HIGH FREQUENCY ROOM ACOUSTIC ANALYSIS USING FAST MULTIPOLE BEM

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1 INTRODUCTION

The boundary element method (BEM) is commonly used for low frequency room acoustics. BEM solves the Helmholtz equation “exactly” (i.e. with exact arithmetic, it converges to the correct solution with mesh refinement) but computational demands increase rapidly with frequency. BEM is complimentary to ray-tracing which becomes more accurate at higher frequency, but does not model diffraction properly. Some acoustic problems are many acoustic wavelengths in extent, and hence large for BEM, but yet diffractive effects are still important, hence it is desirable to solve higher frequency room acoustics problems with BEM. As frequency increases, a finer mesh is required by BEM, and the matrix factorization increases as the 6th power of frequency, for a direct solution. Krylov subspace iterative solution techniques reduce this to 4th power x number of iterations. The computationally demanding part of these algorithms is a matrix-vector multiplication. The fast multipole boundary element method (FMBEM) speeds this up and eliminates the large storage requirement for the matrix. The author has written an FMBEM program. Results from this implementation are compared against a 2.5D approach, for a room of constant height, with good agreement. It is observed that more iterations are required for convergence at higher frequency. Increasing the damping is found to reduce the number of iterations required for convergence.

2 BOUNDARY ELEMENT ANALYSIS FOR ROOM ACOUSTICS

In an enclosed acoustic cavity with an ideal point source at \( x_s \), vibrating at circular frequency \( \omega \), the pressure field in the domain satisfies the equation

\[
\nabla^2 p + k^2 p = -\delta(x - x_s)
\]

within the domain and

\[
\frac{\partial p}{\partial n} = -i\omega p Y
\]

at the boundary, where \( k = \frac{\omega}{c} \) is the wavenumber, \( c \) is the speed of sound, \( p \) is the density, \( Y \) is the surface admittance, \( n \) is the outwards normal, and \( \alpha^i \) is the time variation assumed. Using Green’s second formula, it can be shown that the pressure field satisfies the integral equation

\[
\rho \frac{\partial^2 p}{\partial t^2} + i\omega \rho \frac{\partial p}{\partial n} + i\omega Y g(x, y) \cdot \nabla (x, y) = g(x_s, x)
\]

where \( \Gamma \) is the boundary of the domain, and \( g(x, y) \) is the free space Green’s function.
\[ g(x, y) = \frac{e^{-ik|x-y|}}{4\pi|x-y|} \]  

and \( 4\pi\varepsilon \) is the solid angle in the domain at the point \( x \). If \( x \) is collocated to a smooth point on the surface, then \( \varepsilon = \frac{1}{2} \) and equation (3) is an integral equation relating purely surface quantities. The surface pressure distribution can be determined numerically, by using some basis functions associated with a surface mesh, and generating a linear equation relating the surface pressure freedoms taking \( x \) to be collocated at each nodal degree of freedom in turn\(^1\). The linear equations can be assembled into matrix form

\[
[H] \{p\} = \{p_i\} \tag{5}
\]

where \([H]\) is a square dense complex matrix, \( \{p\} \) is a vector of the pressures at the nodal surface degrees of freedom and \( \{p_i\} \) is a vector of the source term from the right hand side of (3) evaluated at the degrees of freedom.

The results presented in this paper use a surface mesh of triangles, with the pressure assumed constant over each triangle. The pressure field on the surface generally has a characteristic length scale variation of the acoustic wavelength. To represent this it is necessary that each edge of the mesh be no longer than \( \lambda/6 \). Some analysts recommend \( \lambda/10 \), to get better accuracy. According to either criterion, the number of surface degrees of freedom \( n \) (= the length of the vector \( \{p\} \) in equation (5)) increases as the square of the frequency \( f \). The storage of the matrix \([H]\) using complex double precision requires \( 16n^2 \) bytes, and hence is proportional to \( f^4 \). The CPU time required to form \([H]\) has the same frequency dependence.
The time to divide by an \( n \times n \) matrix, using a direct solution algorithm, is proportional to \( n^3 \). This is illustrated in the graph in figure 1, showing some computations using LU factorization on a 2.5 GHz core-i7 processor. Hence the solution of the linear equations in (5) will increase as \( f^6 \). This rapid increase imposes a hard upper frequency limit on problems which can be solved by a direct solution approach.

3 TEST PROBLEM

The results presented below are for a room of constant height 2.94m, and cross section as shown in figure 2.

![Figure 2, cross section of room](image)

Results were computed for two BEM meshes. The coarser mesh, M1, is shown in figure 3.

![Figure 3, mesh M1](image)

Mesh M1 is composed of 16970 constant pressure triangles, with the longest side length of 0.18972m. Mesh M2 is composed of 193192 constant pressure triangle, with the longest side length of 0.05179m. Thus with a \( \lambda/10 \) mesh criterion this would relate to 179 Hz and 656 Hz for meshes M1 and M2 respectively. The \([H]\) matrices for M1 and M2, if formed explicitly, would require 4.4 Gbytes and 570 Gbytes respectively.

The properties of air used are speed of sound = 340 ms\(^{-1}\) and density= 1.2 km\(^{-1}\). The source was taken to be

\[
p_t = \frac{e^{-ikr}}{r}
\]

where \( r \) is the distance from the source point, which is located 0.2m from floor and walls. The microphone point was taken at 0.2m from walls and ceiling.

Two cases are considered:
(1) rigid boundaries, \( Y=0 \)
(2) all boundaries having admittance

\[
Y = 0.000128999 \text{ ms}^{-1} \text{Pa}^{-1}
\]
4 ITERATIVE SOLUTION METHODS

An alternative approach to the direct solution is to use an iterative scheme. In Krylov subspace methods, an initial guess \( \{p_0\} \) is made for the solution of equation (5), and solutions are sought in the vector space spanned by \( \{p_0\}, [H][p_0], [H]^2\{p_0\}, [H]^3\{p_0\}, \ldots \). At each iteration the dimension of the subspace is increased by 1, and a new vector \( \{p_k\} \) is computed. The aim is to reduce the magnitude of the residual vector \( [H][p_k] - \{p_f\} \) below some tolerance value. The most widely used method of this type is GMRES\(^2\). When applied to the solution of BEM equations, such as (5), the most computationally demanding part of the algorithm, is a matrix vector multiplication, which is required at each iteration. If \([H]\) is an \( n \times n \) matrix then this requires \( n^2 \) operations, and so the iterative scheme will be more efficient than a direct solution provided that

\[
N \ll n
\]  

(6)

where \( N \) is the number of iterations required for convergence.

Results were computed, using mesh M1, starting with an initial vector \( \{p_0\} = \{p_f\} \) and iterating until the residual can decreased by to \( 10^{-6} \times \) the initial value. The number of iteration required, as a function of frequency, is plotted for the rigid and absorbent boundary cases in figure 4.

![Number of Iterations](image)

Figure 4, number of iterations required for convergence for M1

It can be seen that for both cases, the number of iterations required for convergence increases with frequency. Also with damping included, fewer iterations are required.

Figure 5 shows the reduction in the residual value for both cases, at frequency 200 Hz.
It can be seen that the undamped case converges very slowly at first. The author believes that this is because the cavity is modally dense, at this frequency and it is necessary for the algorithm to build up a good approximation to all the nearby modal vectors before the rate of convergence can accelerate, whereas in the damped case fewer of the cavity modes are important in approximating the required result.

There are two strategies for speeding up these iterative techniques: preconditioning the equations or using a faster method for the matrix \( \times \) vector operation. Preconditioning, replaces equation (5) by

\[
\begin{bmatrix} M \end{bmatrix} \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} p \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} p_f \end{bmatrix}
\]

(7)

Ideally [M] is a good approximation to \([H]^{-1}\), and multiplication by [M] can be achieved efficiently. This should have the effect of reducing the number of iterations required for convergence. Preconditioning techniques are discussed in the book by Chen\(^3\).

The most widely used method for performing the matrix multiplication more efficiently is the fast multipole boundary element method.

5 FAST MULTIPOLe BOUNDARY ELEMENT METHOD

The author has written a simple fast multipole scheme implementing some of the methods in ref\(^4\) \(^5\). At the moment the multipole translation coefficients \( (S | S)'_{m \rightarrow n}^{im} \) are not very efficiently evaluated. (notation from Gumerov\(^4\)) The current program can solve large problems, but not as rapidly as when some further improvements have been made.

6 RESULTS

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Results are compared against a composite numerical/analytical approach, using a finite element analysis of the cross section and Fourier series in the z direction, valid for rooms of constant height. The 2.5D analysis uses modal damping, whereas the fast multipole BEM code uses surface impedance conditions. Hence comparison is made for the undamped, hard wall case. Figure 6 has 2.5D results from 1 Hz to 200 Hz in steps of 0.1 Hz. The FMBEM results are over the same range but in steps of 0.25 Hz.

\[
\text{SPL at (0.2, 3.11, 2.74)}
\]

![Graph showing SPL comparison](image)

Figure 6, comparison of FMBEM results (mesh M1) and 2.5D analysis

Mesh M1 was used for computing the FMBEM results. The 2.5D analysis used the cross section mesh shown in figure 7, with 18081 linear triangles, and 40 Fourier terms through the height of the room.

As there is no damping, the resonances are infinite. The cavity is also extremely modally dense in the upper part of the frequency range. The agreement between the methods is very good in view of these factors.

The results with the surface admittance condition are shown in figure 8 (FMBEM only).

Figure 9 illustrates the pressure distribution at 650 Hz from mesh M2.
Figure 8, results from FMBEM mesh M1 with damping included

Figure 9, pressure field in phase with source at 650 Hz, computed by FMBEM mesh M2

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7 CONCLUSIONS

The fast multipole boundary element method is capable of extending the computational frequency limit of direct-solution BEM approaches. Higher frequency problems than those shown above will be solvable when some efficiency improvements have been made to the code used in this paper. The fast multipole boundary element method clearly has potential applications in room acoustic analysis.

8 REFERENCES