TRANSIENT ACOUSTIC ANALYSIS OF SIMPLE ROOMS

PC Macey PACSYS Limited, Strelley Hall, Nottingham, UK
KF Griffiths Electroacoustic Design Limited, Bridgend, UK

1 INTRODUCTION

Numerical and analytical techniques are used for the design and setup of small rooms. At low frequency the finite element method (FEM)[1] or boundary element method (BEM)[1] are applicable. These simulation techniques can be used for arbitrary geometries, and have the benefit of solving the wave equation "exactly", in the sense that mesh refinements result in convergence to the true solution with exact arithmetic. Thus diffraction, which is an important phenomenon at low frequency, will be included, as it is predicted by the wave equation. However as frequency increases the required element size decreases, and the computational requirements for CPU time, memory and storage space increase rapidly. Ray tracing, an alternative simulation technique, is not good at modeling diffraction, and hence not appropriate for low frequency analysis, but is useful for high frequency analysis where diffraction effects are negligible. There is clearly benefit in extending the frequency range over which FEM can be used. This paper considers analysis of rooms of constant height, when various hybrid methods, FE/part analytical can be used. It extends previous work[2], employing these techniques in the frequency domain, to transient analysis.

2 MODAL APPROACHES TO TRANSIENT ANALYSIS

2.1 3D Modal Transient Analysis

The pressure field in a fluid medium due to acoustic vibrations satisfies the wave equation.

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -S(\underline{x},t) \tag{1}$$

where c is the speed of sound and $S(\underline{x},t)$ is a source density term. In the current work a single point source is considered.

$$S(\underline{x},t) = \delta(\underline{x} - \underline{x_s}) f(t) \tag{2}$$

The extension to a distribution of sources is straightforward. If the acoustic domain is a cavity V with rigid boundaries, then the pressure distribution can be efficiently described as a linear combination of the cavity modes. Let $\varphi_1, \varphi_2, \varphi_3....$ be the eigenmodes with the Neumann boundary condition, with associated circular frequencies $\omega_1, \omega_2, \omega_3...$ and assume that they are orthonormalized such that

$$\int_V \varphi_n \varphi_m dV = \begin{cases} 1 \text{ if } n = m \\ 0 \text{ if } n \neq m \end{cases} \tag{3}$$

An arbitrary pressure distribution in the cavity, satisfying the rigid boundary condition, can be represented as a sum of modal contributions

$$p(x,y,z,t)=\sum_{n=0}^{\infty}\alpha_n(t)\varphi_n(x,y,z) \tag{4}$$

Substituting this into the wave equation, multiplying by φ_n , integrating over the volume and using the orthonormal properties results in a differential equation for the modal contribution factor α_n.

$$\ddot{\alpha}_n+{\omega_n}^2\alpha_n=c^2\varphi_n(x_s,y_s,z_s)f(t) \tag{5}$$

In the transient analysis results of section 4, the source term will be assumed to be a pulse of duration τ given by

$$f(t)=\begin{cases}\dfrac{12\pi t^2(\tau-t)^2(\tau-2t)}{(\tau/2)^6} & \text{for t} \le \tau \\ 0 & \text{for } t>\tau\end{cases} \tag{6}$$

The differential equation (5) can be solved using standard techniques.

$$\alpha_n=\begin{cases}A_n\cos(\omega_n t)+B_n\sin(\omega_n t)+g_n(t) & \text{for } t\le\tau \\ A'_n\cos(\omega_n t)+B'_n\sin(\omega_n t) & \text{for } t>\tau\end{cases} \tag{7}$$

The particular integral is $g_n(t)$, a 5th degree polynomial. The trigonometric terms are the complimentary function. The constants are chosen to satisfy the initial and continuity conditions.

The above approach can be modified to include absorption either as modal damping or by computing complex modes for a cavity with absorbent boundaries.

2.2 2.5D Approach

If the room has constant height h, then the 3D room modes are easily constructed from the 2D modes of the cross section. If $\varphi_1,\varphi_2,\varphi_3....$ are the cross section modes, with radian frequencies $\omega_1,\omega_2,\omega_3...$ and the φ_n are scaled to be orthonormal with respect to integration over the cross section area, then the 3D modes are given by

$$\Phi_{nm}=\sqrt{\frac{\varepsilon_n}{h}}\varphi_n(x,y)\cos\left(\frac{m\pi z}{h}\right) \tag{8}$$

with associated circular frequencies

$$\omega_{nm}=\sqrt{{\omega_n}^2+\frac{m^2\pi^2c^2}{h^2}} \tag{9}$$

where

$$\varepsilon_n = \begin{cases} 1 \text{ if } & n = 0 \\ 2 \text{ if } & n > 0 \end{cases} \tag{10}$$

Computing the 2D cross section modes is a significantly smaller computational task than computing the 3D modes.

2.3 Inverse Fourier Transform Approach

An alternative approach to compute a linear transient solution, is to compute results initially in the frequency domain and then use an inverse Fourier transform. The frequency domain analogue of the wave equation is the Helmholtz equation, with a source term included this is

$$\nabla^2 g + k^2 g = -\delta\left(\underline{x} - \underline{x_0}\right) \tag{11}$$

The modal contribution factors can be determined as

$$\alpha_m = \frac{c^2}{\omega_m^{\ 2} + 2i\gamma_m \omega_m \omega - \omega^2}\varphi_m\left(\underline{x}\right) \tag{12}$$

As above, for a room of constant height, rapid computation can be achieved using equations (8) and (9) as in previous work[2].

3 FOURIER SERIES APPROACH

An alternative 2.5D transient approach is to express the pressure distribution and source function as Fourier series through the height, rather than using a modal expansion.

$$p(x,y,z,t) = \sum_{n=0}^{\infty} p_n(x,y,t)\cos\left(\frac{n\pi z}{h}\right) \tag{13}$$

$$S(\underline{x},t) = \sum_{n=0}^{\infty} S_n(x,y,t)\cos\left(\frac{n\pi z}{h}\right) \tag{14}$$

After substituting into the wave equation (1) and using the orthogonality of the trigonometric terms, a modified wave equation for the cross section distributions can be derived

$$\nabla_{xy}^2 p_n(x,y,t) - \frac{n^2\pi^2}{h^2} p_n(x,y,t) - \frac{1}{c^2}\frac{\partial^2 p_n(x,y,t)}{\partial t^2} = -S_n(x,y,t) \tag{15}$$

Thus it is possible to solve for the $p_n(x,y,t)$ distributions using 2D FEM and combine to obtain the full 3D pressure distribution using equation (13).

4 ANALYTIC VERIFICATION

A cuboid room with rigid walls can be analysed using the method of mirror images. The point source is reflected repeatedly in all 6 walls to obtain a lattice of image sources. The solution is obtained as a sum of contributions from this infinite set of sources. However, for a particular receiver point, and time t, only a finite number of terms need to be considered. Using this method, an analytic solution was produced for a 1m x 1m x 1m cavity, with source at (0,0,0), pulse duration 0.0015 seconds and receiver points (1,0,0), (1,1,0) and (0.5,0.5,0.5). The speed of sound was taken as 340 ms^{-1} for both the cube analysis and the room in the next section. The analytic results are shown in figure 1.

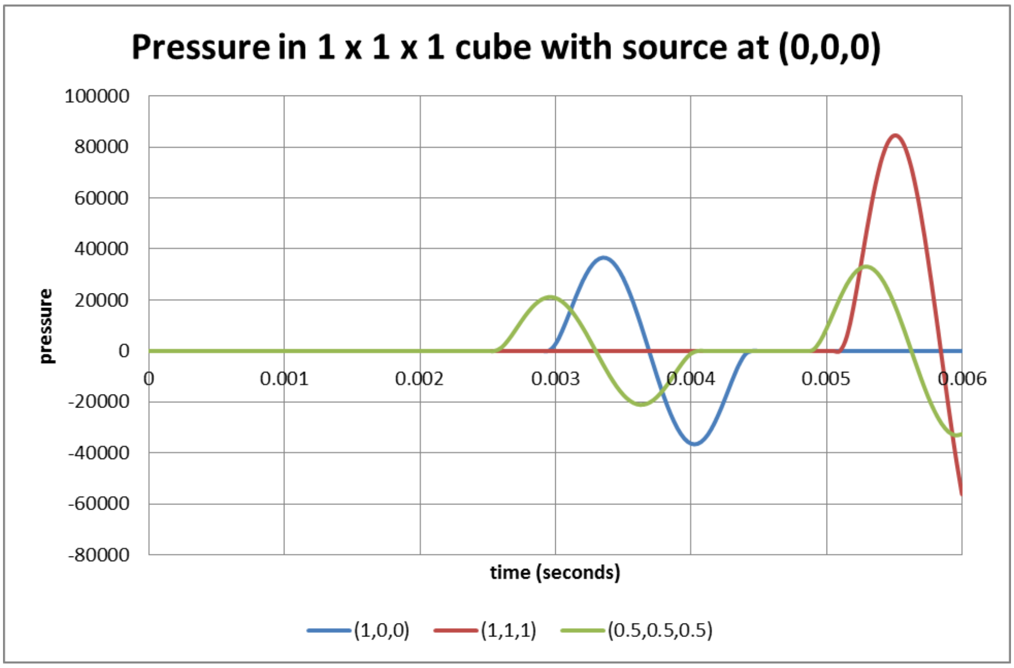


Figure 1 analytical pressure in cubical cavity

Results were computed using both modal and Fourier 2.5D approaches, using the cross section mesh shown in phase 2, consisting of 3200 linear triangles and 1681 nodes.

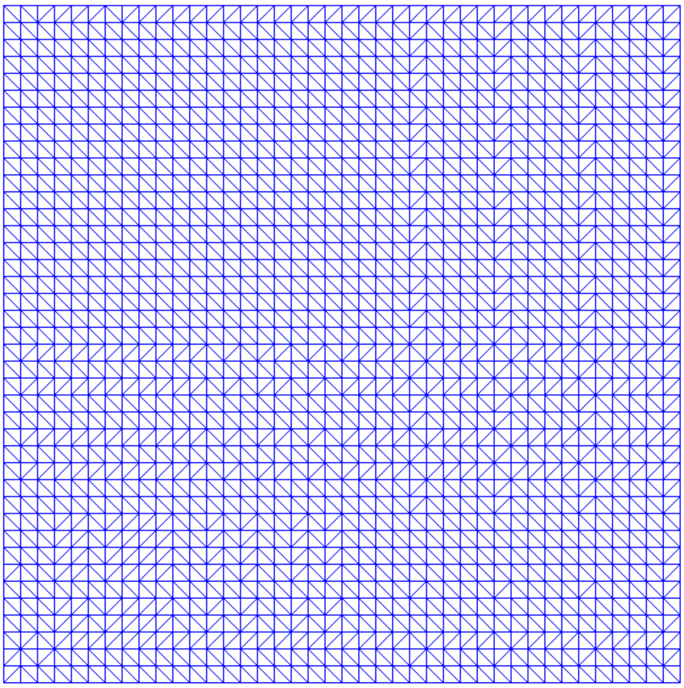


Figure 2 cross sectional mesh for cuboid problem

For the 2.5D Fourier approach, a time step of 0.00005 seconds was used. Both Fourier and modal approaches used a maximum of 20 half wavelengths through the height. For the modal approach 400 cross section modes were used. The results fro the Fourier and modal approaches are shown in figures 3 and 4 respectively.

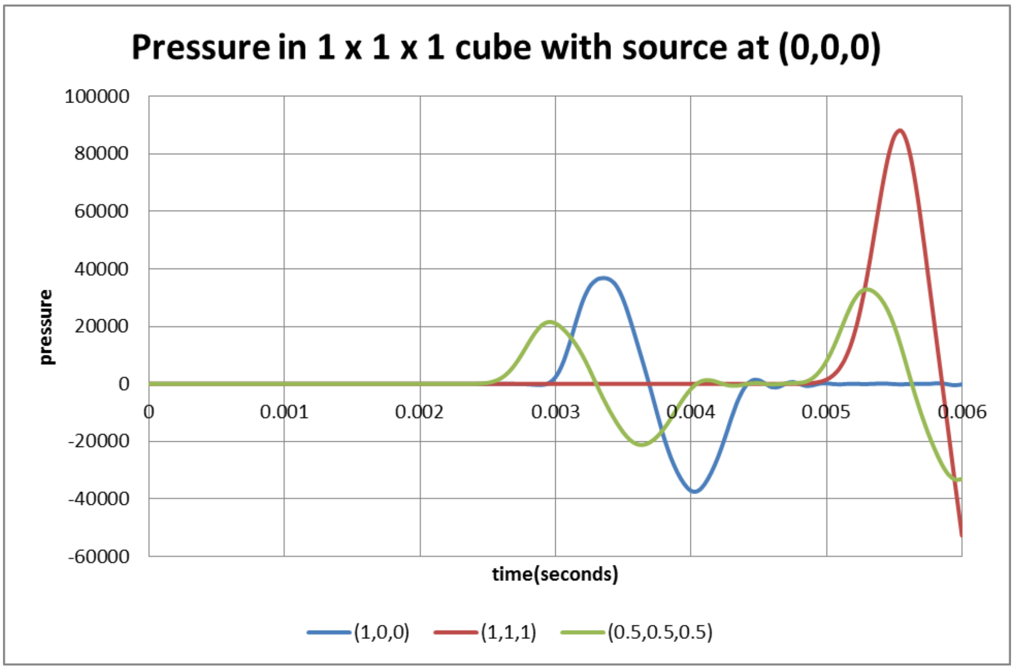


Figure 3 2.5D Fourier method results for cube

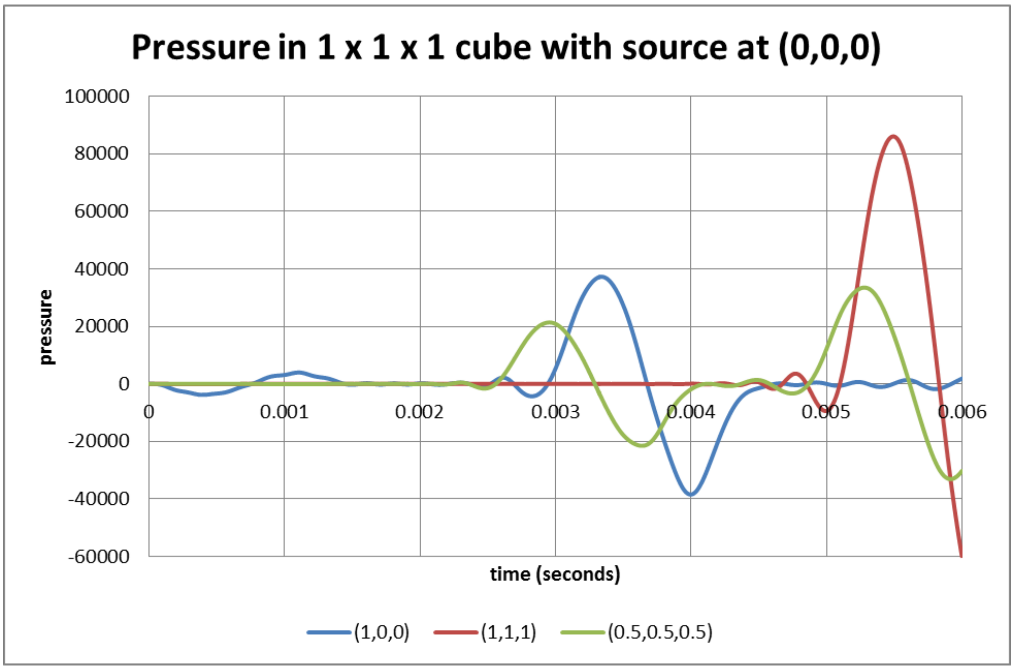


Figure 4 2.5D modal method results for cube

The Fourier 2.5D results are in excellent agreement with the analytical solution. Ther modal result at (1,0,0) has some initial spurious oscillations, at low amplitude, which need further investigation, but the modal results generally are good.

5 IRREGULAR CROSS SECTION EXAMPLE

A transient acoustic analysis was performed for a more irregular-shaped room. The cross section is shown in figure 5. The height used was 2.94m and the source was taken to be a distance 0.2m from neighbouring walls and floor. The pulse duratiomn was 0.006 seconds. The receiver point was taken at ceiling height in the corner. Comparison is made with a 3D finite element model, using a direct time marching scheme. The finite element model, shown in fugure 6 comprises of 21252 quadratic elements and 90275 nodes. 2.5D results, using both Fourier and modal approaches, were computed using the cross section mesh in figure 7, comprising of 5901 linear triangle and 3067 nodes, and taking a maximum of 30 half wavelengths through the height. The three sets of results are shown in figure 8. The agreement is good, particularly at early times. The two 2.5D approaches agree more closely with each other than with the 3D model. This is to be expected since they use the same mesh.

The 2.5D approaches were several orders of magnitude faster than the 3D computation.

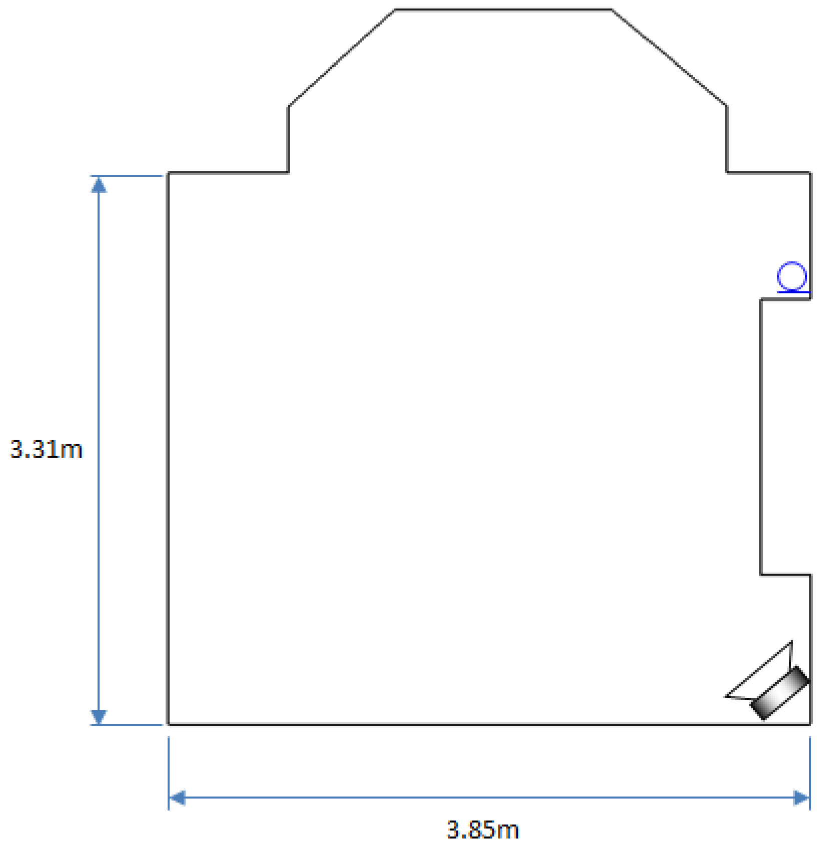


Figure 5 Cross section of room

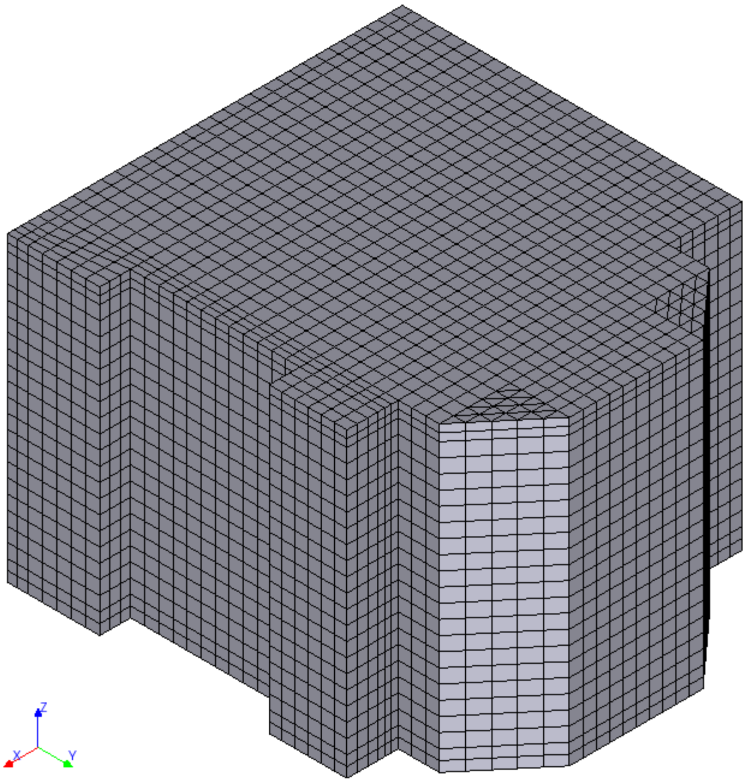


Figure 6 3D FE model of room

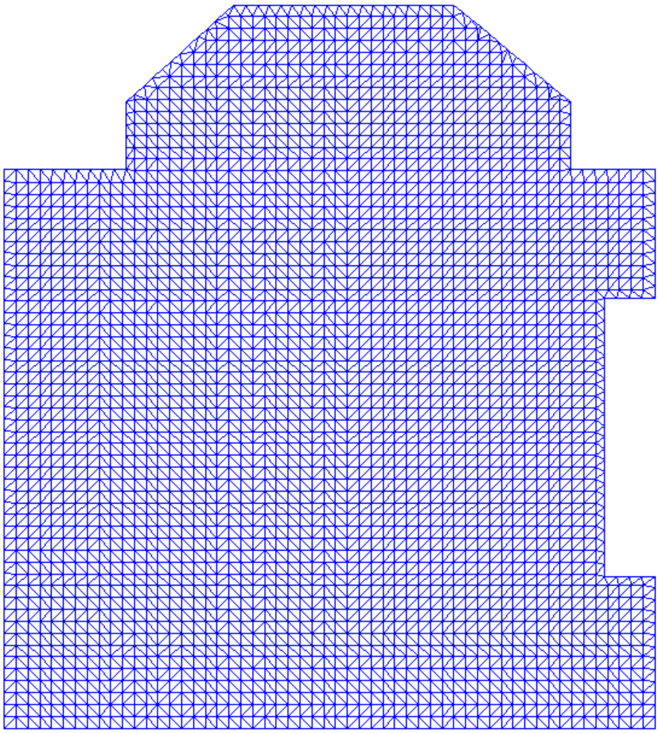


Figure 7 Cross section mesh for irregular room

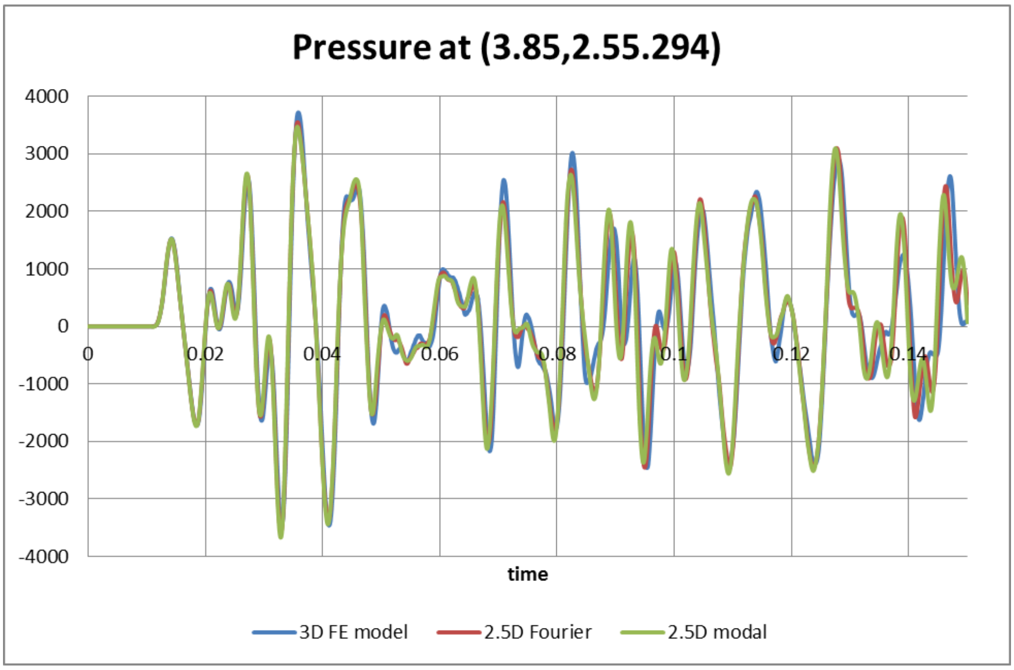


Figure 8 Comparison 3D FE and 2.5D results

6 ROOMS OF ALMOST CONSTANT HEIGHT

In practice many rooms are not precisely of constant height. To investigate the effect of minor deviations from the constant height assumption, for the irregular room, the 3D model was reanalysed, but with a reduction of 0.15m at the front as shown in figure 9. The com-parison of results is in figure 10. At early times the responsesd are identical. There is a tendency for the discrepancy to increase as time progresses, but there are some well matched peaks at quite late time. These may relate to mul;tiple reflections off surfaces in the section containing the source, where the height is constant.

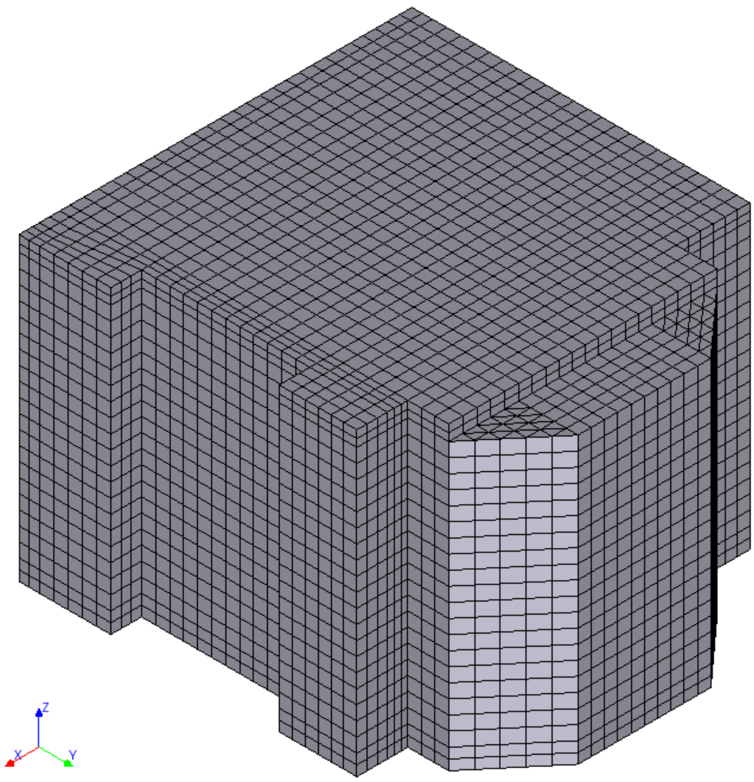


Figure 9 3D FE model with variation in height

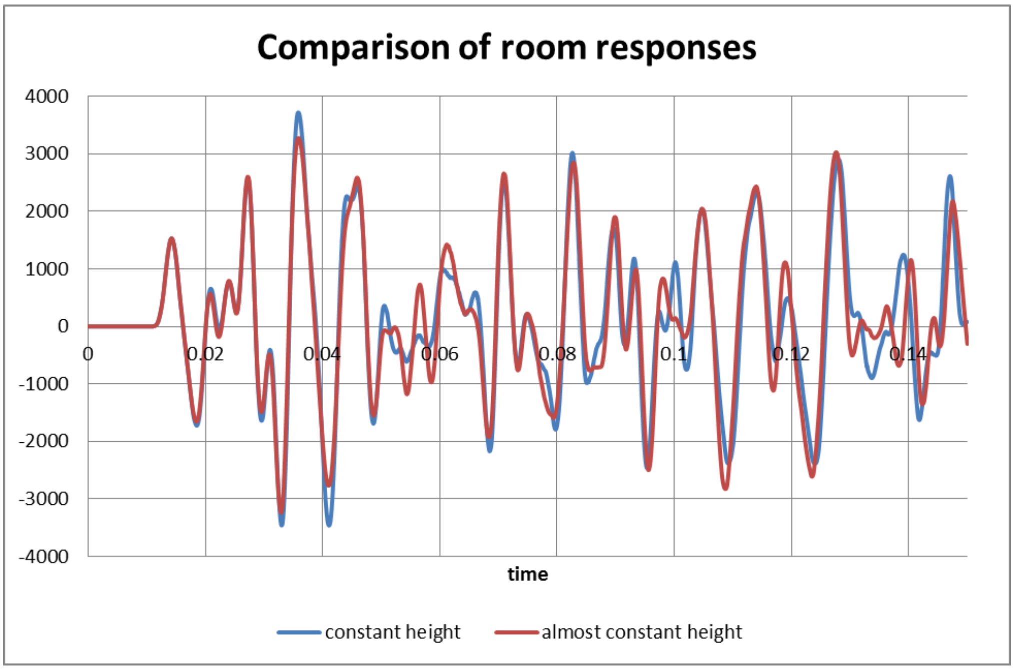


Figure 10 minor deviation from “constant height”

7 AURALISATION

7.1 Auralisation of Output from 2.5D Analysis

Auralisation enables acoustic engineers to make a more direct assessment of aspects of sound quality than visually observing features on a frequency response graph. It has been a feature of numerous room simulation products based on ray tracing but is seen to a lesser extent in FEM/ BEM predictions. A probable reason for this is because FEM/ BEM approaches become increasingly expensive as frequencies are increased in a large problem domain which has, in a room acoustics context relegated the method to low frequency analysis only. Using the 2.5D approach, these problems are largely overcome making it possible to return solutions extending much higher in frequency thus permitting a more complete audible assessment of room/ system designs.

The results presented below are based on an inverse Fourier transform of frequency domain results, as in section 2.3. A modal damping coefficient of 0.02 was used. Further work will consider the use of direct transient analysis results for auralisation.

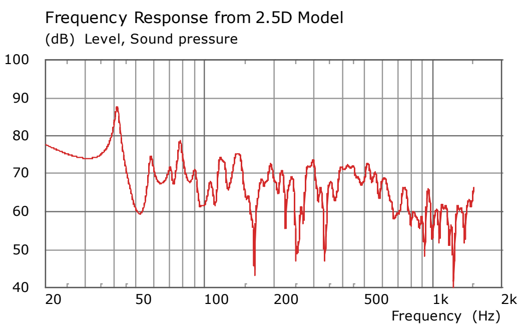


Figure 11

- Inverse Fourier Transform

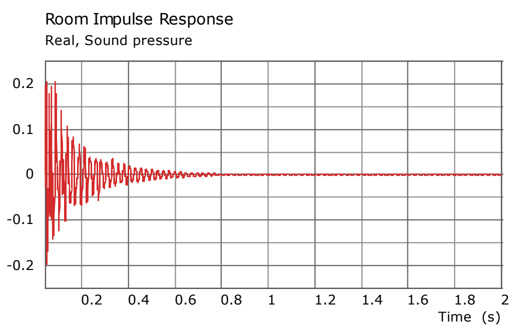


Figure 12

- Normalising and Resampling (for .WAV conversion)
- Extraction of data from Audio Excerpt (.WAV format)
- Convolution of Room Impulse Response with Audio Excerpt
- Export of result in .WAV Format ready for playback

7.2 Comparison with Measured Room Responses

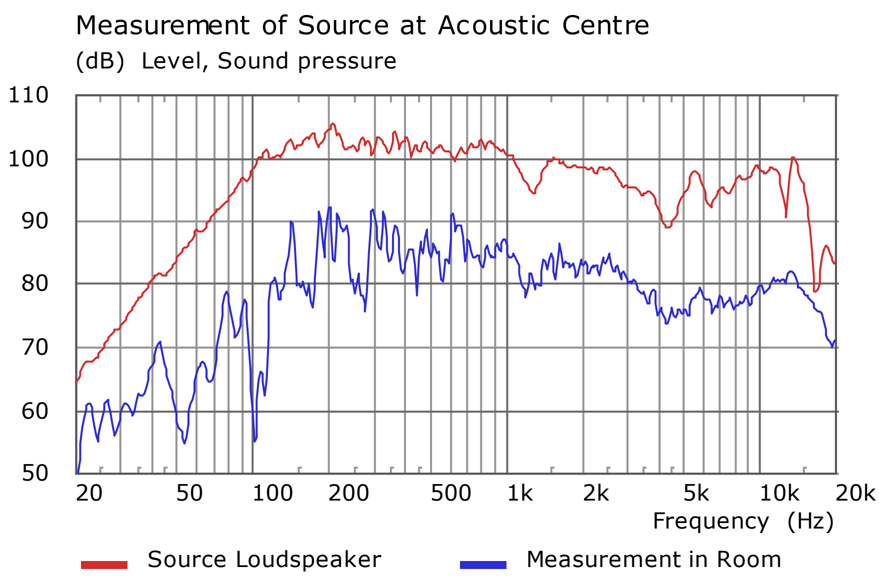


Figure 13 – Source Determination and Room Measurement

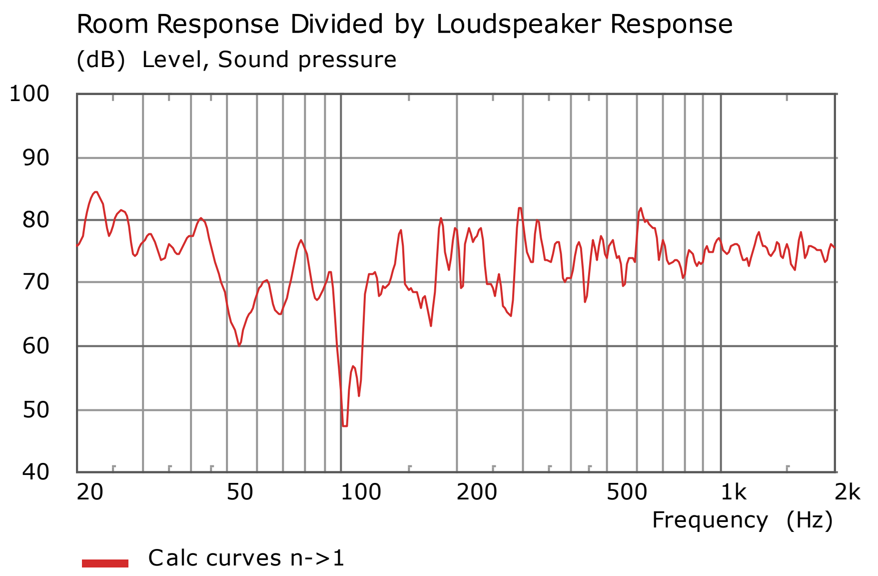


Figure 14 – Room Response with Loudspeaker Response Removed

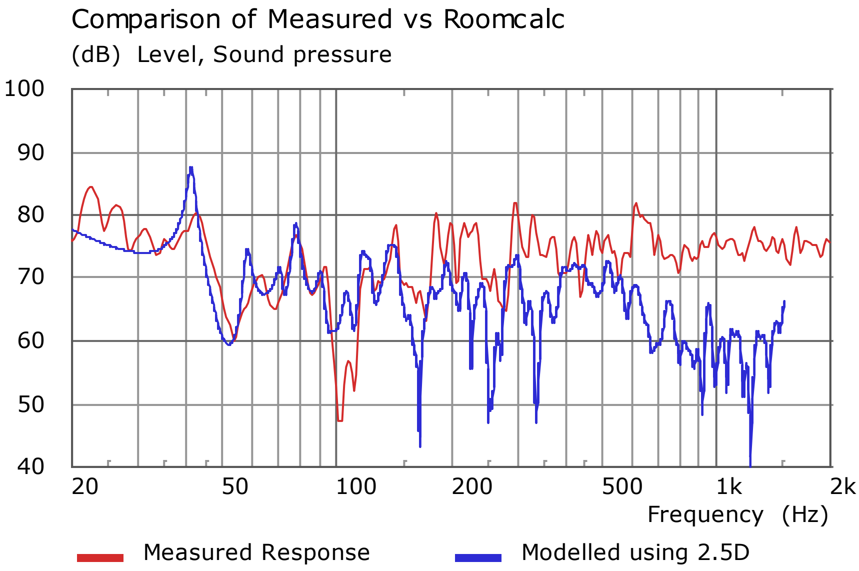


Figure 15

One source of mismatch between the synthesised and measured data is that the source is non-ideal. A small 2 way loudspeaker was chosen based on what was available. A positive aspect of the choice was that it was relatively small and of sealed box design which simplified matters. The bass extension was limited meaning that background noise would be an inevitable consequence at the lowest frequencies which is evidenced by spurious peaks in the response between 20 and 30Hz.

Despite efforts to locate its acoustic centre at low frequency in a succession of measurements around the cabinet (which was determined to be 70mm in front of the bass driver), the loudspeaker used in the measurements clearly does not behave as a simple monopole source throughout the full frequency range of interest and the comparison between experimental and simulated scenarios can only be considered suitable below ~500Hz in the case presented. The comparisons in the appendix further illustrate the divergence between the experimental and simulated data suspected to be a combination of near-field measurement of the 2 way loudspeaker in front of the bass driver, slight deviations in the room geometry (not exactly constant in cross section) and the result of increasing directivity in the loudspeaker used for measurements.

7.3 Areas of Applicability

Automobile audio system design would be an ideal application of auralisation using the 2.5D method. The problem domain is limited in size and therefore solutions can be returned that extend to higher frequencies at lower computational cost. Phenomena such as diffraction are automatically considered which would account for the presence of objects in the vicinity of the listener (seats, head rests etc.). A limitation would be in establishing the axis in which there are negligible variations in the cross sectional detail.

Critical listening spaces where a specific listening position is defined may benefit from this approach and the ability to place absorbent layers in the space may help designers converge more rapidly on room layouts.

8 FURTHER WORK/CONCLUSIONS

An efficient method for the computation of the frequency response of a room and its subsequent application to auralisation was described. The work has shown an application of the 2.5D approach in a small room reaching frequencies where conventional 3D finite or boundary element methods would become prohibitively expensive. The 2.5D method provides a useful development tool for designers who may require numerous runs in order to converge on a design and expect solutions to become available rapidly.

A further refinement would be to produce auralisations based upon the output of transient 2.5D analysis.

The aural assessment of rooms where only part of the spectral content is available is not ideal and an area of further study might be to combine the results from the lower frequency range with a higher frequency analysis, perhaps from a ray-tracing model. Taking this approach would preserve the predictive aspect of simulation driven design whilst enabling the design to select the most efficient method.

Although the results presented here are based on a single receiver point and are therefore mono, the integration of a matrix of these responses with a sophisticated playback system such as that described by Blauert[3] is entirely possible. As a predictive counterpart to measurements that might normally be performed using a mannequin in an existing space, there is scope to supply a database of room impulse responses of a nonexistent room in an array corresponding to small head rotations. A model of the human head and ear detail could also be applied as a post processing step (head related transfer functions). These may in turn be used in a head tracked playback system addressing motional aspects of spatial hearing.

Despite being able to define numerous sources, currently the only source type that can be specified is an ideal point source. There are few (if any) loudspeakers that radiate sound in such a way beyond the lowest frequencies and a future enhancement could be to make the sources more realistic in the model by including specific frequency response and directivity descriptors.

9 REFERENCES

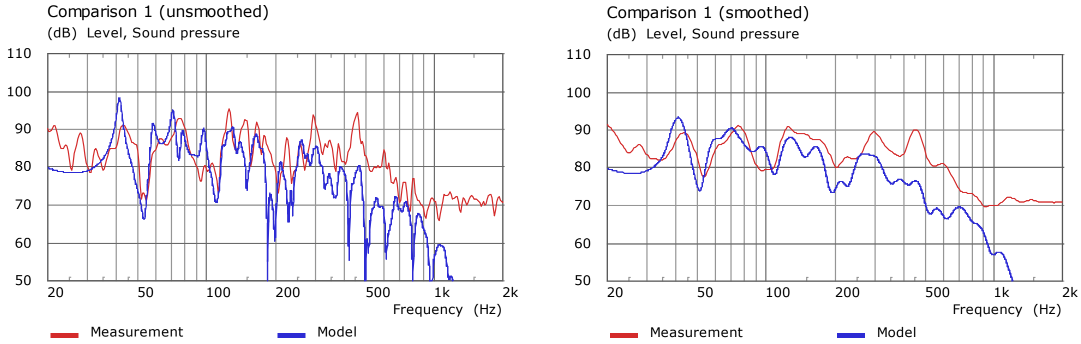
1. S. Marburg and B. Nolte, Computational Acoustics of Noise Porpagation in Fluids, Springer (2008) ISBN 978-3-540-77447-1.
2. P.C. Macey and K.F. Griffiths, Room acoustic analysis using a 2.5 dimensional approach, Proc. IoA, Vol. 33, Pt 6 (Brighton 2011).
3. J.Blauert, Spatial Hearing, Revised Edition, MIT Press, (1997) ISBN 0-262-02413-6

10 APPENDICES

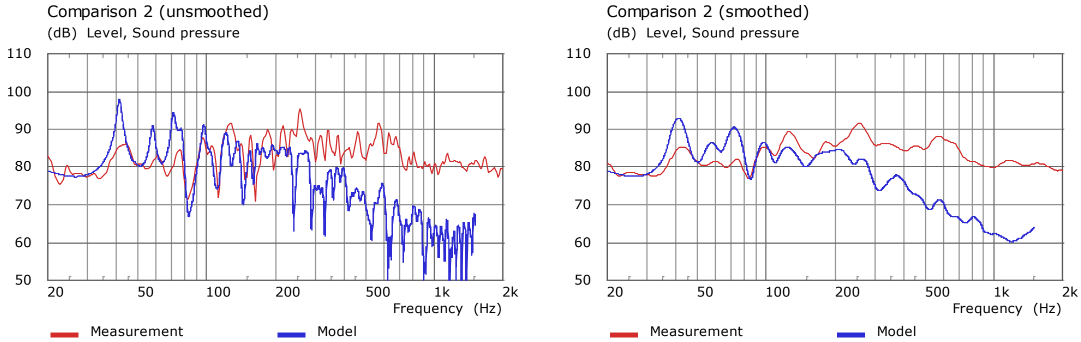
10.1 Comparisons of Modelled and Measured Room Frequency Responses (unsmoothed and 3rd octave smoothed)

The comparisons below illustrate what happens when the source and receiver are moved to different locations in the room.

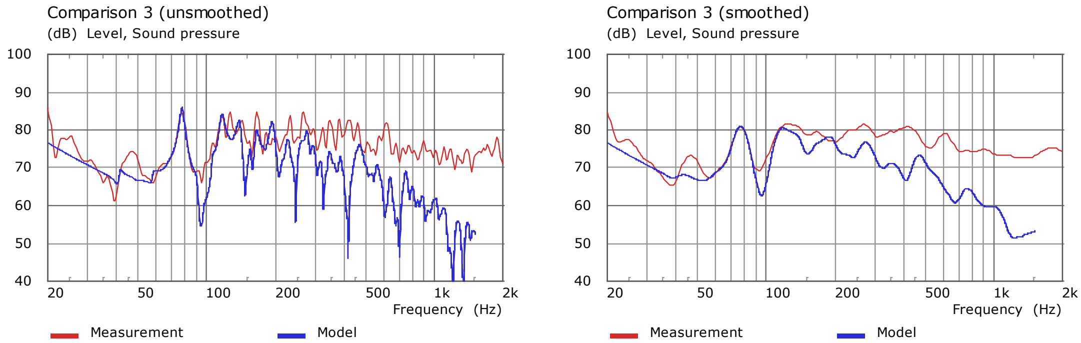
(a)



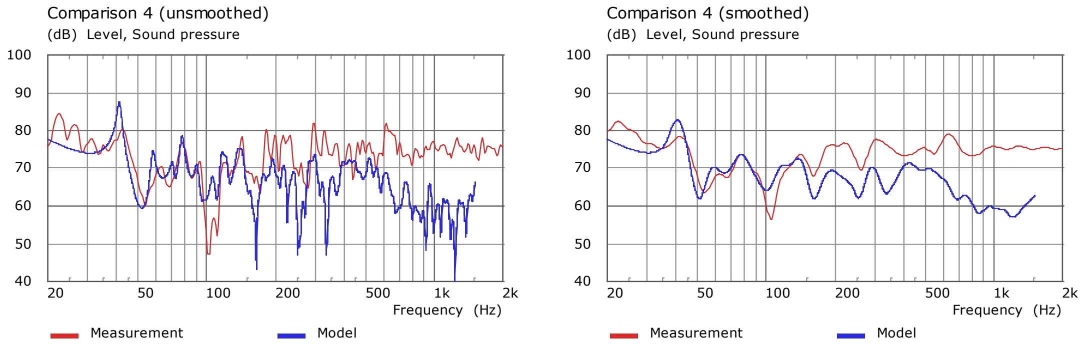
(b)



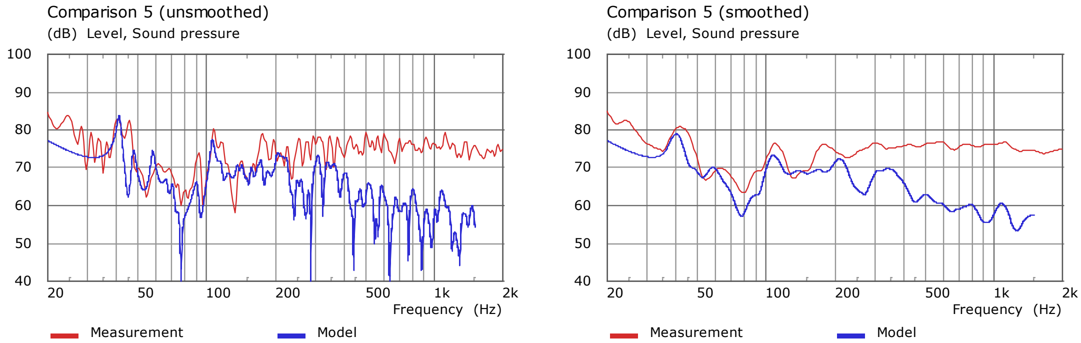
(c)



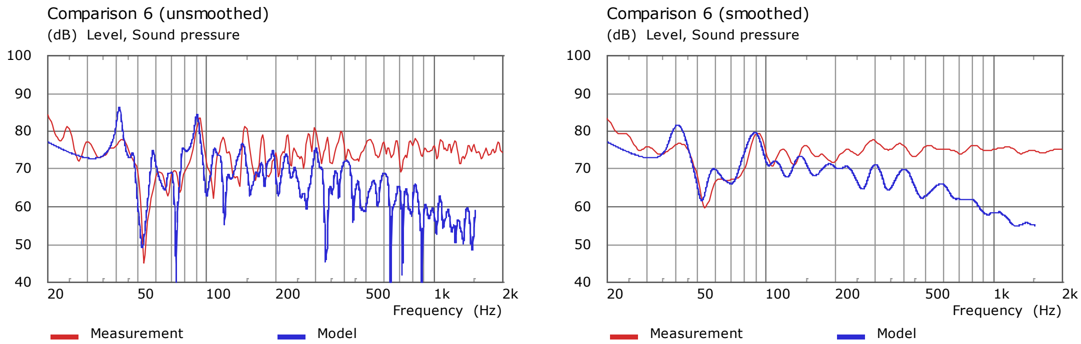
(d)


Comparison 4 (unsmoothed)
(dB) Level, Sound pressure
100
90
80
70
60
50
40
20
50
100
200
500
1k
2k
Frequency (Hz)
Measurement
Model
Comparison 4 (smoothed)
(dB) Level, Sound pressure
Frequency (Hz)
Measurement
Model

(e)


Comparison 5 (unsmoothed)
(dB) Level, Sound pressure
100
90
80
70
60
50
40
20
50
100
200
500
1k
2k
Frequency (Hz)
Measurement
Model
Comparison 5 (smoothed)
(dB) Level, Sound pressure
Frequency (Hz)
Measurement
Model

(f)


Comparison 6 (unsmoothed)
(dB) Level, Sound pressure
100
90
80
70
60
50
40
20
50
100
200
500
1k
2k
Frequency (Hz)
Measurement
Model
Comparison 6 (smoothed)
(dB) Level, Sound pressure
Frequency (Hz)
Measurement
Model