PORT / STARBOARD DISCRIMINATION WITH TOWED ARRAYS

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1. INTRODUCTION

The advantages of towed arrays are well known, and with the low operating frequencies currently envisaged for both active and passive sonars they constitute almost the only option for the receiving array. Towed arrays, however, suffer from one major drawback: a simple line array is one dimensional; so it cannot distinguish between targets on either side of the towing direction. The only way to resolve this ambiguity is to carry out a manoeuvre that will affect targets to port and targets to starboard in different ways.

There are a number of potential solutions: the hardware options include hydrophones with intrinsic directivity, directional transmitters, the use of two or more transversally separated hydrophones per receiving element, and the use of two parallel arrays. Then, for each hardware option there are several feasible signal conditioning and processing choices, the selection depending partly on whether the requirement is for independent port and starboard beamformer outputs or simply a port/starboard target indication.

All these approaches rely fundamentally on the ability to measure a phase or time differential over a small fraction of a wavelength and are thus very sensitive to tolerances and errors throughout the system. Many of the errors may be minimised by use of appropriate technology, but the precision achievable in a practical system still represents a limit on the lowest frequency at which port/starboard discrimination can be achieved.

In what follows a number of these options will be compared, with attention being concentrated on solutions based on a single receiving array - directional transmitters, multiple arrays and systems achieving discrimination outside the array will not be considered.

2. SINGLE TRANSDUCER TECHNIQUES

Two single transducer techniques have been considered. The first borrows from microphone technology to provide a single transducer with intrinsic directivity and the second uses synthetic aperture methods to obtain directivity from a single transducer that is in motion.

2.1 Transducers with Intrinsic Directivity

The general principle of an electro-acoustic transducer having intrinsic directivity was first proposed by Bauer [1] who established the relationships between geometrical and physical parameters required to provide the desired behaviour. Since then microphones have been produced providing various directivity patterns (omnidirectional, cardioid, figure-of-eight, etc.) from a single sensor. This is done using acoustical elements such as holes (inductance), cavities (capacitance) and grids (resistance) to provide filter networks controlling the pressure seen by either side of a diaphragm. Studio quality microphones based on these ideas can achieve >20 dB front-to-back ratios from 100 Hz to 10 kHz. In principle the techniques can be applied to hydrophones, although the transduction mechanism must have a higher impedance for use in water than the moving coil and electret capacitor elements usually used in microphones. A prototype cardioid hydrophone was described by Marciniak [2] in 1971, and more recent developments have resulted in an operational device with outer

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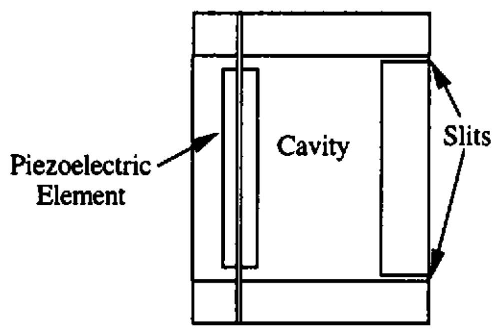

Fig. 1 Sketch of cardioid hydrophone (after [3]).

dimensions of about 3 cm and displaying a front-to-back rejection ratio of more than 12 dB in the frequency range from 200 Hz to 20 kHz [3].

Such a hydrophone is shown schematically in Figure 1, and is made up of a bilaminar or trilaminar piezoelectric element open to acoustic pressure on one face and to the same pressure suitably delayed by an acoustical filter network on the other face. The filter is realised by mounting the active element on an oil-filled cavity that includes a thin slit to allow pressure transfer between the surrounding water and the oil. The oil is retained by a rubber membrane. In equivalent circuit terms the cavity acts as a capacitor and the slit as an inductor and resistor. Fine 'tuning' of the parameters results in an exact cardioid pattern at only one frequency, but judicious adjustment can result in a large bandwidth with good front-to-back rejection ratio over most of the band.

This device seems to be eminently suitable for port/starboard ambiguity resolution - it can produce a good cardioid over a wide frequency range, and the simple construction indicates reliability and low production costs. There are, however, a number of drawbacks. The device as developed so far is not pressure compensated, so depth of operation would be limited. The performance is also very dependent upon the temperature of the oil in the cavity. Further research could overcome these problems but, more importantly, the cardioid pattern is fixed relative to the axis of the device, so some means of orientation would be required in an array. A vertical ring of such hydrophones is feasible, with some means of switching in the two most nearly horizontal and pointing port and starboard This, however, would result in a large array element. The hydrophone described here has outside dimensions of about 3 cm and, because both the sensitivity and the low frequency cut-off depend largely on the size, significant reductions are unlikely. Even if the size can be tolerated, the fixed directivity pattern would still provide optimum discrimination only for broadside signals, with performance falling off as the array is steered towards end-fire.

2.2. A Synthetic Aperture Approach

Anticipating the discussion of multiple sensor solutions (Section 3), it may be accepted that a unidirectional response can be obtained by the combination of the outputs from two or more (possibly omnidirectional) sensors with suitable phasing. Such techniques are commonplace, for example, in radio direction finders or 'coincident' stereo microphones. Success, however, is strongly dependent upon the two elements being well matched in both amplitude and phase responses, and this presents problems when using practical hydrophones.

This limitation might well be overcome if the signals from a single omnidirectional sensor, observed at different times and at different locations, could be combined in a suitable manner. This is effectively a synthetic aperture, and would require movement of the hydrophone between observations - in particular, it would require transverse motion to obtain port/starboard discrimination in a line array. To date, such an approach does not seem to have been applied to forming a cardioid, but a dipole (figure of eight) directivity pattern has been obtained [4].

This approach seems attractive at first sight; it removes errors due to differences between sensors, and it removes the need for multiple hydrophones along with the associated multiple switching and signal conditioning channels. There are, however, drawbacks. Transverse motion of the towed array is implied, and it must be measured with a precision of, perhaps, 1% of a wavelength. At a frequency of 100 Hz this is 15 cm, reducing to

1.5 cm at 1 kHz, and represents a very small displacement in an array that may be several hundred metres long. The other major drawback is the nature of sonar signals. The combination of temporally displaced signals in this way implies a high degree of temporal coherence. Active sonar signals are subject to fluctuations in the medium and relative motion between the sonar and target. Overall, this approach does not seem to represent a feasible solution.

3. MULTIPLE TRANSDUCER TECHNIQUES

This concept is straightforward, and may be considered from two points of view. If only an indication of whether a signal is coming from port or starboard is required then, given two spatially separated receivers aligned transverse to the line of the array, the wavefront arrives at the one nearest the source first. The phase or time delay between the transducer outputs can be measured and the leading side gives the direction of the target. If, on the other hand, two output channels are required, one carrying signals arriving from the port side and the other from the starboard side, the two transducer outputs can be summed with suitable phasing to cancel the signal from the unwanted direction.

Although the idea is simple, application is more difficult. Firstly, the array is free to roll, so the two transducers cannot be assumed to lie horizontally, and their alignment cannot be known without instrumentation to sense the array orientation. Secondly, the space available within a typical towed array means that the transducers can only be separated by a few centimetres. A generous spacing of, say, 10 cm represents a delay of 67 μs. This is 0.067 wavelengths at 1 kHz, or 24° of phase, and only 2.4° of phase at 100 Hz. Errors due to mismatch between the transducers, their positions, or the associated amplifiers and filters must be smaller than these values.

3.1. Direction 'Flag' Methods.

In principle there are many methods that might be used to measure the delay between two signals. Timing between zero crossings of the two waveforms is notoriously susceptible to noise, and may be ruled out. Cross-correlation is limited by the width of the correlation peak. Some more promising techniques may be briefly discussed, along with their accuracy in the presence of noise. Other errors, for example those arising from the uncertainty in transducer positioning, will be considered later in Section 3.2.1, because they are common to both these direction 'flag' methods and to methods giving separate port and starboard signal outputs.

3.1.1. <u>Integrated Zero-Crossing Detection.</u> A refinement of the zero-crossing detection technique is shown schematically in Figure 2. The input waveforms are squared by the input comparators (ie 1 bit sampling) and the following AND gate produces pulses that are high only when both signals are high. The width of these pulses is

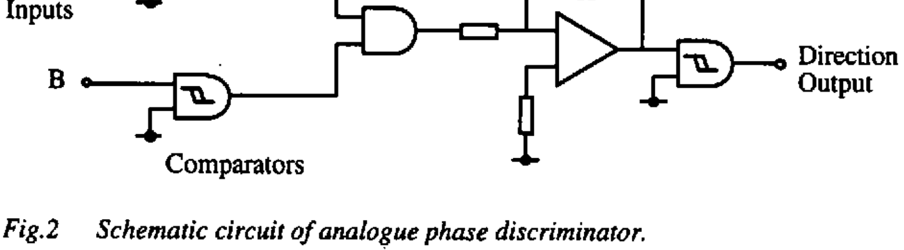

Fig.2 Schematic circuit of analogue phase discriminator.

the time lag; they are integrated to reduce the effects of noise, and produce a running average of phase difference that is passed to another comparator to give a port-starboard indication. Unfortunately, the technique is confused near zero delay, and this is overcome by inverting one signal so that a 50% duty cycle represents zero. This approach is primarily suited to analogue implementation, but is then subject to the drifts and tolerances of such circuitry. One advantage is that it is relatively immune to both amplitude and frequency variations.

3.1.2. Synchronised Sampling. It may be shown that if the received signals are sampled at a rate synchronised with the transmission so there are precisely four samples per period, then if these signals have amplitudes A and B and a phase difference $\Delta\phi$, multiplying the two signals and averaging over a time short compared with phase difference variations yields

$$\langle AB \rangle \cos(\Delta\phi)/2 \tag{1}$$

along with terms that are negligible if the averaging is over several periods. The angle brackets represent averaging. Delaying one signal by one sample (1/4 period) and duplicating the multiply and average process gives

$$\langle AB \rangle \sin(\Delta\phi)/2 \tag{2}$$

Obviously $\tan(\Delta\phi)$ and hence $\Delta\phi$ can now be obtained. For port/starboard discrimination it is only necessary to know whether $\Delta\phi$ is positive or negative, and this can be found from the sign of $\tan(\Delta\phi)$ or the signs of both (1) and (2) above. This is a simple processing technique that is again immune to amplitude variations, but only works with CW signals, and would be degraded by Doppler shifts.

3.1.3. Spectrum Calculation A method involving slightly more computation is the calculation of ratio, F_{AB}, of the discrete Fourier transforms (DFTs) of the two signals at a single frequency, f, as follows:

$$F_{AB} = \sum_{n=1}^{N} W_n S_A \exp(-i2\pi f t_n) \Big/ \sum_{n=1}^{N} W_n S_B \exp(-i2\pi f t_n) \tag{3}$$

where S_A and S_B are the two signals, Wn is a suitable window function, and $t_n = n\Delta_t$, the time corresponding to the nth sample. Phase is obtained from the arctangent of the ratio of the imaginary and real parts but, once again, it is only necessary to know their signs to obtain a port/starboard indication. This method can give a reliable result from only two or three cycles and, because it is based on a ratio, many errors cancel. Because of this, it is not necessary to know f precisely, and Doppler shifts and chirps can be handled.

3.1.4. Noise Performance All these methods can give arbitrarily accurate results with 'clean' signals, but are degraded by noise. An estimate of the signal to noise ratio (SNR) required for unambiguous port/starboard indication may be obtained from examination of the phasor diagram sketched in Figure 3. Two signals are shown, S_A and S_B. Noise phasors, N, are added and, because the noise has random phase, these can lie anywhere within the circles shown.

Although in isotropic ambient noise, the noise at two closely spaced sensors would be highly correlated, this cannot be guaranteed, but it can be said with confidence that a reliable estimate of which is the leading signal can be obtained if the two circles do not overlap. Thus, the minimum SNR is given by

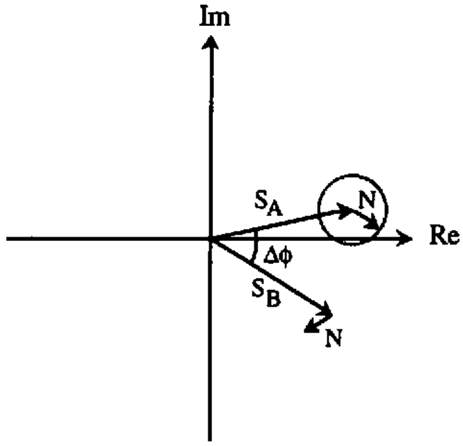

Fig.3 Phasor diagram showing the effect of noise on phase measurement.

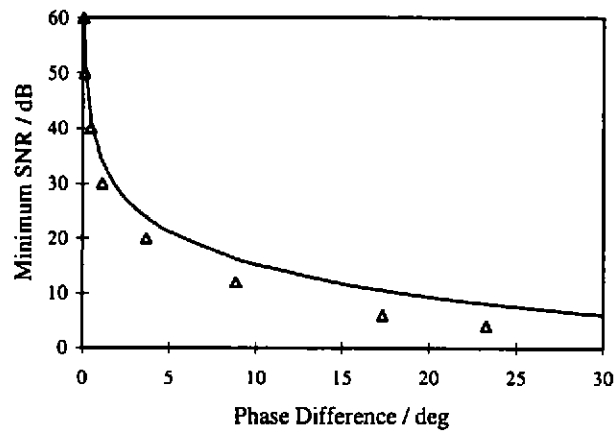

Fig.4 Minimum SNR for reliable phase discrimination from Eq.(4) (solid line) and from simulation (symbols).

$$SNR_{min} = -20 \log[\sin(\Delta\phi/2)] \tag{4}$$

Minimum *SNR* from (4) is plotted against the phase difference $\Delta\phi$ in Figure 4. Also shown are results obtained from a simulation in which artificial signals were generated with a known phase difference and added broadband noise. The phase difference was estimated using (3), and the symbols show the *SNR* at which the maximum error found in a large number of runs was equal to the difference. It is apparent that (4) is slightly pessimistic, but gives a good indication of the noise requirements - 20dB *SNR* allows reliable discrimination with a 5° phase difference and 10dB is adequate with 20°.

This performance is of course degraded by differences in the phase responses of the two transducers and the associated amplifiers and filters, as well as by uncertainties in their positioning and orientation. On the other hand, it is improved by the gains associated with an array of several elements and by temporal averaging. These factors will be discussed in Sections 3.2.1 and 3.2.2.

3.2 Two Transducer 'Cardioids'

The outputs from two or more sensors can be combined to provide various directivity patterns, and such techniques are common in radio antenna and stereo microphone technologies. In the present case it is required to produce two outputs, one of which rejects signals arriving from the port direction, and the other from starboard. This can be achieved with the arrangement shown schematically in Figure 5.

The geometry is shown in Figure 5(A). For each array element two transducers, *A* and *B*, are located transverse to the line of the array, separated by a distance *d* and nominally horizontal. In practice, this may be attained in various ways but a straightforward technique is to arrange several transducers around a circle and to use a gravity sensing switch to select the two that are nearest to the horizontal. Whatever approach is adopted, the transducers will be offset from horizontal by a roll angle *f*, and this angle may not be known but is limited. The target bearing is θ, relative to the tow ship's heading.

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The transducer signals are combined in a network like that shown in Figure 5(B). The output from each transducer is subtracted from a delayed version of the other to provide port and starboard outputs. Assuming an incident plane wave, the transducer outputs are, at *A*:

$$V_A = S\, e^{-j[\omega t - (kd/2)\sin\theta\cos\phi]} \qquad (5)$$

and at *B*:

$$V_B = S\, e^{-j[\omega t + (kd/2)\sin\theta\cos\phi]} \qquad (6)$$

where *S* is a constant encompassing the strength of the incident signal and the transducer sensitivity, $\omega = 2\pi f$, and $k = 2\pi f/c$, *f* being frequency and *c* sound speed. The port output, neglecting the constant *S*, is then

$$V_{port} = e^{-j[\omega t - (kd/2)\sin\theta\cos\phi]} - e^{-j[\omega t + (kd/2)\sin\theta\cos\phi - \omega\tau]} \qquad (7)$$

where τ is the delay time. It is apparent from (7) that signals from the starboard side are rejected when $\tau = (d/c)\sin\theta\cos\phi$, and in particular, for $\theta = 0°$ (broadside) and $\phi = 0°$ (zero roll), when $\tau = d/c$. If τ is fixed at d/c, and the roll is zero, the port directivity pattern is as shown in Figure 6(A). This is a true cardioid with a maximum at port broadside and a null at starboard broadside. Port/starboard discrimination is complete for broadside signals, and reduces towards endfire. For any other condition (ie. $\tau \neq d/c$ or $\phi \neq 0$) a modified cardioid is produced, as seen in Figure 5(B), with nulls in some other direction. With no roll, this direction is obtained from $\tau = (d/c)\sin\theta$.

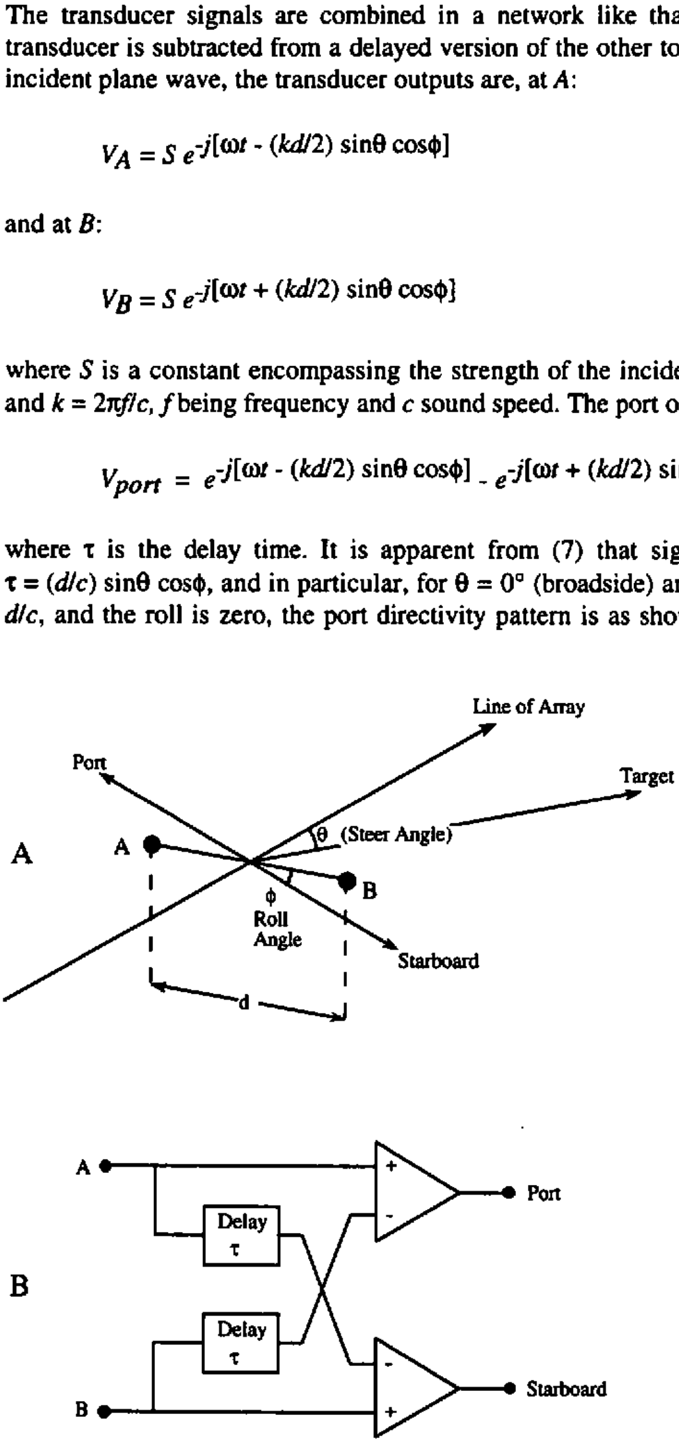

Fig.5 Two transducer cardioid arrangement showing (A) geometry and (B) electrical network.

It is noted here that, for a towed array with a number of steered beams, because of the rotational symmetry of the directivity pattern about the array axis, the rejection is required in the 'complementary' direction shown in Figure 7. This is best achieved by varying the delay time τ with the beam steer angle to direct nulls in complementary direction. Setting $\tau = (d/c) \sin\theta$ with zero roll produces a pattern as shown in Figure 8(A), where it is seen that the rejection to starboard is complete at all steer angles, although the port output falls away within 20-30° of endfire. This would be overcome in practice by a compromise that restrained the delay to minimum in these steer directions. The effect of a constant delay error is shown in Figure

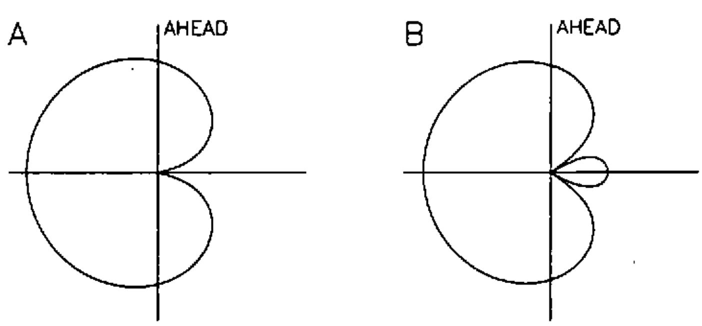

Fig.6 Directivity patterns for port output with arrangement shown in Figure 5 for (A) zero roll and $\tau = d/c$ (true cardioid), and (B) a finite roll angle (modified cardioid).

8(B), where it is seen that the broadside rejection is degraded in the same way as in Figure 5(B).

These examples show that, in principle, it is possible to obtain port/starboard discrimination limited only by ambient noise with a two transducer cardioid system, but the effects of errors, along with frequency-dependent variations, must be considered to determine the applicability of this scheme to a low frequency sonar.

3.2.1. <u>Errors and frequency dependence.</u> Early analogue implementations of the network of Figure 5(B) used RC phase shift circuits, and were therefore suitable only for narrow band applications, but using precise time delays implies, in the absence of other errors, that the discrimination performance is frequency independent. However, because this discrimination is obtained by subtracting one signal from another with a fixed time (rather than phase) relationship, the output level does vary with frequency.

Inspection of the geometry and network of Figure 5 shows that the broadside output is at a maximum of 6 dB above the level from a single transducer when the propagation time between sensors *A* and *B* plus the delay τ add up to half the signal period (or an odd multiple), or $f = c/4d$. As the frequency is reduced, this output falls at 6 dB/octave. With a 10 cm spacing the maximum output is at about 3.8 kHz, and it has fallen to 10 dB below a single transducer by 350 Hz.

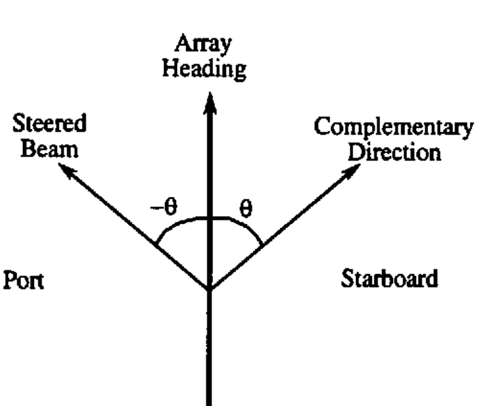

Fig.7 Sketch showing steered beam and complementary direction in relation to array axis.

In order to consider the effects of errors and tolerances, a few simplifying assumptions may be made:

i) With modern electronics, the amplifiers and filters associated with transducers *A* and *B* can be matched to within say 2.5° in phase and 0.1 dB in amplitude. The amplitude tolerance can be ignored in a first approximation, but the phase is significant.

ii) Errors due to the digitising process are negligible, and with appropriate interpolation or other algorithms, the time delay τ can be made arbitrarily precise.

iii) Selection during assembly allows transducers *A* and *B* to be matched to within 5° in phase and 1 dB in amplitude. The amplitude tolerance can be ignored in a first approximation, but the phase is significant.

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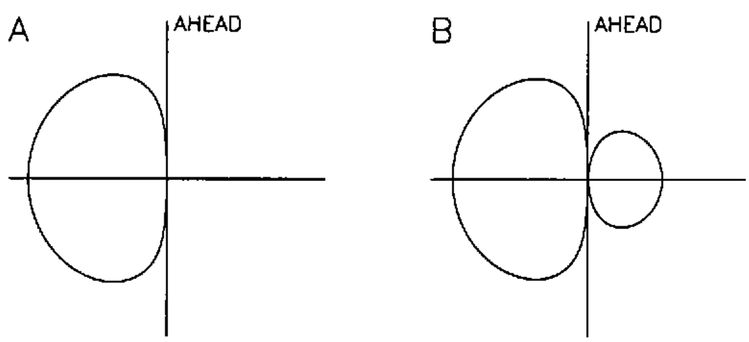

Fig.8 Directivity patterns for port output with arrangement shown in Figure 5 for (A) zero roll and $\tau = d/c \sin\theta$ and (B) a finite roll angle, but delay still varied as $\sin\theta$.

iv) The roll angle is not known, but can be constrained to, say, ±30°, and this also is significant. It has the effect of changing the effective spacing by a factor $\cos\phi$.

The effect of phase errors may be assessed as follows: when two signals of magnitude X, and phase difference ψ are subtracted, the magnitude of the resultant is simply $2X\sin(\psi/2)$. With a phase error ψ and roll angle ϕ, the rejection ratio ρ (or ratio of port to starboard outputs) is given approximately by

$$\rho = \frac{\sin(\omega\tau\cos\phi + \psi/2)}{\sin(\psi/2)} \tag{8}$$

All angles are small, so the approximation $\sin x \approx x$ may be used, and the rejection ratio in dB, R, with y in radians becomes

$$P = 20\log\left[1 + \frac{\omega\tau\cos\phi}{\psi/2}\right] \tag{9}$$

This approximate result shows that discrimination increases with frequency and time delay (which increases with transducer separation but reduces as the array is steered away from broadside) and is reduced by both roll angle and phase errors. Given a minimum rejection ratio and values for worst case roll angle and phase error, a low frequency limit can be determined for any separation and steer angle. If a rejection of 10 dB is acceptable, the low frequency limit f_L is just

$$f_L = \frac{\psi}{2\pi\,\tau\cos\phi} = \frac{\psi}{2\pi\,(d/c)\sin\theta\cos\phi} \tag{10}$$

Figure 11 shows f_L plotted against spacing for a maximum phase error of 7.5°, taken from (i) and (iii) above, but no steering and no roll angle (solid line), with 7.5° phase error and 30° roll (dashed line), and with 7.5° phase, 30° roll, and steered 60° from broadside (dot-dash). With 7.5° phase error, the lower limit is 600 Hz with 5 cm spacing or 300 Hz with 10 cm, and it is seen from (10) that f_L is inversely proportional to spacing. It is also seen from (10) that f_L is directly proportional to the phase error, so halving phase errors can gain an octave. What is also clear is that roll or steer angles up to 30° have little effect, but larger angles do. Combining errors, the limit is 1.4 kHz at 5 cm spacing and 692 Hz at 10 cm.

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In operational systems, a wide range of beam-steer is required, restraining roll to ±30° is straightforward, and space available for transducer separation is limited. The only way to improve the performance is by reducing phase errors. The values quoted here are probably approaching the best available in production systems, but it should be noted that these are for an isolated cardioid element - the gains available from an array of such elements must also be considered.

3.2.2. Array gain. There are two ways that an array of cardioid elements can be combined: The discrimination may be performed within the array element and a single signal from each element fed to a conventional beamformer, or there may be a pair of beamformers, one operating on the '*A*' transducer signals and the other on the '*B*' signals, with the discrimination being performed after beamforming.

In the first case, the result is to average the variations in the individual cardioid directivity patterns, whilst in the second the phase errors are averaged before cardioid formation. The overall effect is similar in either case, and depends upon the error distributions, but generally the phase errors averaged over an array of *N* elements should tend towards zero mean, with a standard deviation $1/\sqrt{N}$ times the standard deviation of the individual errors. The same is true of roll errors, but these must be treated separately because of the $\cos\phi$ dependence.

Generally, the low frequency limit is lowered by a factor of the order of $\sqrt{N}$, and for typical towed arrays this factor will be between 5 and 10.

4. DISCUSSION AND CONCLUSIONS

Various approaches to achieving port/starboard discrimination in line arrays have been discussed, with an emphasis on operation at low frequencies. Some of the options considered are obviously non-starters, but a few seem to be capable of achieving the desired discrimination, and the most appropriate choice would depend upon the detailed requirements of a specific application.

A single-element cardioid hydrophone looks attractive at first sight. Further development is required to produce units capable of operating at depth, but the main drawback is the fixed directivity pattern. For applications not requiring beam-steering the cardioid hydrophone may be an option, but in the more general case where full azimuth coverage is needed, adequate discrimination may not be possible beyond about 30° from broadside.

Two transducer systems giving a simple port/starboard indication seem capable of robust performance with *SNR*'s of 10 dB - subject to phase error limitations similar to those discussed in 3.2.1, but with an

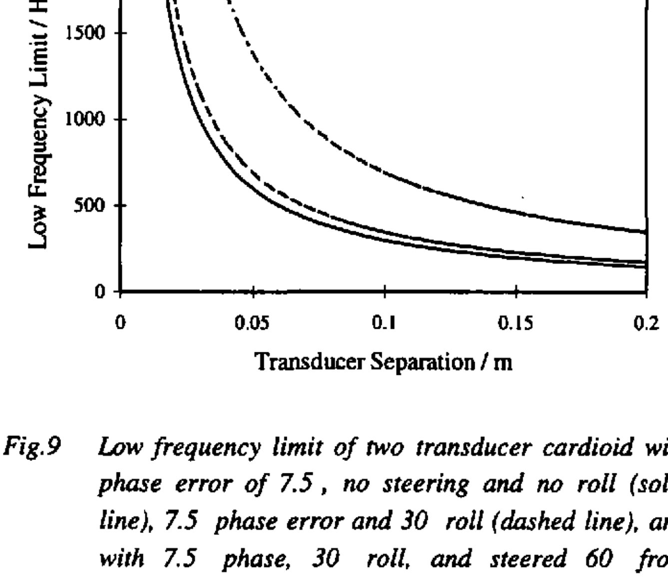

Fig.9 Low frequency limit of two transducer cardioid with phase error of 7.5 , no steering and no roll (solid line), 7.5 phase error and 30 roll (dashed line), and with 7.5 phase, 30 roll, and steered 60 from broadside (dot-dash).

improvement in both noise and error performance of order $\sqrt{N}$ in an N element array. The advantage of such an approach is that it separates the detection and discrimination processes, so detection is not limited by the 6 dB/octave low frequency roll-off associated with the two transducer cardioid. The disadvantage is that it can only give reliable discrimination on one target per beam - it would not function well in a cluttered scenario.

The two transducer cardioid is probably the most generally applicable approach. With realisable phase error constraints, a single element is in principle capable of 10 dB port/starboard discrimination, over steer angles up to 60° from broadside, down to about 1.4 kHz with 5 cm transducer spacing. The averaging effect of an array would in most cases lower this limit to below 500 Hz.

5. REFERENCES

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