## FINITE ELEMENT MODELLING OF OPTICAL FIBRE HYDROPHONES

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#### 1. INTRODUCTION

Optical fibre hydrophones have been developed in recent years as an alternative to piezoelectric devices for use in a wide variety of sonar systems. The Defence Research Agency (DRA) has been involved in the development of an optical hydrophone system based on reflectometric time-division multiplexed interferometry [1]. Optical fibre technology offers a number of advantages compared with conventional techniques, including the electrically passive nature of the hydrophone, and the ability to operate large numbers of hydrophones without the need for underwater multiplexing electronics. A large part of the work in this area has been aimed at the development of optical system architectures, but the design of the hydrophones themselves has also been addressed. Previous work carried out by GEC-Marconi under DRA funding has included the construction and testing of a number of experimental hydrophone designs, together with the development of an analytical model for prediction of acoustic sensitivity for several simple hydrophone configurations [2]. This work showed that good agreement between modelling and experiment could be achieved for these simple designs, but the model used was restricted because it was valid only for completely symmetric circular designs and at acoustic wavelengths much greater than the hydrophone dimensions. This paper describes the development of a new modelling approach based on the use of finite element techniques. A finite element model offers much greater flexibility than the analytical model, and can be applied to a wide range of different hydrophone designs and frequencies. Although the full potential of the approach has not yet been exploited, this paper describes its application to the modelling of simple cylindrical hydrophones, and shows that excellent agreement with experiment is observed.

### 2. OPTICAL FIBRE HYDROPHONE OPERATING PRINCIPLES

Although a number of optical hydrophone techniques have been suggested employing a variety of transduction mechanisms, this paper concentrates on an interferometric technique which is regarded as being generally the most sensitive. This technique is based on the conversion of the acoustic signal into a phase change in light travelling through an optical fibre. In its simplest form, the hydrophone consists of a coil of optical fibre (typically 100m in length) wound onto a mandrel and potted in an encapsulation material such as epoxy resin or polyurethane. An acoustic signal acting on the coil can be regarded as causing a sinusoidal change in the pressure field around the hydrophone. This change in pressure leads to a corresponding change in the strain within the fibre. The hydrophone can therefore be considered to be acting as a strain gauge. If light is passed through the coil, the strain in the fibre can effect the light in two ways. Firstly, there is a change in the physical path length of the light through the fibre, which is determined by the longitudinal strain in the fibre. Secondly, the additional strain causes a change in the refractive index of the glass (an effect known as the elasto-optic effect) which changes the speed of light in the fibre. The change in refractive index is related to the various components of the strain through the

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elasto-optic tensor. The total phase change in a fibre of length L is then given by

$$\Delta \Phi = knLs - \frac{1}{2}n^3kL\left(s_1p_{11} + s_2p_{12} + s_3p_{21}\right).....(1)$$

where n = refractive index of glass, k is the optical wavenumber,  $s_x$  etc. are the components of the strain and  $p_{xy}$  are the elasto-optic or Pockel's coefficients which relate the strain to the change in the refractive index.

Here, the first term is the length change term and the second represents the elasto-optic effect. In many hydrophone designs, it is found that the two terms have opposing sign, and that the length change term is the largest. The elasto-optic term therefore has the effect of tending to reduce the hydrophone sensitivity.

In general when subjected to an acoustic field, there will be a pressure gradient across the hydrophone coil. However, at low frequencies where the acoustic wavelength is large compared to the coil diameter, the pressure field will be uniform across the coil, and the pressure can be considered to be hydrostatic (this also implies that the mass of the hydrophone is negligible compared to the mass of the driving fluid). In this simplest, hydrostatic, case, the pressure field will therefore act on all surfaces of the encapsulated coil i.e. the curved outer surface, the flat end surfaces and the inner surface (in the case of a hollow mandrel). The pressure acting on the inner and end surfaces will tend to oppose that acting on the outer surface and hence reduce the hydrophone sensitivity. The sensitivity of the hydrophone can therefore be increased by removing pressure from the inner and end surfaces. This may be achieved using endcaps, together with an air-filled central cavity.

### 3. ANALYTICAL MODEL

The analytical model described in 1 was based on calculating the radial, axial, and hoop stresses and strains produced by a hydrostatic pressure acting on a three-layered cylinder (Fig 1). The three layers of the cylinder represented respectively the mandrel, the fibre/encapsulant composite and the outer encapsulant layer. The strains calculated at the position in the cylinder corresponding to the centre of the fibre coil were then assumed to be transmitted to the fibre, and the hydrophone sensitivity was calculated by inserting the relevant strains in eqn. 1. The model was designed so that the hydrostatic pressure could be applied to any combination of the inside, outside and end surfaces of the hydrophone, or to all three. This allows hydrophones with various types of endcaps to be simulated. The model made the following assumptions:

- a. The pressure field was constant across the hydrophone (i.e. the hydrostatic approximation);
- b. The cylinder was of infinite length so that end effects could be neglected, apart from the imposition of pressure-stress boundary conditions at the ends;
- c. The fibre/encapsulant composite was an isotropic material with mechanical properties given by the arithmetic mean of the mechanical properties of the fibre and encapsulant;

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d. The strains at the centre of the fibre layer were uniform throughout the length of the fibre.

These assumptions limit the overall effectiveness of the model. For certain hydrophone configurations (in particular, long thin cylinders with a thin fibre layer) the approximations are good and accurate results are obtained. However, for very short hydrophones, the accuracy of the model is reduced because the end effects become significant. The accuracy is also reduced in designs where the fibre layer is spread through a significant portion of the radius of the cylinder, so that the variation in strain through the fibre layer is significant. More seriously, the model cannot be applied to more complex hydrophone designs, and is only valid at low frequencies (for typical hydrophone dimensions, it is valid below approximately 5 kHz). It is therefore impossible to use the model to predict hydrophone frequency responses.

In [2], the results of the model were compared with experiment. Hydrophone sensitivity was calculated for a test hydrophone design with a variety of central cavity sizes, and the results compared with experiment. The design of the test hydrophone is shown in fig. 2. It consists of a simple cylindrical design, with a length of 5cm and a diameter of 7 cm. The encapsulation material is an araldite epoxy. The central cavity was air-filled, the air being retained by flexible membranes at each end of the hydrophone. The model predicted that the presence of the air cavity would increase the sensitivity of the hydrophone by increasing the collapse of the coil for a given acoustic pressure on the outer surface of the coil. As the size of the central cavity increased, this enhancement would have a steadily larger effect, so a continuous increase in sensitivity should be observed. The model results were found to be within 2 dB of the experimental results, showing that the model is accurate even for the test design, which was a fairly stubby cylinder. The results showed that the analytical model is a very useful design tool, despite the limitations in its applicability which are described above.

### 4. THE DRA MODEL

As the optical hydrophone development programme has progressed, it has become clear that a more flexible and powerful model is required to deal with the greater variety and complexity of hydrophone designs. A new model has therefore been developed within DRA based on a different modelling approach. This model has the following features:

- a. A finite element representation of the hydrophone which is used to calculate dynamic stresses and strains within the structure. This is based on the PAFEC Level 7.4 F.E. modelling package [4];
- b. A program to calculate the mean strains within the fibre layer, taking into account the variation in strain through the radius of the hydrophone and along its length. The program then inserts the mean strains into eqn. 1 to calculate the hydrophone sensitivity.

This model has the scope to cover all hydrophone configurations, including non-symmetric geometries, and to take account of the anisotropy of the fibre/encapsulant layers. It can also take account of a complete range of loading conditions, and can be used to predict hydrophone frequency responses. However, as a first stage, the model has been restricted to axisymmetric cylinders with isotropic materials in each layer, subjected to hydrostatic pressure fields. This is equivalent to the system modelled using the analytic approach. The purpose of the first stage of the modelling is to establish that accurate hydrophone sensitivities can be calculated for practical hydrophone configurations, using F.E. based methods.

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Even for this specific case, use of a F.E. based model has a number of advantages. Firstly, the model provides an accurate calculation of the radial, axial and hoop strains at any point in the hydrophone structure, taking into account the finite length of the cylinder and therefore any end effects. Secondly, the calculation of the mean fibre strain allows an accurate determination of the total strain in the fibre even in those cases where there is a large variation in strain through the fibre layer, which is not uncommon. The DRA model, is therefore potentially a highly accurate modelling technique, with the main limitation in the accuracy arising due to uncertainty in measurements of material properties. In addition, a F.E. model gives detailed information about the distribution of stress and strain throughout the structure which is valuable in understanding the operation of the hydrophone and in optimising designs. This information is not available from the previous model.

An F.E. mesh for a typical hydrophone geometry is shown in Fig. 3. The mesh used represents a longitudinal section through the hydrophone. Although finer meshes were used, they did not lead to significantly higher accuracy. The F.E. calculation requires values of Young's modulus E and Poisson's ratio v for each of the three layers of the cylinder. The second layer is effectively a three phase composite, consisting of the glass fibre, its plastic coating and the surrounding encapsulant. Representative values of E and v longitudinal and transverse to the direction of the fibre were calculated for this layer, according to [3], using arithmetic means calculated from the volume fractions of the three phases. However, the PAFEC model does allow actual values for E and v in each direction to be used [4].

### 5. COMPARISON OF RESULTS

The performance of the F.E. model can be compared with that of the analytical model using the results contained in Fig. 4 of [2]. This plots experimental and theoretical sensitivities for a test hydrophone (shown in Fig. 1) as a function of central cavity size. This is a well characterised hydrophone design for which the properties of the materials in the various layers are well established. In this section, we use the DRA model to generate sensitivities for these test hydrophone configurations, and compare them with the figures quoted in [2]. We also discuss how the output of the DRA model gives some useful insights into the mode of operation of these hydrophones.

The sensitivity of the hydrophones as calculated using the DRA model is shown in Fig. 4, together with the earlier predictions and experimental results. It can be seen that the agreement between all 3 sets of results is good. The errors involved with the earlier modelling are of the order of +/- 2dBs, while the experimental measurements, are thought to have an error of around +/- 1 dB. The uncertainty in the new predictions is again estimated at +/- 1 dB, arising due to uncertainties in the values for the relevant material parameters. It is therefore apparent that all 3 sets of results agree within the specified error limits, implying that for this specific case, the DRA model performs at least as well as the analytic model.

The agreement between the analytical and F.E. models seems to indicate that end effects in this hydrophone design are not significant. This is confirmed by Tables 1 below and 2, which shows the three strain components at the end surface of one of the test hydrophones (with 3 cavity sizes), and also the calculated mean strain within the fibre layer, after integration by the second part of the program. The radial and hoop components of the strain are very similar in the two cases, indicating that end effects are not significant for these strain components. In the case of the axial strain, some contribution due to end effects is apparent, particularly with the 30mm cavity diameter. However, although the difference in the strain is around 80% in this particular case, the actual effect on the sensitivity is quite small because the axial strain makes a much smaller contribution to the sensitivity than does the hoop strain. This does

confirm the validity of the analytic model for many practical situations.

Table 1: Strains at centre of fibre layer

Central cavity diameter/mm	Axial	Radial	Ноор
10	-6.71 x10-5	+7.24 x 10-6	-1.12 x 10-4
30	-4.02 x 10-5	+4.54 x 10-5	-2.06 x 10-4
60	+1.27 x 10-4	+2.38 x 10-4	-7.10 x 10-4

Table 2: Corrected weighted mean strains in fibre layer

Central cavity diameter	Axial	Radial	Ноор
10	-5.59 x10-5	+8.57 x 10-6	-1.21 x 10-4
30	-2.83 x 10-5	+4.80 x 10-5	-2.19 x 10-4
60	+1.28 x 10-4	+2.39 x 10-4	-7.25 x 10-4

The test hydrophone design used for these comparisons is a typical optical hydrophone configuration, and it is instructive to consider the mode of operation of this hydrophone. This is shown in fig.5, where the deformation of the mesh is shown for the case of the 10mm central cavity and the 60 mm cavity. Because of the way in which the model scales the displacements for display, the magnitudes cannot be compared directly, but the displacements give a good indication of the way in which the hydrophone deforms The deformation plotted is for a positive pressure change In both cases, compression of the coil is evident. In the 10mm case, the compression is greater at the end of the coil than at the centre, so that some end effect is evident, although it is quite small. In the axial direction, the axial compression (due to the pressure acting on the ends of the coil) is greater in the encapsulant region than in the coil region, due to the stiffening effects of the coil. For the 60mm hole, the radial compression is very uniform along the length of the coil, so that end effects are very small. This is probably because the compressive force due to the pressure gradient between the outside and inside of the hydrophone is so large that all other effects are overwhelmed. The radial compression is also uniform, because the area of the coil is a large proportion of the hydrophopne area, compared to the case of the 10mm cavity, and so the stiffening effect of the coil extends throughout the area of the hydrophone.

#### 6. FUTURE WORK

The results described in Section 5 show that the for the limited range of hydrophone designs considered, the new DRA model is at least as good as the analytical model. For the next stage of work, it is intended to use the model to look at hydrophone designs which cannot be modelled by the original analytical

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model. These will include hydrophones with multiple coils, non-cylindrical outer surfaces and more complex endcap arrangements. The results obtained will be compared with experiment to determine the validity of the model. If the validity of the model under these conditions is proved, it will be used to examine a wide variety of practical hydrophone designs.

A number of other refinements to the model will also be addressed. Of particular interest is the effect of including the non-isotropy of the fibre layer in the sensitivity calculation. The accuracy of results obtained so far suggests that this will cause only a small correction to the sensitivity, but this will be checked by comparing results calculated using both methods.

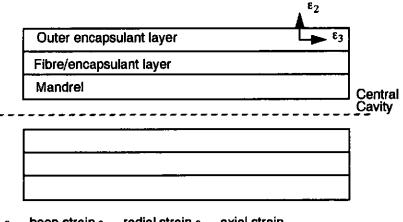
In addition, the model will be used in the dynamic strain mode to consider higher frequencies where the acoustic wavelength is comparable to the hydrophone dimensions, and so the pressure field is non-uniform. This will allow the prediction of frequency responses, which is an essential part of hydrophone design. This dynamic model will also be able to include static loading due to ambient pressures, therefore allowing the modelling of hydrophone performance at various water depths.

#### 7. CONCLUSIONS

A new model has been described for the prediction of optical hydrophone sensitivities. The model, which is based on the use of finite element techniques, shows good agreement with results obtained from experiment and from a previous, analytical, model. However, the model is potentially much more powerful than the previous model, and is expected to be used as a tool for the design of a wide variety of practical optical hydrophones. The model also gives valuable insights into the physical operation of optical hydrophones.

#### 8. REFERENCES

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 $\epsilon_1$  = hoop strain  $\epsilon_2$  = radial strain  $\epsilon_3$  = axial strain  $\epsilon_1$  acts into paper

Figure 1: Longitudinal cross-section through 3 layer hydrophone

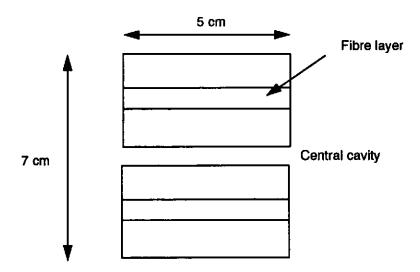


Figure 2: Cross-section through test hydrophone

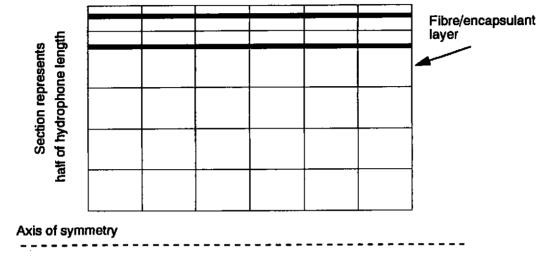
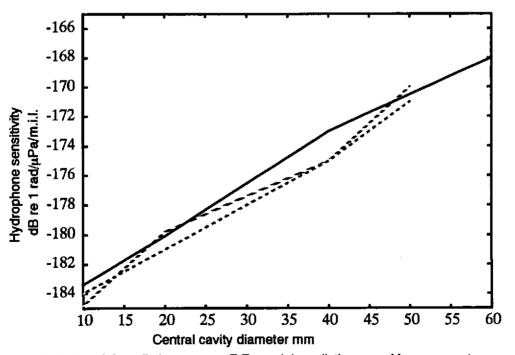
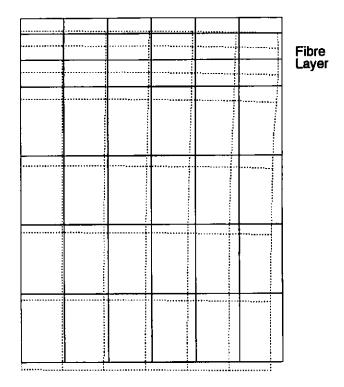


Figure 3: Mesh for typical hydrophone



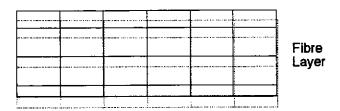
---- Analytical model prediction — F.E. model prediction -- Measurement Figure 4: Comparison of predicted and measured hydrophone sensitivities (from [2])

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Axis of symmetry

Figure 5: Mesh showing hydrophone deformation for a. 10 mm central cavity and b. 60 mm central cavity



Axis of symmetry