CONE SHAPE OPTIMISATION FOR AN IMPROVED RADIATED SOUND FIELD

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1 INTRODUCTION

As one of an increasing number of design tools employed to develop drive units, vibroacoustic FEM models are used to compute the response of the loudspeaker at frequencies where flexible diaphragm components "break up" into multiple orders of resonance.
Diaphragm "break up" modes in direct radiator loudspeakers may generate audible undulations in the frequency response which would ideally be smooth on and at angles off the driver axis.
The aim for the designer would be to engineer undesirable features out of the acoustic response providing a drive unit capable of good quality sound reproduction.

As engineering processes have developed to accommodate FEM into common practice, it is evident that some improvements in efficiency are within easy reach. Loudspeaker design, both physical and virtual is an iterative process (nicknamed a "black art" by some), exploring numerous permutations and requiring constant guidance by the engineer to produce good results. Experience has shown that in an organised and well stocked prototyping lab, it can be quicker to build a physical prototype than it is to simulate one – the real loudspeaker being "accurate" by default. This challenges the ideal situation of reaping a worthwhile return on the financial and intellectual investments in simulation activities.
One way of ensuring a tangible output from simulations is to ensure that vast, but unattended computations bear the brunt of the design effort while revealing new, unexplored trends.

If the platform exists to simulate loudspeakers, and to automatically cycle through models, then much of the iterative effort could be accommodated by a computer. The missing element is the engineering judgement required following each iteration to steer the design towards specified targets. Methods of investigating parameter sensitivity for loudspeaker modelling [2] could help identify the components which are most responsive to variation, otherwise experience and knowledge must make the initial evaluation of which factors are important in achieving a desired result. An optimisation scheme with an objective function that captures the essence of the performance required could supply this missing guidance a priori. Once prepared, the basis of a fully automated loudspeaker development tool is formed, requiring only target specifications, model parameters and realistic design freedoms from the user, the rest is computed.

2 ACOUSTICAL PERFORMANCE TARGETS

The optimiser requires that some descriptors of good (and bad) performance are offered so that it can incrementally improve the loudspeaker design. A flat frequency response measured in a semi-anechoic chamber on the axis of the driver is a basic indicator of good loudspeaker performance.
Simply flattening and smoothing the on axis response curve might seem to be a one dimensional approach particularly if one assumes that little else matters. It is however, from an optimisation perspective a straightforward starting point and more complete metrics could be included later to provide further clues as to how the loudspeaker might sound.

3 AUTOMOTIVE TRANSDUCER DESIGN - WOOFERS

Automotive loudspeakers have numerous design constraints. A key problem is that of package space requiring that loudspeakers that must produce wide bandwidth performance with relatively high efficiency must incongruously have small dimensions to allow it to be inconspicuously fitted for example, into a door. Making the cone shallow has the effect of reducing the structural rigidity and the frequencies of the axisymmetric "breakup" modes and the scope for extending the response higher in frequency is diminished.

Altering the shape of the cone can alter the distribution of these modes. A simple change in shape from a cone which has a straight profile is to make the profile curved. As explained in [1], for a given loudspeaker this generally has the effect of reducing the frequency of the first modes but increases the frequency of the upper modes and changes the location of the structural nodal lines that determine the proportion of polarised radiating surface that can effectively contribute to far-field radiation.

3.1 Experimental Investigation into Cone Shape

In order to validate the simulations, physical prototypes were built reflecting a simple geometric variation to the cone with all other aspects of the loudspeaker kept constant. If the simulations are able to return acoustical results consistent with the experimental data for this simple cone shape change, then it would be expected that the output from the optimiser would also be faithful to the specified target when physically realised.

3.1.1 Creation of Controlled Prototypes and Validation Models

Having selected a suitable demonstration loudspeaker, two differently profiled cones were tooled – one incorporating a 90mm radius and one straight sided. The depth of the cones was fixed in accordance to packaging constraints.

In the physical domain, things remain within the realms of feasibility but in simulation models, it is easy to create fictitious scenarios and therefore extra care needs to be taken to offer models realistic parameters to get plausible data.

The cone material was a specific paper pulp and additional, uniform "off process" specimens were requested so that mechanical parameters could be measured. The process of cone manufacture called "felting" involves straining mixed paper fibres in pulp form over a shaped perforated mould and then forming a component by closing the tool applying both heat and pressure. The compression of mulched paper fibres in a non-uniform tool inevitably means that the material in the final component is not entirely homogeneous but has variations in density, elastic modulus and loss tangent throughout the component with the effect of making them somewhat individual.

The two cones were attached to identical rubber surrounds. The surround was formed specifically for the flared cone and therefore due to the change in angle where the cone meets the surround a small gap had to be tolerated. This was filleted slightly with a light adhesive. It is noteworthy that this type of "build noise" is not uncommon where sundry components are physically trialled for suitability. This is an inconvenience that does not have to carry over into the domain of virtual prototyping and it is conceivable that more representative predictions of finished product performance could emerge from simulations.

Two loudspeaker models were created to mimic the real scenario outlined above and to ensure that the acoustic effects of making a simple geometric change were equivalent.

3.1.2 Results

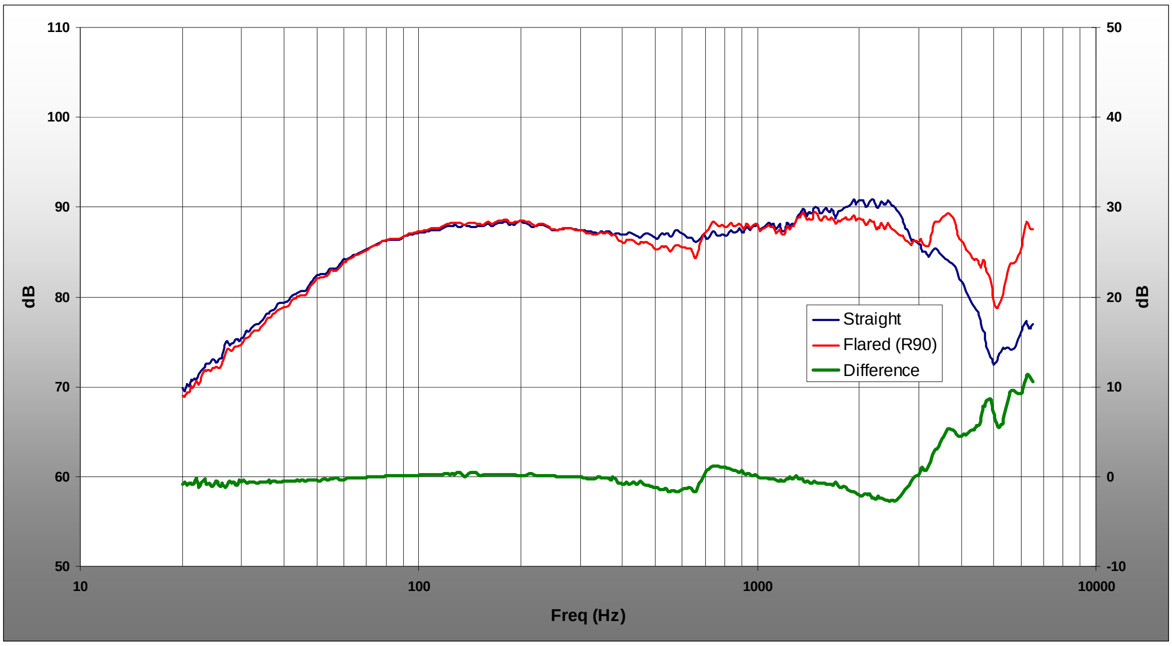


Figure 1 Experimental Comparison between Straight Sided and Flared Cones

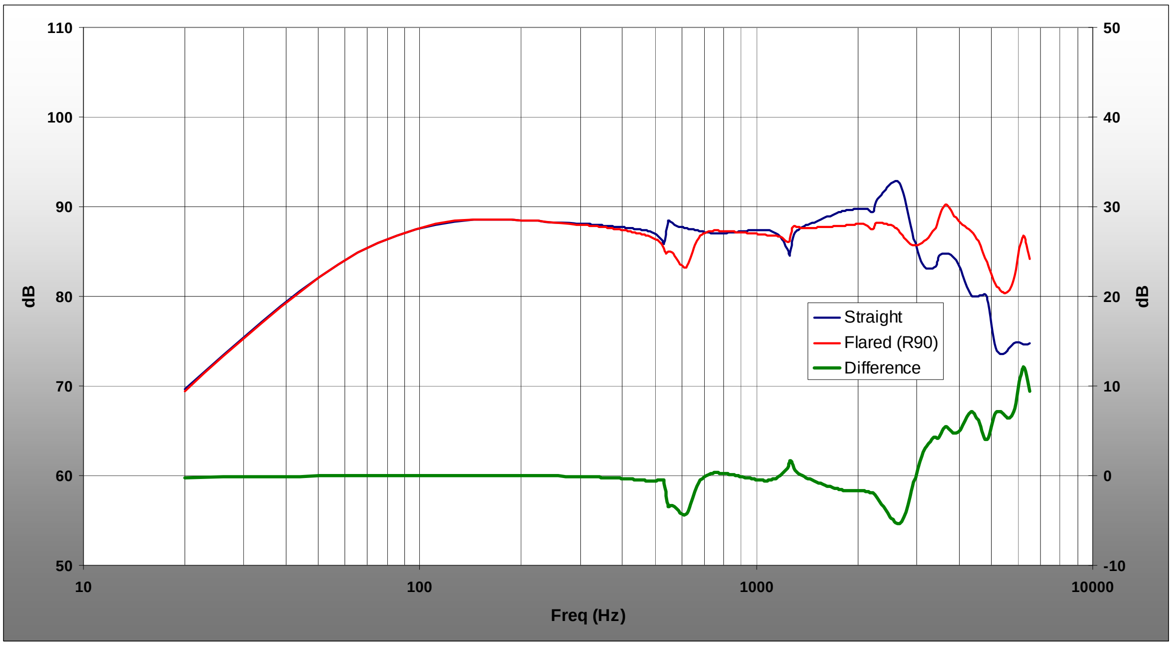


Figure 2 Simulated Comparison between Straight Sided and Flared Cones

The frequency responses in Figures 1 and 2 illustrate the change in acoustic response due to the change in cone profile from flared to straight sided. The flared cone clearly extends the response to roll off at higher frequency however the first breakup frequency is significantly decreased from 1.2kHz to 650Hz and creates a wiggle in the response of about 4dB as opposed to 1.5dB.

The difference between the cones borne out in the simulation reflects the experimental difference reasonably well. A further check was a comparison of the structural modeshapes using a scanning laser Doppler vibrometer. This confirmed that up to at least 6kHz, the structural response for both

the straight sided cone and the flared cone was faithfully reproduced by the models. This is important because we will be optimising on the basis that the changes to the structural model are linked exclusively to changes in the acoustic response, all other things remaining fixed. Therefore if a target acoustic curve is presented, complementary cone geometry will be returned from the analysis. The laser scan also produced a surprise result in that non-axisymmetric “bell modes” were present in the flared cone but not in the straight sided cone which is the opposite trend of what was previously observed in [1].
It would appear in the comparison between figures 1 and 2 that the simulated response is less damped than the experimental case. An explanation for this may lie in observing the experimental situation, beyond the drive unit itself using a laser Doppler vibrometer.

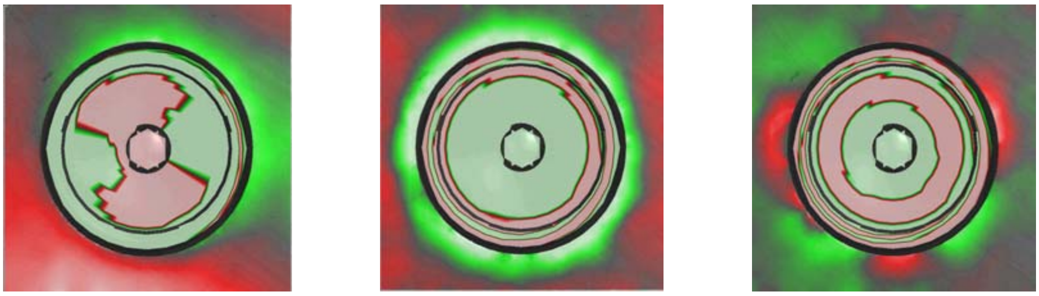


Figure 3 Vibration Patterns on Loudspeaker Baffle at 650Hz, 2.9kHz and 4.1kHz

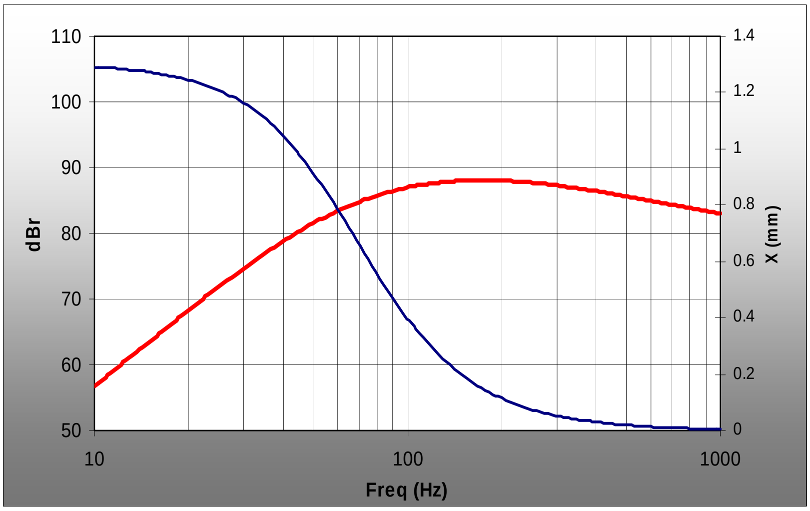
Figure 3 illustrates a point previously made in [1] where a clear discrepancy exists in the idealised boundary conditions of a model and those in a real loudspeaker where energy is able to migrate into the surrounding apparatus – a loss mechanism not accounted for in the simulation model.

4 LOUDSPEAKER DESCRIPTION

The flared loudspeaker described in section 3.11 was used as the basis of the optimisation. The lumped parameters are shown in the table below:

Table 1 Small Signal Parameters for Basis Model

BL (N/A)	4
Re (Ω)	1.9
Mms (kg)	0.016
Kms (N/m)	2300
Sd (m^2)	0.0139
Fs (Hz)	60
Qms	4
Qes	0.7
Qts	0.6
Le (mH)	0.21
L2 (mH)	0.3
R2 (Ω)	1.46



As illustrated in Section 3, the cone is for many drive units the most influential component defining the upper range frequency response of the drive unit and for this reason it has been chosen as the candidate for variation. A close examination of the frequency response and structural deformations

enables us to determine that not only the cone can create wiggles in the frequency response but the spider also has a role in its lower order modes although these are much lower in magnitude.
The structural deformation corresponding to features in the frequency response for the flared cone are illustrated below:

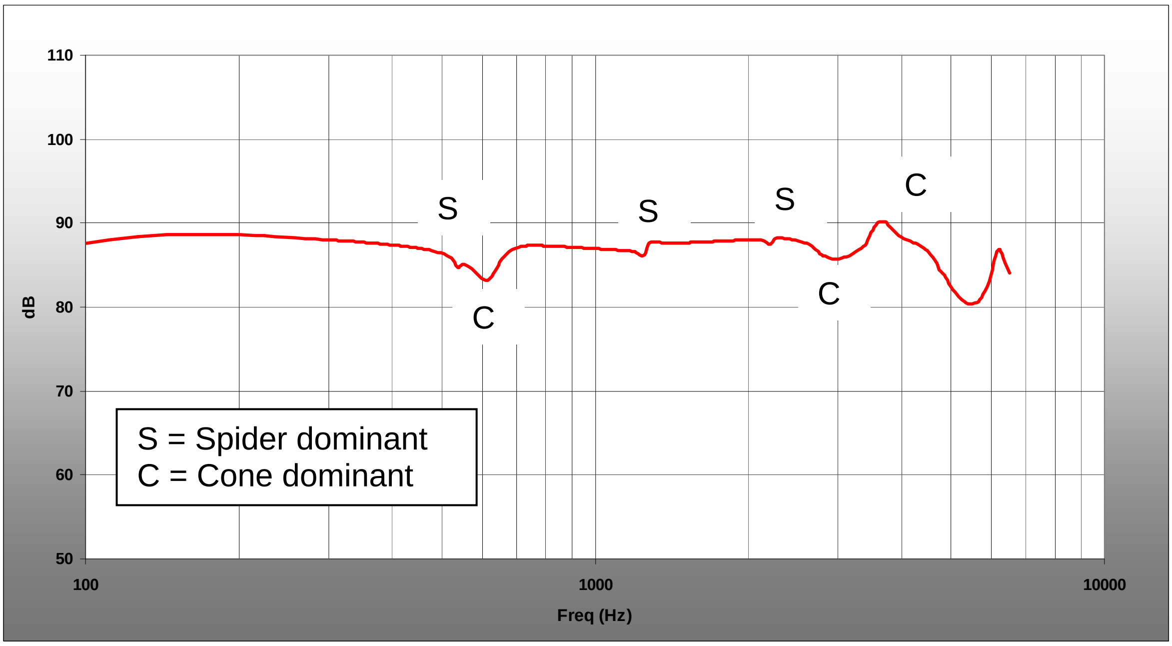


Figure 4 **Component Resonances through Frequency Range**

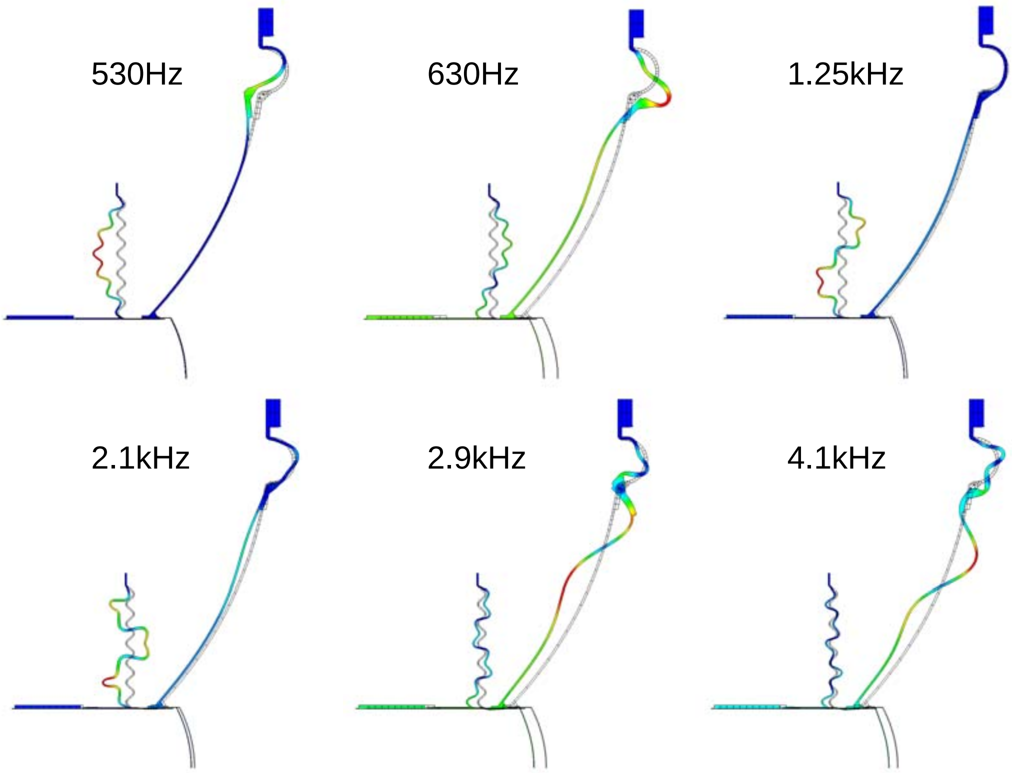


Figure 5 **Structural Deformation of Components**

The important modes at 630Hz, 2.9kHz and 4.1kHz will be the target for optimisation as they create the most significant undulations in the response, determine the upper limit of the frequency response and, as the flared versus straight sided comparison has indicated, they are most directly influenced by the cone shape.

4.1 Performance Requirements from this Driver

This loudspeaker has a acoustic response problem due its first axisymmetric breakup mode, sometimes referred to as “cone-surround mismatch”. A common fix to improve the structural behaviour is to create a fillet, often with adhesive with highly damped mechanical properties, where the geometry changes at the beginning of the surround roll coming from the cone. This additional material braces this geometric weakness and reduces the magnitude of the cone edge displacement associated with this modeshape. In the cone under consideration, the fillet is moulded into the surround.
The fillet only partly improved matters but the change of cone profile to straight sided suggested that shape changes could almost remove all evidence of the problem from the frequency response (section 3.1.2). Making the cone straight however meant that the response at higher frequencies was curtailed and therefore a solution was required where the first breakup mode could be improved while maintaining the SPL up to 4kHz.

5 CONE SHAPE OPTIMISATION

Two optimisation strategies to adjust the cone shape were considered, one based on attempting to improve the radiated sound directly, and the other more indirectly by raising the frequency of the first cone break up mode (630 Hz in figure 5) whilst maintaining the frequencies of the next two cone break up modes (2.9 kHz and 4.1 kHz). For both approaches it is necessary to have a definition of the generalised cone shape controlled by a few parameters. The optimisation procedure adjusts the parameters, performs a FEM(/BEM) analysis, evaluates the design using an objective function and determines further parameter changes so as to try to minimise the objective function. The design engineer needs to construct a suitable objective function. In the current work a simple optimisation procedure, searching over a lattice of points in design space was used to search for a minimum.

5.1 Cone Shape Definition

The central line in the cone cross section consists of a straight line, running axially along the former, a sharp curvature, assumed constant radius bending away from the former and the main curve, extending out radially to the surround. If the main curve is a circular arc, then it can be shown, by solving some trigonometric equations, that the depth, cone diameter and radius of curvature determine the angle turned through by the fillet radius. Thus the general curve defining the cone centre line can be defined by a radius and an appropriate radial stretching function, which varies with angle.

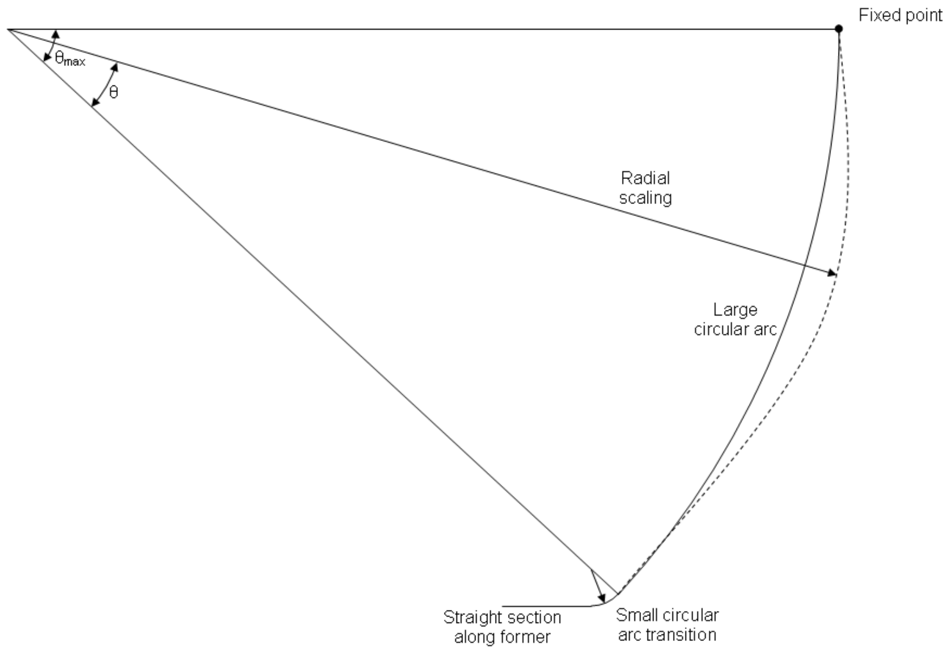


Figure 6 Definition of cone shape

To ensure the cone has the correct depth and diameter the radial stretching function g must satisfy the condition

$$g(\theta_{\max}) = 1 \tag{1}$$

To ensure that it meets up with the transition arc without an abrupt change in gradient, the conditions

$$g(0) = 1 \tag{2}$$

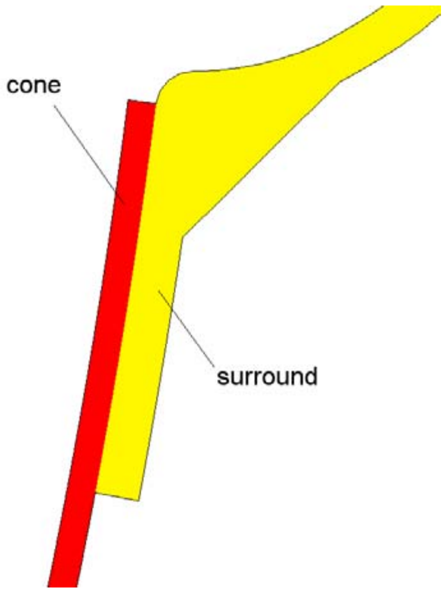
$$\frac{dg}{d\theta}(0) = 0 \tag{3}$$

are required. The general function satisfying these constraints can be written as

$$g(\theta) = 1 + \frac{\theta^2(\theta_{\max} - \theta)}{\theta_{\max}^3}\left(\gamma_0 \frac{l}{R} + \gamma_1 \frac{l\theta}{R\theta_{\max}} + \ldots\right) \tag{4}$$

where l is the axial length of the curved section and R is the radius of the main curved part, before scaling. In the optimisation below the variable parameters were R and γ_0 or R, γ_0 and γ_1. Taking additional coefficient from equation (4) would give a greater design space, but the optimisation procedure would require more computation.

In the design being optimised, the surround is packed out with a triangular part in the cross section as in figure 7



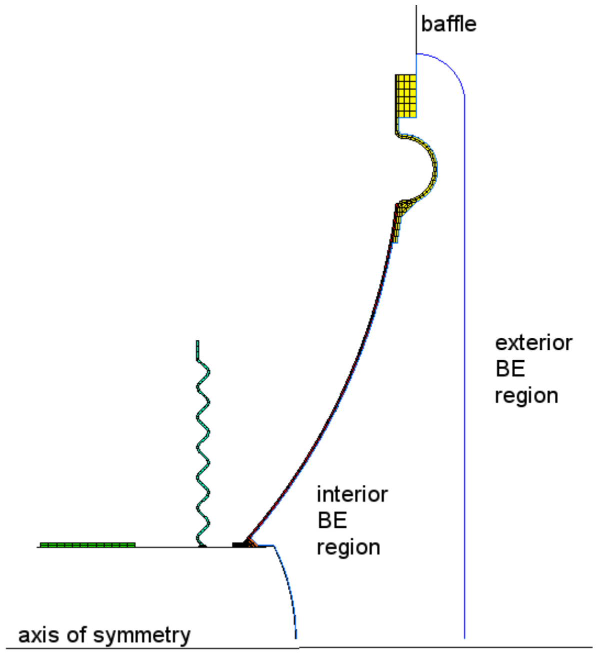


Figure 7 Cone/surround junction **Figure 8 FEM/BEM model**

Thus as the cone shape is altered, and the gradient changes it is necessary to make an assumption as to how the surround is changed. The angle at the start of the triangular section was taken as constant, if possible, otherwise the packing was assumed to be tangential to the roll of the surround. In principle it would be possible to consider the shape of the cone/surround junction as an additional optimisation variable. This contrasts with an experimental approach, where working with an existing surround would be practical during a design stage.

5.2 Direct Optimisation of the SPL on Axis

From inspection of figure 2 and the requirements stated in section 4.1, it seems that the on axis response can be improved by attempting to smooth the response in the frequency range 500 – 1500 Hz and raise the average level in the range 2 - 5 kHz. Hence the objective function used was taken to be

$$F(R,\gamma_0) = A\,\mathrm{var}(\mathrm{SPL\ on\ axis\ 500\text{-}1500\,Hz}) - B\,mean(\mathrm{SPL\ on\ axis\ 2\text{-}5\,kHz}) \qquad (5)$$

where the constants A and B can be adjusted to give greater weight to one objective or the other.

Computation of the SPL required modelling the air in front of the diaphragm. This was done using a boundary element with 2 zones, as in figure 8. There were 4699 structural degrees of freedom and 103 boundary element degrees of freedom. Evaluating the objective function in (5) was done using 70 frequencies. The optimisation procedure can be speeded up by using a coarser model or using fewer frequencies, at the risk of losing accuracy.

A comparison of the original flared cone result and the optimized result is given in figure 9.

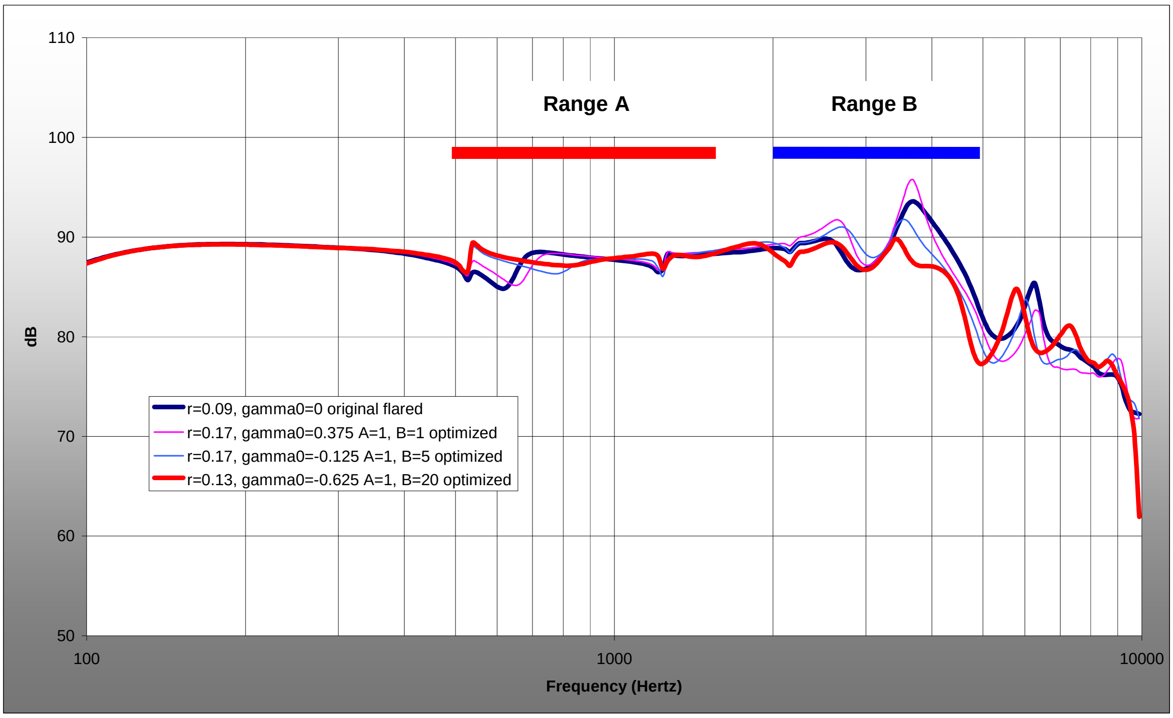


Figure 9 **Comparison of Original and Optimised Responses**

The corresponding cone profiles keeping the overall depth fixed are shown below to illustrate the shape changes.

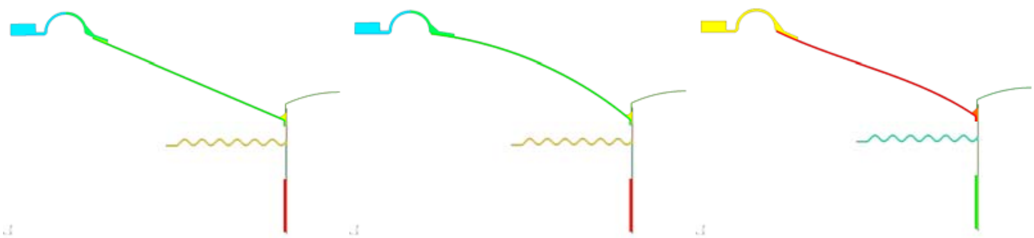


Figure 10 **Straight, Flared and Optimised Cone Profiles**

5.3 Indirect Optimisation via Modal Analysis

The alternative approach considered was to modify the cone shape to increase the first cone break up frequency, whilst preventing the upper cone break up frequencies from dropping. An objective function such as

$$F(R,\gamma_0,\gamma_1)=\begin{cases}0 & \text{if } f_{2c}<\overline{f}_{2c} \text{ or } f_{3c}<\overline{f}_{3c}\\ -f_{1c} & \text{otherwise}\end{cases} \qquad (6)$$

where f_{1c}, f_{2c} and f_{3c} are the frequencies of the first three cone break up modes and $\overline{f}_{2c}$ and $\overline{f}_{3c}$ are minimum acceptable values for the second and third cone break up frequencies.

As the cone shape is adjusted and the frequencies change, the ordering of frequencies is likely to change. In order to implement the objective function of equation (6) it is necessary for the optimisation program to correctly identify the modes. This can be done using the fact that the modes are orthonormal with respect to the system mass matrix. If $\{u_i\}$ and $\{u_j\}$ are the (i)th and (j)th modes of the original, flared cone, then

$$\{u_i\}^T[M]\{u_j\} = \begin{cases} 1 \text{ if } i = j \\ 0 \quad \text{otherwise} \end{cases} \tag{7}$$

Thus if $\{u'_j\}$ is the (j)th mode of the perturbed system, then by choosing j to maximise the absolute value of $\{u'_j\}^T[M]\{u_i\}$ the mode corresponding to the (i)th mode in the original can be identified.
A modal analysis of the structure only is substantially quicker than a fully coupled structure/acoustic analysis scanning over many frequencies. Thus this approach has the potential benefit of producing new designs more rapidly.

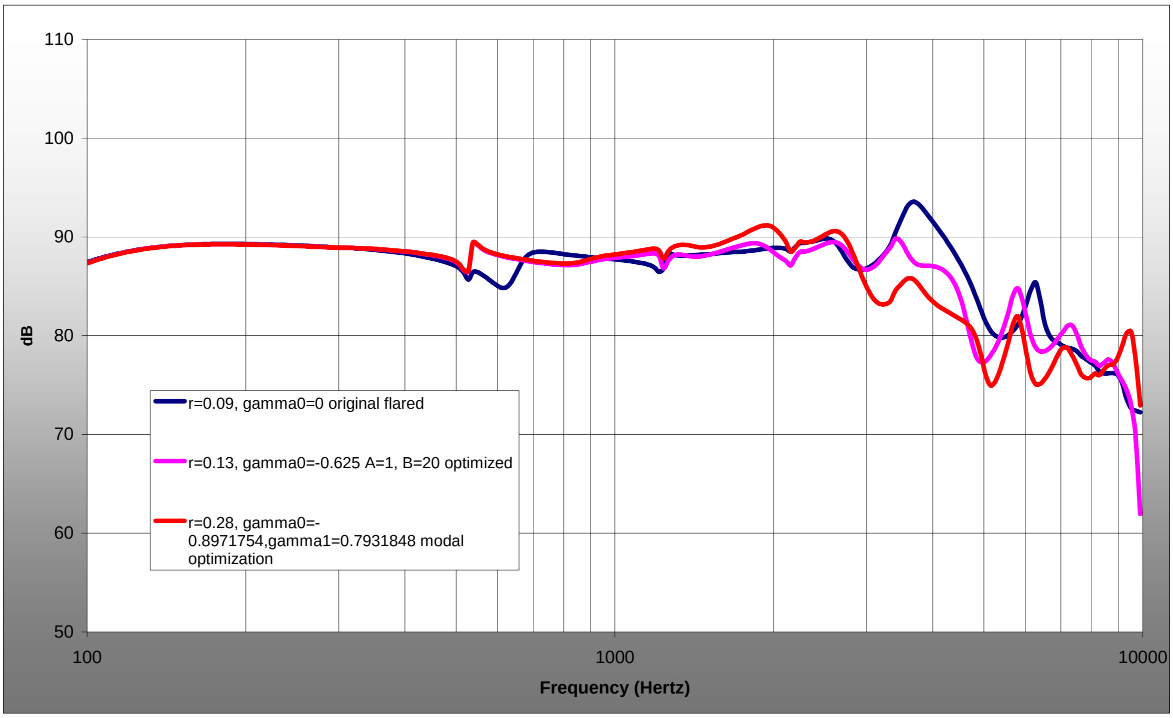


Figure 11 **Comparison of Original, Direct and Indirect Optimisation Results**

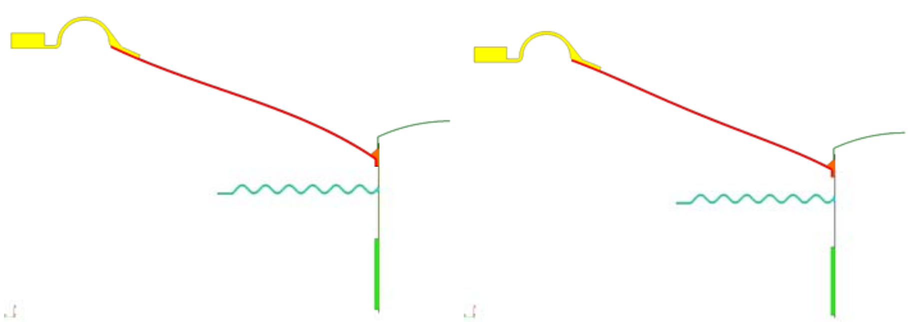


Figure 12 **Cone Shapes from Direct and Indirect Optimisations**

6 DISCUSSION OF RESULTS

The direct optimisation of SPL produced a number of response curves presented in section 5.2. The result that was most appropriate to the specific performance requirement is the red curve. This generally flat response produced good output up to 4kHz where it would then be crossed over to another transducer to cover the remainder of the upper frequency range.
The jogs in the response at 530Hz, 1.2kHz and 2.1kHz as discussed in section 4 are spider resonances and as the experimental results indicate, are more heavily damped in reality which means that the optimised SPL data would be even smoother.
The data in the lower optimisation range was particularly interesting because the dip in response due to the first breakup mode in the flared cone had almost completely disappeared into the more general response shape. This perhaps indicates that the nodal line which separates two portions of cone area moving in opposite directions in a mode superimposed upon the general axial motion of the diaphragm, has changed position with the cone shape change. The result appears to be a more "balanced" system where the first cone breakup mode exhibits self cancelling behaviour.

The indirect optimisation using a modal approach resulted in an improvement in response in the lower frequency range 500 – 1.5kHz but in the upper range 2kHz to 5kHz the results were not as smooth as with the direct optimisation. The cone shapes from each optimisation method were different, the indirect method returning a cone close to straight sided. The task given to the optimiser in each case was quite different. In the direct approach, frequency response smoothness was set as a criterion however in the indirect approach, positioning of the cone modes was set as the target. The indirect optimisation problem was newly set up at the time of writing the paper and these are very early results, as such, more work may be needed to tweak the performance of the optimisation.
The results from the direct approach took 10 times longer to generate and there is obvious motivation to pursue an optimisation based on modal analyses perhaps based upon different objective function.

7 CONCLUSIONS/ FURTHER WORK

A key motivation for FEM/ BEM optimisation is to develop the best possible loudspeakers but also to ensure that the bulk of the effort is undertaken by computers ensuring that human intervention in the form of physical prototyping is minimised.
A useful optimisation procedure has been demonstrated enabling loudspeaker designers to specify target acoustical performance criteria and within realistic design constraints, suitable component geometry could be computed.

The acoustical effects of making a simple geometrical change to the cone profile has been investigated both experimentally and using vibroacoustic FEM/ BEM simulations and the data was shown to be in reasonable agreement. These important results meant that more general changes could be made using a model optimisation scheme with a degree of confidence that the acoustical effects sought would be realisable into a product.

Using direct optimisation of SPL on the axis of the drive unit it was possible to create a more complex cone profile that met performance targets that were probably unachievable by simply changing the flare radius. This work has shown that unintuitive design problems can be solved without the time consuming practice of trialling numerous models or physical prototypes. With the judicious definition of an objective function, a favourable balance of manned data preparation and unattended computational effort to converge upon a suitable design can be established.

A valuable next step would be to manufacture a real cone based upon the geometry returned from the optimiser and compare with the simulated data. Exploring different ways to speed the optimisation up by incorporating modal methods would make the process even more efficient.

Further work is also required to include more radiating components and other aspects of loudspeaker behaviour in the optimisation that would characterise its performance more completely and lead to a generally more improved product. These may include acoustic power and directivity index and indicators of mid-band distortion.

8 ACKNOWLEDGEMENTS

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9 REFERENCES & BIBLIOGRAPHY

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