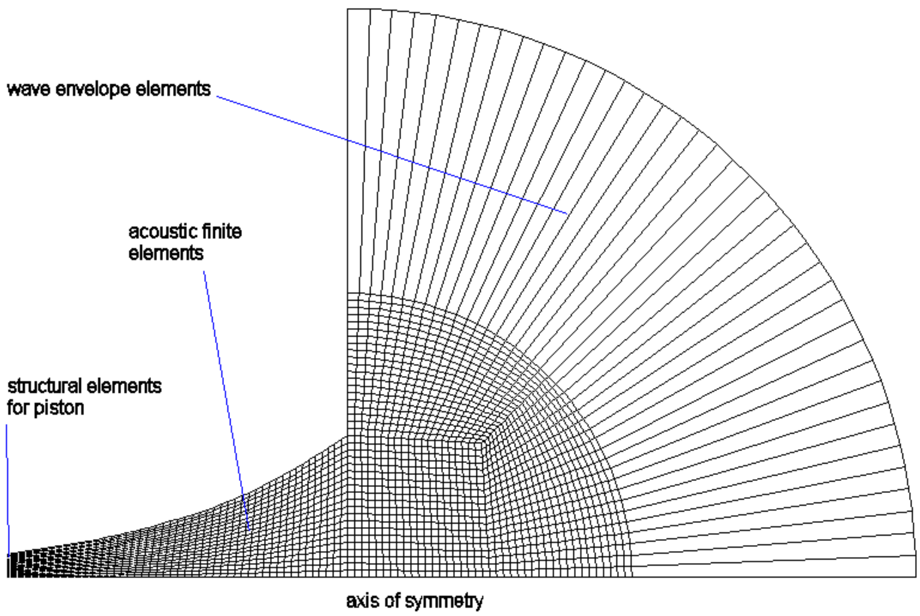

Figure 6 Finite element model of axisymmetric horn

The wave envelope elements have radial order 4. The computed pressure on axis in the mouth of the throat is shown in figure 7.

The intention had been to use a mesh for an explicit solution created by subdividing the mesh in figure 6. However, as constructed, the acoustic finite elements in the throat of the horn have very small side lengths in the radial direction. This has resulted in explicit solutions being unstable except for using very small time steps. To get a fair comparison between the methods, it seems necessary to construct a mesh more carefully, so as to control the minimum element side length and consequently the time step.

Another complication is that the wave envelope element technique, which works well for implicit solutions cannot be used in the explicit case. A simple approach would be to put a ρc impedance on the truncation surface. In order to obtain sufficient accuracy, it may be necessary to model up to greater distance with the acoustic finite element mesh. A more sophisticated non-reflecting boundary condition would reduce the mesh required for external problems.

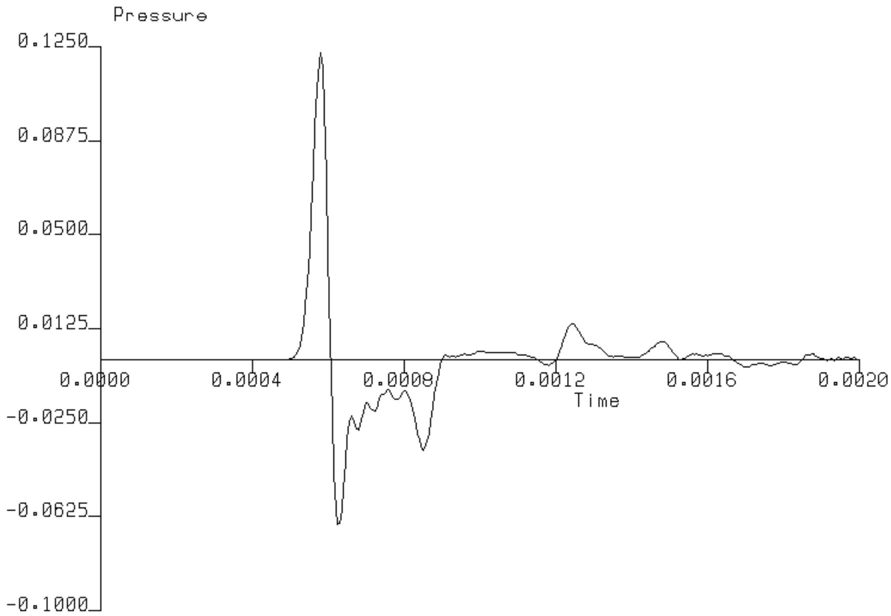

Figure 7
Pressure on axis
in mouth of horn

5 CONCLUSIONS AND FURTHER WORK

Further work is in progress to compare the implicit and explicit solutions for the horn problem.

It is expected that the explicit solution will prove to be a more efficient technique for very large problems. For small/medium sized problems an implicit approach is probably better. If the equations become nonlinear, such as including the distortion effects that occur when the dependence of the wavespeed on the amplitude are included, then the explicit approach seems better.

For the explicit solution more careful mesh construction is likely to be required to control element size, and prevent instability.

6 REFERENCES

1. P.C.Macey, Hybrid modelling techniques using acoustic boundary and finite elements, COMAC95, 213-220. Southampton (1995).
2. N.M.Newmark, ‘A method for computation of structural dynamics’, Proc Am Soc Civ Eng 85(EM3), 67-94 (1959).
3. R.J.Astley, J-P Coyette and L.Cremers. ‘Three-dimesnsional wave-envelope elements of variable order for acoustic radiation and scattering,. Part II. Formulation in the time domain’, J.Acoust. Soc Am. 103(1), 64-72 (1998).
4. P.Jacob and L.Goulding, An Explicit Finite Element Primer, NAFEMS Ltd 2002, ISBN 1 874376 45 X