ROOM ACOUSTIC ANALYSIS USING A 2.5 DIMENSIONAL APPROACH

PC Macey PACSYS Limited, Strelley Hall, Nottingham, UK
KF Griffiths Consultant, Bridgend, UK

1 INTRODUCTION

This paper considers some numerical techniques for low frequency room acoustic analysis, for simple cases where the room has constant height. It is well known that the prediction of the frequency response of a room is complicated and efficient computation requires that different methods are employed for two distinct frequency regions. At low frequency, either modal analysis or a direct numerical solution of the wave equation calculates a relatively coarse spatial variation of pressure which, in a room with little damping, results in distinct peaks and troughs in the frequency response. These solution techniques grow rapidly in computational expense with increasing frequency. At higher frequencies, methods such as ray tracing are used to calculate the response efficiently, assuming that sound propagates in a ray-like way and is reflected in a specular manner. These assumptions become worse at longer wavelengths. Separating these two regions is a transition region, often distinguished by the Schroeder frequency

$$f_s = 2000.\sqrt{\frac{RT_{60}}{V}} \qquad (1)$$

where RT_{60} is the reverberation time and V is the volume.

Toole[1] describes a method for predicting the perceived room response based on anechoid measurements of loudspeakers. These predictions are shown to be accurate above the Schroeder frequency, but below this, modes dominate the response and make otherwise general room response predictions specific to individual rooms. It would therefore be valuable to be able to easily and rapidly predict the low frequency response to “complete the curve” whether the upper band prediction is based upon anechoic loudspeaker measurements or the output of ray tracing analysis.

In the low frequency region, for a simple cuboid region, analytical modes are easily determined. For a few other simple geometries analytical solutions are available. However this is not general enough for many practical situations, and hence numerical methods are required. For small rooms/low frequencies the finite element method[2] (FEM) or boundary element method[2] (BEM) are applicable and can be used for rooms of arbitrary geometry. FEM and BEM are exact, in the sense, that they converge to the true solution as the mesh is refine, assuming exact arithmetic. However as frequency increases, the required element size decreases, resulting in a rapid increase in computational requirement. The current work considers a hybrid approach, partly FE-based , and partly analytical, applicable to rooms of arbitrary cross section, but constant height.

2 THEORETICAL BACKGROUND

2.1 Modal Approach for Undamped Cavity

Consider an acoustic cavity V with rigid boundaries. For acoustic vibrations at frequency f, the pressure field satisfies the Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \tag{2}$$

where $k = \dfrac{\omega}{c}$ is the wavenumber, c is the speed of sound and $\omega = 2\pi f$ is the circular frequency. We wish to obtain the solution for the Green's function $g(\underline{x}, \underline{y}, \omega)$ for this geometry with rigid boundaries, i.e. an ideal omni-directional point source at $\underline{x_0}$ satisfying

$$\nabla^2 g + k^2 g = -\delta(\underline{x} - \underline{x_0}) \tag{3}$$

within the domain and

$$\frac{\partial g}{\partial n_y} = 0 \tag{4}$$

at the boundaries.

Let $\varphi_1, \varphi_2, \ldots$ be the cavity modes, with associated frequencies $\omega_1, \omega_2, \ldots$ and assume that they are orthonormal such that

$$\int_V \varphi_n \varphi_m dV = \begin{cases} 1 \text{ if } & n = m \\ 0 \text{ if } & n \neq m \end{cases} \tag{5}$$

It can be shown, using the divergence theorem and the properties of the Green's function, that

$$g(\underline{x}, \underline{y}, \omega) = \sum_{m=0}^{\infty} \alpha_m \varphi_m (\underline{y}) \tag{6}$$

where the coefficients are given by

$$\alpha_m = \frac{c^2}{{\omega_m}^2 - \omega^2} \varphi_m (\underline{x}) \tag{7}$$

The modes and frequencies for applying this procedure might be obtained analytically for simple geometries, or numerically using FEM or less commonly BEM for more complex geometries. Alternatively FEM or BEM can be used to solve equation (3) and (4) directly.

2.2 Cuboid Case

For a cuboid cavity $\{(x, y, z): 0 \le x \le L_x, 0 \le y \le L_y, 0 \le z \le L_z\}$ standard separation of variable techniques can be used to determine the modes

$$\varphi_{n_x n_y n_z} = \sqrt{\frac{\varepsilon_{n_x} \varepsilon_{n_y} \varepsilon_{n_z}}{V}} \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) \tag{8}$$

and frequencies

$$\omega_{n_x n_y n_z} = c\pi \sqrt{\frac{n_x^{\ 2}}{L_x^{\ 2}} + \frac{n_y^{\ 2}}{L_y^{\ 2}} + \frac{n_z^{\ 2}}{L_z^{\ 2}}} \tag{9}$$

where

$$\varepsilon_n = \begin{cases} 1 \text{ if } & n = 0 \\ 2 \text{ if } & n > 0 \end{cases} \tag{10}$$

The non-negative integers n_x, n_y and n_z are the numbert of half wavelengths in the x, y and z directions respectively.

2.3 Hybrid 2.5D Modal Approach

When the room has constant height, the 3D cavity modes can be constructed from the 2D cross section modes. If $\varphi_1, \varphi_2, ...$ are the cross-section modes, with associated circular frequencies $\omega_1, \omega_2, ...$ and the modes are orthonormal with respect to integration over the cross sectional area, then the 3D modes are

$$\Phi_{nm} = \sqrt{\frac{\varepsilon_m}{h}} \varphi_n(x, y) \cos\left(\frac{m\pi z}{h}\right) \tag{11}$$

with associated frequencies

$$\omega_{nm} = \sqrt{\omega_n^{\ 2} + \frac{m^2 \pi^2 c^2}{h^2}} \tag{12}$$

Computing the 2D cross-section modes is much faster than computing the modes of a 3D model. Thus using equations (11), (12), (6) and (7) offers a vary fast method for doing 3D room acoustic analysis, when the room has constant height.

2.4 Including Damping

A major difficulty in modeling rooms is developing a model for the losses. The real-life physics of room losses is complex and highly resistant to complete justification. At low frequencies, the room boundaries vibrate and energy is both transmitted away from the room to other parts of the building or re-radiated into the space, diminished and delayed. At higher frequencies, smaller room features become significant and surface finishes determine the energy loss upon each reflection.

An easy way to add damping to the above approach is to assume modal damping. If the modal contribution for the nth mode decays as $e^{-\gamma_n \omega_n t}$, where γ_n is the proportion of critical damping, then equation (7) becomes modified to

$$\alpha_m = \frac{c^2}{\omega_m^{\ 2} + 2i\gamma_m \omega_m \omega - \omega^2} \varphi_m(\underline{x}) \tag{13}$$

where $i = \sqrt{-1}$. This is easy to implement in a modal scheme, but effectively assumes that damping occurs within the body of the body of the air volume, and furthermore damping does not cause coupling between cavity modes, and hence is not close to the actual physics.

One way of including a damping phenomenon specifically at part of the boundary would be to use a complex impedance condition. This could be incorporated into the above approach by solving a complex eigenvalue problem. The modes satisfy a Helmholtz equation with complex wavenumbers within the domain, and the impedance condition at the boundary. Equation (7) would be used, but with complex ω_m values. It would be necessary to assume that absorbent regions stretch from the ground right to the ceiling, rather than covering part of the height. It would also be necessary to make a simple assumption, e.g. impedance varying linearly with frequency, for the approach to work. Furthermore, in practice the impedance of an absorbent surface, may in practice vary with the angle of an incident wave, and this effect could not be included. This method has not been investigated further.

In many cases an absorbent medium is porous. Soujnd can propagate through the pores, but viscous effects are important at small length scales. The energy loss can be modeled by using complex acoustic properties. The theory of sound propagation in porous media is well developed[4]. Some formulations assume a rigid structural frame. Other proroelastic approaches and include vibration of the fibres or cell structure of the frame medium.

The methodology outlined in the following section makes a rigid frame assumption. The complex properties can be computed using one of a number of schemes. The Delany Bazley formulation[5] has been widely used historically. This empirical model is straightforward to use as it simply requires the flow resistivity, which can be determined by a simple experiment measuring the static pressure drop across a specimen. The more recent formulation by Allard and Champoux[6] can be used over a greater frequency range.

Modelling the sound field within the absorbent medium should be more accurate than a simple impedance condition, as a variation of impedance with angle will quite naturally be taken into account. Furthermore it is possible to consider problems such as transmission through an absorbent panel from one room to another.

2.5 Fourier Series Approach

If the domain has several acoustic media, then equations (2) and (3) are still valid, but now the wavenumber k is a function of x and y the position in the cross section, but is assumed independent of the height z. Between different acoustic media, there is continuity of pressure and normal velocity. The pressure field in the three dimensional region can be expressed as a sum of cross section distributions as

$$p(x, y, z) = \sum_{m=0}^{\infty} p_m(x, y)\cos\left(\frac{m\pi z}{h}\right) \quad (14)$$

because of the rigid condition at z=0 and z=h. The cross section distributions satisfy the equations

$$\nabla_{xy}{}^2 p_m + \left(k^2 - \frac{m^2\pi^2}{h^2}\right)p_m = -\frac{\varepsilon_m}{h}\cos\left(\frac{m\pi z_0}{h}\right)\delta(x - x_0, y - y_0) \quad (15)$$

Hence it is possible to solve for the 2D cross section distributions independently, usaing a 2D finite element model of the cross section. Note that when

$$m > \frac{2fh}{c} \tag{16}$$

The effective wave number for the cross section in the air region is imaginary, giving rise to an evanescent wave field, decaying exponentially away from the source point. AS m increases further, the exponential decay becomes more rapid. Thus the convergence of the series in equation (14) would be expected to become more rapid as the separation between the source and receiver increases.

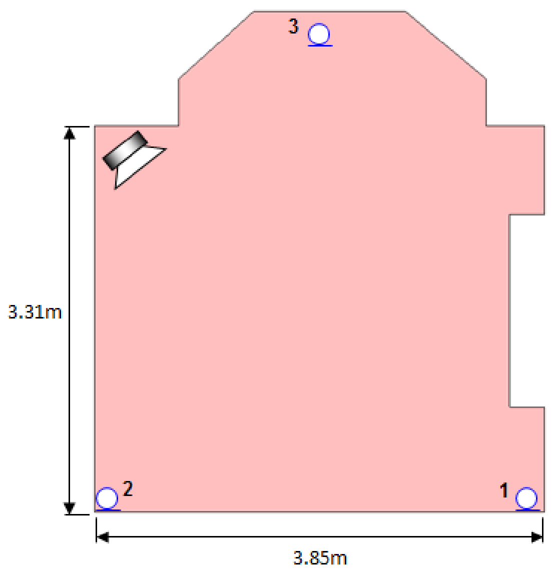
If the series in equation (14) is truncated at m=M, then M+1 2D finite element solutions are required for each frequency analyzed. This is significantly less than the computational requirement of a full 3D analysis, but is not as fast as the modal approach, where the calculation is trivial once the cross section modes have been computed.

3 TEST PROBLEM

3.1 Room Description

Previous work[7] considering the hybrid modal method compared the 2.5 dimensional approach with 3D finite and boundary element simulations of a room of constant height. The results were very accurate. The current work makes a comparison with measurements, for a more realistic "almost constant height" room.

Figure 1 illustrates the cross section of the room used for measurement. It was a lounge, with a bay window at the front and a fireplace on the right hand side.



The source height, i.e. centre of loudspeaker, was 0.2m from the floor. The source position was kept fixed. Measurements were made at three positions. The heights for these were 2.82m, 0.1m and 0.1m respectively.

The room was not perfectly constant height. The main differences were the bay window, with different ceiling height, the gasmeter cupboard, cornice, ceiling fan and lights and the source loudspeaker. The simulations assumed rigid walls. The actual surface materials were plaster except for the window and door.

Figure 1 source and receiver positiions

The source was a pair of loudspeakers mounted in opposite faces of a 0.41m x 0.41m x 0.41m enclosure, see figure 2.

Figure 2 Loudspeaker source with drivers in two opposite faces.

3.2 Simulation Results and Comparison with Measurement

The finite element mesh used to compute modes for the cross section is shown in figure 3. It comprises of 2944 3-noded linear triangles and has 1554 nodes. The number of cross sectional modes used was 40, and the maximum number of half wavelengths through the height was taken as 10. The damping coefficient γ was varied linearly from 0.05 at 30 Hz to 0.03 for 130 Hz.

The comparison of results are shown in figures 4, 5 and 6 respectively. The agreement is good at low frequency. At higher frequencies, where the deviations in geometry from constant height are greater, as measured in wavelengths, the level of agreement decreases.

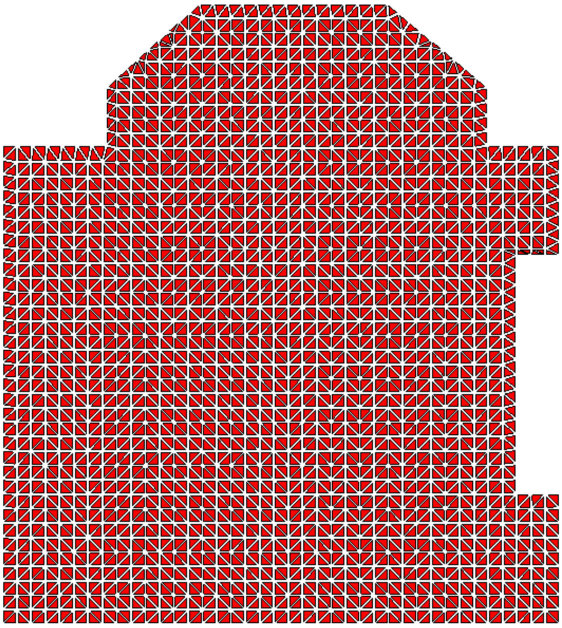


Figure 3 finite element mesh of cross section

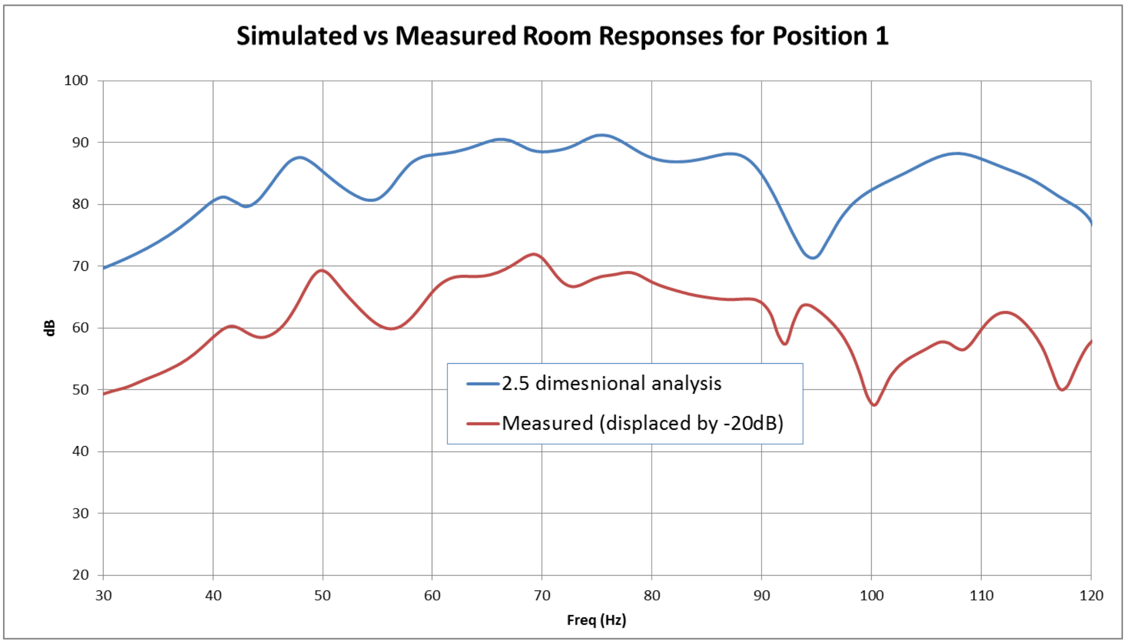


Figure 4 comparison for position 1

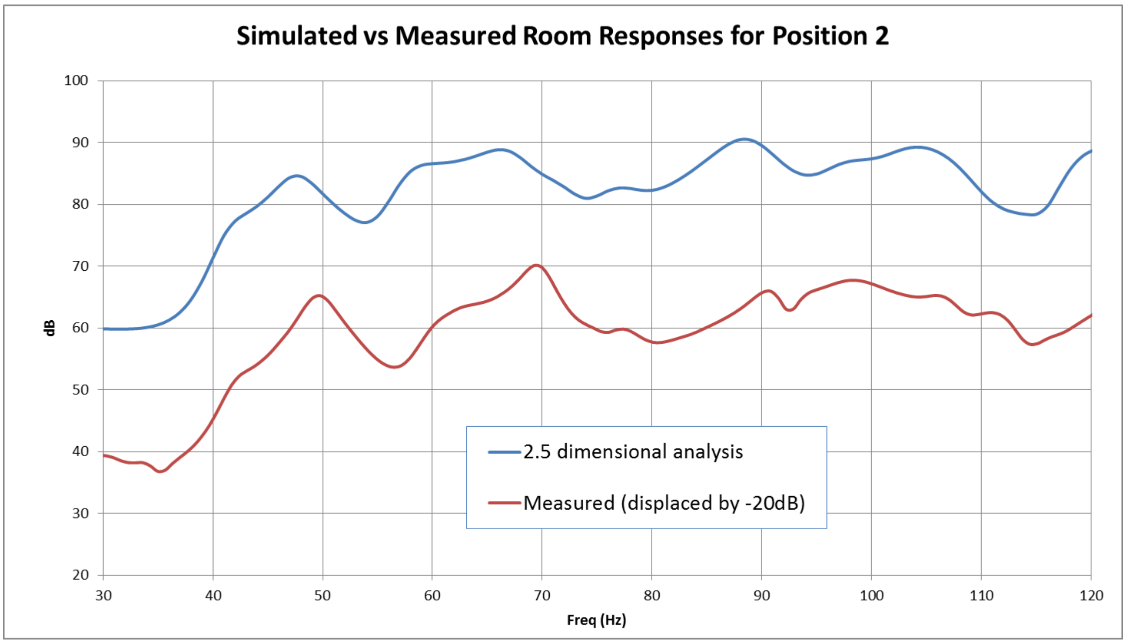


Figure 5 comparison for position 2

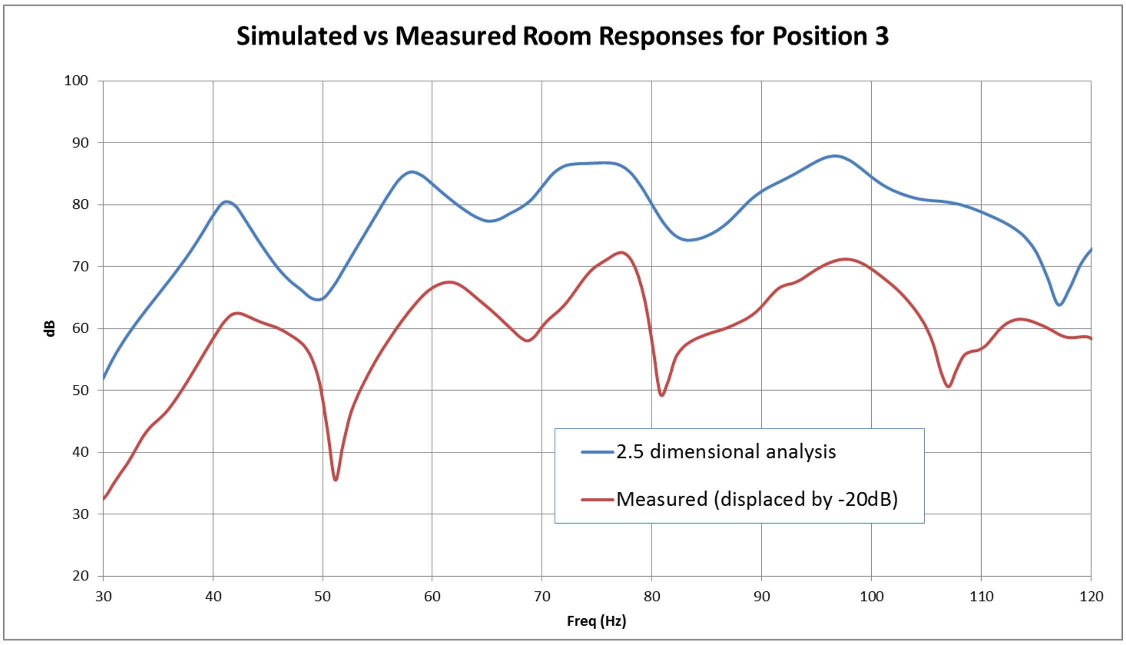


Figure 6 comparison for position 3

4 CONCLUSIONS

The hybrid modal method has shown good agreement with experiment, especially at the lower frequency end of the spectrum for a room of almost constant height, yet is much faster by orders of magnitudre than a full 3D finite element analysis. For appropriate rooms, this allows the possibility of optimization of source placement, strength and phase.

5 FURTHER WORK

The Fourier series methodology could be extended to poroelastic problems in the frequency domain, and time domain analysis, further extending the utility of the 2.d dimensional approach.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

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