CHAOTIC SIGNALS IN A MULTIPATH CHANNEL

P.R. Atkins[1], A.J. Fenwick[2]

[1] Department of Electronic, Electrical & Computer Engineering, University of Birmingham, Edgbaston, Birmingham, UK, B15 2TT p.r.atkins@bham.ac.uk

[2] UPS, QinetiQ, Winfrith Technology Centre, Dorchester, UK DT2 8XJ AJFENWICK@qinetiq.com

1. INTRODUCTION

The concept of characterising systems using drive signals derived from the apparently random behaviour that can be generated by simple non-linear oscillators has been known about for some time. These signals have close analogies with the so-called chaotic behaviour that has been found to occur in fields as diverse as biological neural networks, control systems and population dynamics. In underwater acoustic applications, ray propagation in media with sloping boundaries is chaotic. Likewise, turbulence and large amplitude water waves can also be wholly or partly explained using non-linear models.

Chaotic signals, that is functions whose time variation is governed by a non-linear system, are used in covert communication systems. Because they behave like noise, they may be used in place of conventional maximal-length sequences to generate pseudo-random signals for radar and sonar transmitters. As shown in [1,2], they may be transmitted continuously (see also [3] for more recent work in radar) and so give continuous unambiguous range coverage. They have a thumb-tack ambiguity function. There is a need to establish the technological limitations of such signals through measurements at sea.

An opportunity to take some preliminary measurements was provided when the University of Birmingham was invited to take part in a series of trials funded by the US Navy, entitled 'SignalEx-E Experiment' near Ship Island, Mississippi Sound, Gulf of Mexico 24-25 October 2001. Although primarily aimed at underwater digital communications developments, the concept of including signals with higher dimensionality characteristics for both channel-probing and communication developments (RAKE) were deemed to be compatible with the aims of the trial. Of particular interest was the requirement to transmit multiple, low-bandwidth signals through a quasi-stationary channel using conventional, power-efficient, analogue communication techniques. A set of periodic and chaotic solutions of the Duffing equation, one of the standard non-linear systems, were used to modulate a number of carriers and transmitted over a multipath channel. This paper presents a sub-set of the results obtained from the analysis of the recorded data and gives an assessment of the difficulties likely to arise in recovering information from the received signal. The overall aim was to examine alternative techniques for automatic channel probing. The requirements of such a system would include multi-static and forward scattering characterisation as well as the higher-order nature of the channel (dynamic non-linear behaviour).

In the next section, a channel model is presented, then in section 3, the chaotic signals used and the transmit setup are discussed. A practical means of testing for non-linearity in a channel using bistatic soundings is demonstrated in section 4. With this type of system, performance measures such as signal-to-noise ratio are unsatisfactory. Alternative quality measures are given in section 5.

2. CHANNEL PROPERTIES

The experiment used a transmitter and a receiver that were allowed to drift apart. The separation started at 1 km and opened-out to 10 km over a period of 3 hours, corresponding to an average velocity of about 0.9 metres per second. The depth of water was of the order of 20 m. A number of users were allocated one-minute time slots for their experiments, the transmissions being repeated at about twenty-minute intervals. A channel sounding chirp preceded each one-minute transmission.

The received pressure would generally be expected to be characterised by an exponential decay law:-

$$p = \frac{\hat{P}}{r^n} \exp(-\alpha r) \exp[j(\omega t - kr)] \tag{1}$$

where p is the received pressure, $\hat{P}$ is the peak transmitted pressure, r is the operating range, n is the geometrical spreading constant (measured experimentally as approaching unity for very short ranges and 0.7 for the majority of transmissions), α is the absorption constant expressed in nepers per metre and k is the wave number. Thus, as the range separation increases during the course of the experiment, a factor of ten variation in the received pressure in each of the ray-paths would be expected.

The pressure at the receiver, p_{RX}, will be the sum of many individual delayed contributions. Allowing for a non-linear contribution, *f(p,t),* (e.g. scattering from bubble clouds under the influence of surface waves) the received pressure is given by:-

$$p_{RX}(t) = \sum_{i=1}^{N} \beta_i p(t - \tau_i) + f(p,t) \tag{2}$$

Under typical shallow-water conditions, the number of paths, *N*, that may be identified often exceeds thirty. The contribution, β_i, of each of these paths can vary enormously over a few minutes and the direct path is frequently not the strongest. The range of variation of the time delays, τ_i, is usually most extreme at shortest ranges and typically spans 10 ms for the results presented here.

The distortion introduced by such a channel is complex to solve analytically, Panter [4]. However, simple analytic solutions based on a number of equal amplitude arrivals would indicate that a deep, narrow-band fade would be expected with a period of about 13 seconds (based on a 20 m deep channel and a velocity of 0.9ms^{-1}). Similarly, the 10 ms time-spread, multipath arrivals encountered in this channel would result in a 10% distortion of the demodulated signal when using a phase modulated carrier with a bandwidth exceeding 9 Hz (based on pessimistic assumptions).

Traditional propagation modelling approaches have assumed that the channel is linear. This assumption is largely based on the lack of experimental evidence indicating the presence of non-linear artefacts within a channel. Alternatives to the conventional CW and chirp transmission signals are required in order to detect such non-linear behaviour [6].

3. CHAOTIC MODULATION OF A CARRIER

The chaotic signal was generated by a Duffing oscillator which has a cubic term in the restoring force and models the behaviour of a hardening spring. The spring is driven by a cosinusoidal force with constant amplitude and frequency, ω. The behaviour of this system can only be obtained by numerical integration in a process which generates both position and velocity. For particular combinations of the spring constants, driving frequency and amplitude, the oscillation is chaotic, having the following properties:-

the generated signal, x, is aperiodic

the generated signal is sensitive to initial conditions

the curve (x, $\dot{x}$) converges to a strange attractor

the generated signal has a continuous spectrum

The curve given by (x, $\dot{x}$) is known as the phase portrait and is characteristic of the system.

The Duffing system used here is

$$\ddot{x} + 0.06\dot{x} - ax + 0.75x^3 = 0.28\cos(\omega t) \qquad \text{where} \quad \omega = 1.15 \qquad (3)$$

The solution starting from the point x = 0 and $\dot{x}$ = 0 is periodic for *a* = *0.1*, but becomes chaotic before *a* reaches *0.2* and remains chaotic up to at least *a* = *0.6*. This change in the character of the solution is known as a bifurcation.

The peak-to-standard deviation ratio of a transmission signal is of particular importance in battery-operated equipment and impacts on the power consumption, efficiency and physical size of the transmitter. This ratio is minimised (1.414) for a continuous sinusoidal (CW) transmission. Typical channel probing signals based on pseudo random, binary phase shift keyed signals with a fractional bandwidth of 20% and Hann weighted transmission bits, result in peak-to-standard deviation ratios of about 2.4. This ratio increases to about 2.6 for a practical QPSK underwater transmission link. By comparison, the Duffing oscillator with *a* = *0.5* results in a peak-to-standard deviation ratio of about 1.8 - thus providing significant reductions in the weight and size of the transmission unit.

The transmitted signal was formed by summing six groups of four modulated carriers, as shown in figure 1. Each group corresponds to a value of the parameter *a* and an equivalent value of ω obtained by time compression. The time-compression applied to each of the groups is equivalent to scaling the driving frequency in the range $19.1 \le \omega \le 115$ radians per second, thus requiring an increasing separation between carriers as the frequency is increased. Within a group, each carrier frequency corresponds to an amplitude or phase modulation of x or $\dot{x}$. The lowest frequency is 8.3 kHz and the highest 16.4 kHz The spectrum of the combined signal is given in figure 2.

The reasons for choosing the transmission signal are very similar to those for commercial OFDM radio transmissions (e.g. Digital Radio Mondiale). A set of pilot carriers, in this case the carriers associated with the amplitude modulation (AM) signals, are used to provide channel estimators and local phase references. A second set of phase modulated (PM) signals is used to convey the modulation source data from the transmitter to the receiver in a form that is relatively immune to the multipath characteristics of the channel. Frequency diversity is also employed to provide additional robustness for the x and $\dot{x}$ components.

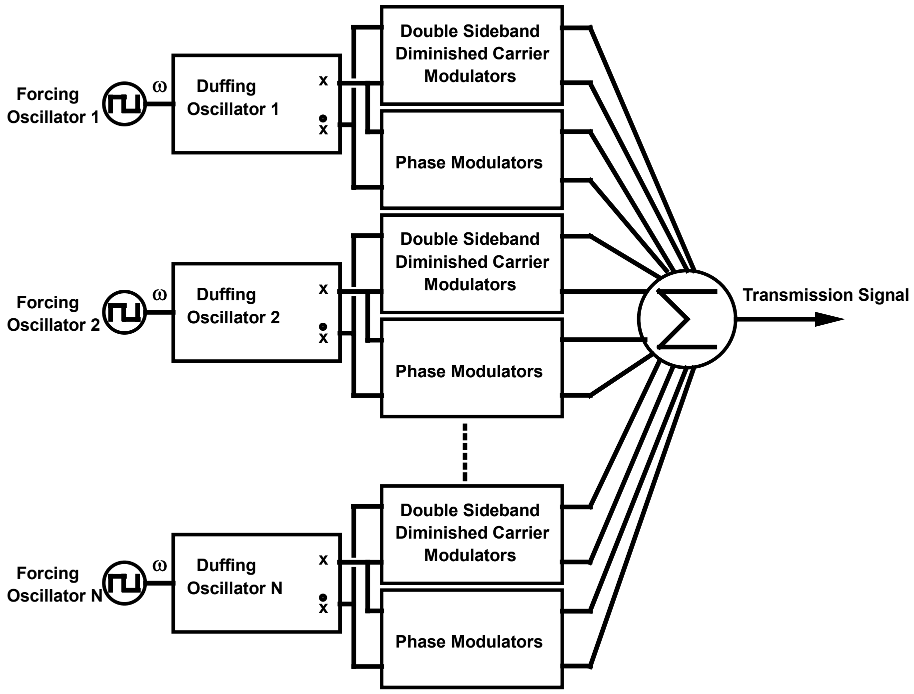

Figure 1: Multi-carrier and multi-modulation transmitter

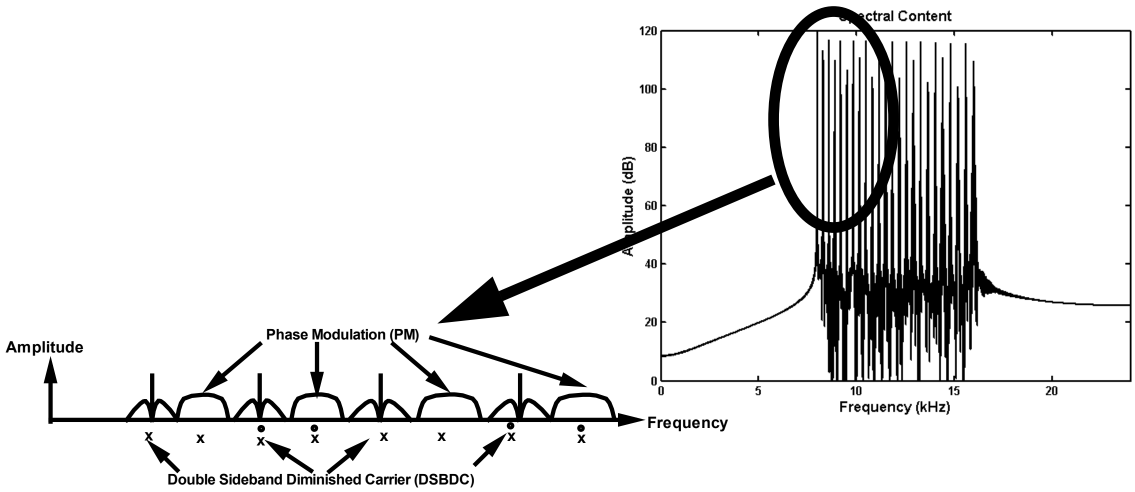

Figure 2: Spectrum of transmitted signal

4. THE USE OF BIFURCATIONS FOR DETECTION

The processing to be discussed is based on the hypothesis that a non-linear channel may be identified by a combination of transmission signals capable of slowly interrogating the state space. The most striking example is if a chaotic transmitted signal is received as a periodic oscillation, as shown using phase portraits (plot of x, $\dot{x}$) in figure 3. This is known as a bifurcation, with the variation parameter related to the observation time. Conversely, since bifurcations of the type considered do not occur in linear systems, their absence is indicative of linear channel behaviour.

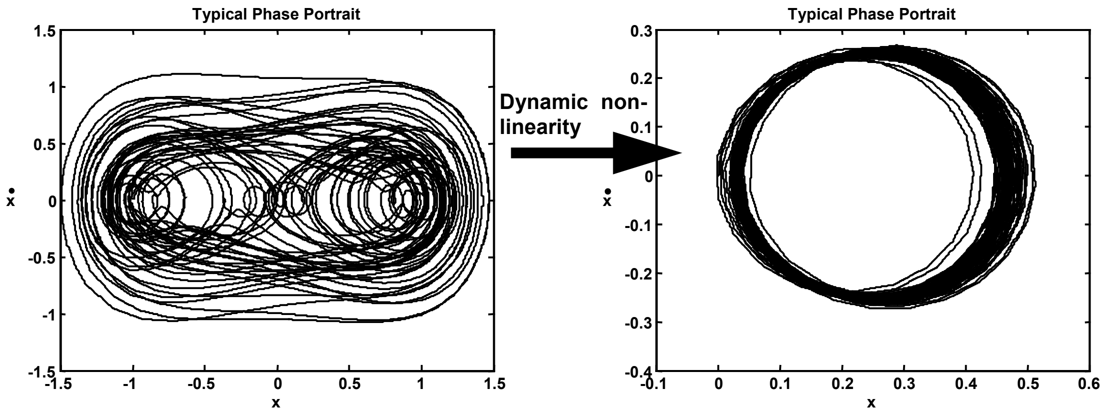

Figure 3 : Extreme example of a bifurcation visualised using phase portraits (plot of x, $\dot{x}$)

Bifurcations are indicated by a change in the phase portrait. A typical bifurcation detector may be implemented by synchronously sampling either the x or $\dot{x}$ components and plotting the sampled values with respect to a system variable, in this case the observation time.

In any practical sonar system, it is difficult to achieve time synchronisation between the transmitter and receiver, especially when the source and receiver are drifting and the transducers are subject to surface wave induced accelerations. For the experiments described here, timing information is conveyed by means of the phase modulated (PM) transmissions. The block diagram of the receiver is as shown in figure 4. The phase modulated signals are also used within the receiver to determine Quality-of-Service metrics. The receiver structure shown in figure 4 is repeated for each of the six Duffing oscillators.

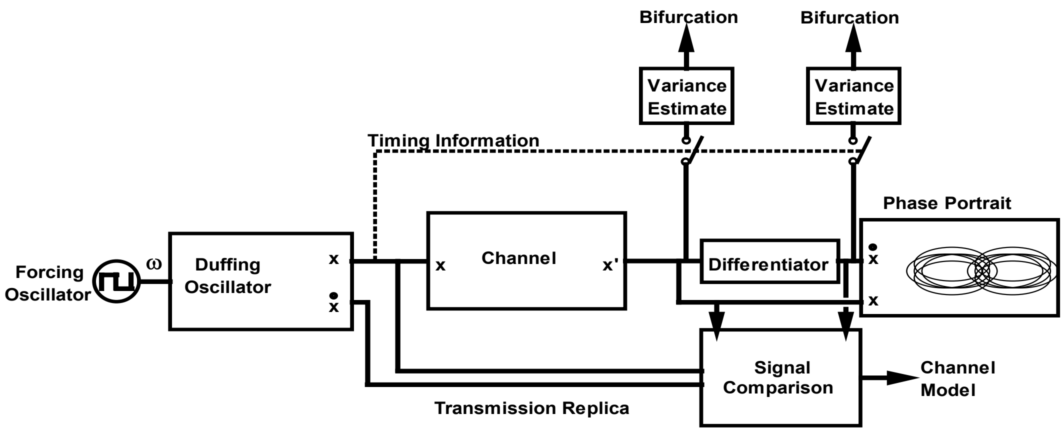

Figure 4 : Typical single-channel receiver structure

Although this may be satisfactory when measuring non-linear systems in the laboratory, it is unsatisfactory for use in a practical channel probing application. The variation in range between the transmitter and receiver will lead to amplitude and phase variations within the received signal, but more importantly, the multipath characteristics of the channel will lead to deep fading of the narrow-band signal. As an illustration of the magnitude of the problem, figure 5a shows the demodulated signal received when transmitting one of the almost periodic signals. It should be noted that the desired phase portrait is that shown in the right hand plot of figure 3.

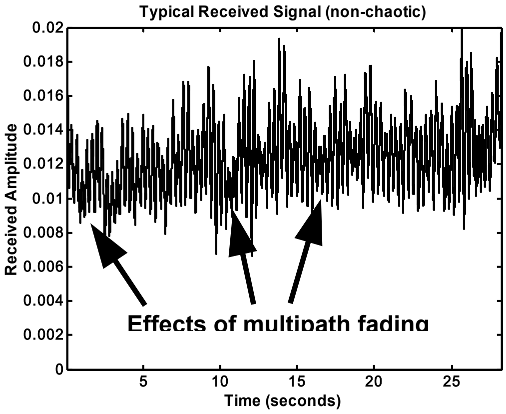

Figure 5a : Typical received signal from a non-chaotic transmission

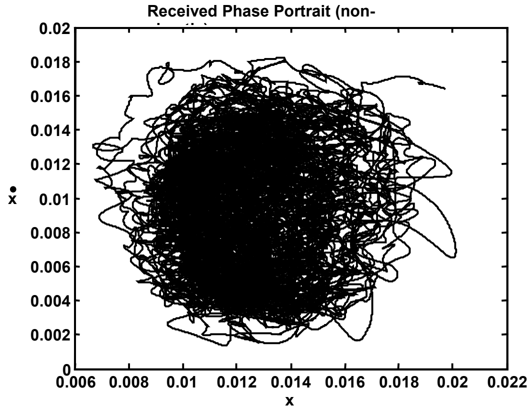

Figure 5b : Phase portrait of received signal (annular plot expected)

Synchronous sampling of the data shown in figure 5b would lead to the classical Rician fading distribution and an automatic classifier would fail to detect bifurcations. This clearly demonstrates the shortcomings of the detector shown in figure 4.

A detector which is capable of normalising amplitude changes due to both blue-water and shallow-water propagation conditions is required. Also, the bifurcation detector should be capable of countering the group delay effects of the channel and be able to provide a smoothed display estimator for the operator given the very low primary drive rate of the Duffing oscillator (3Hz for the results presented in figure 5).

Such a detector is the normalised covariance operator

$$C_{xy}(m)=\frac{\sum_{n=0}^{N-|m|-1}\left[x(n+m)-\frac{1}{N}\sum_{i=0}^{N-1}x_i\right]\left[y^*(n)-\frac{1}{N}\sum_{i=0}^{N-1}y_i^*\right]}{\sqrt{\sum_{n=0}^{N-|m|-1}\left[x(n+m)-\frac{1}{N}\sum_{i=0}^{N-1}x_i\right]^2\sum_{n=0}^{N-|m|-1}\left[y^*(n)-\frac{1}{N}\sum_{i=0}^{N-1}y_i^*\right]^2}} \quad (4)$$

where *x* is the demodulated signal transmitted as an amplitude modulation and *y* is the reference demodulated signal transmitted as a phase modulation. The number of points in the observation window, *N*, was typically chosen to correspond to between one and two seconds. Multiple overlapping observation windows, with overlapping ratios of 90 – 95%, were used to provide the required smoothing for display. The normalised covariance is calculated for all values of lag, *m*, and the maximum value selected. Values of the normalised covariance approaching unity indicate that bifurcation has not occurred and values approaching zero indicate the presence of bifurcation or the loss of demodulated data. Erroneous conclusions may be avoided by ensuring that there is sufficient signal-to-noise ratio at the input to the processor.

The improvements obtained by using the normalised covariance bifurcation detector are shown in figure 6. A non-chaotic transmission signal is used and therefore the output of the normalised covariance detector would be expected to approach unity. Evidence of a deep-fade is visible at about 10 seconds. The amplitude modulation process results in two sidebands carrying identical information mirrored in frequency. The receiver extracts each sideband independently resulting in 12 parallel processing channels. It can be seen that the adverse signal-to-noise ratio caused by the deep fade at 10 seconds only affects the upper sideband (USB). The results of the sampling bifurcation detector (crosses) are overlaid on this figure, illustrating the poor performance of this type of detector in practical situations.

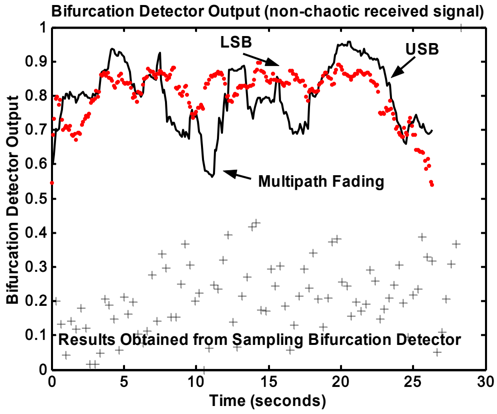

Figure 6 : Comparison of synchronous and normalised covariance bifurcation detectors

Figure 7 shows the composite results for six Duffing sources with values of *a* varying from 0.1 to 0.6 in increments of 0.1. The continuous lines (black) represent the upper sideband transmission and the dotted lines (red) represent the lower sideband transmission of the same signal. Under the trial conditions, it is expected that the channel is linear, so no bifurcation should be detectable (output values should approach unity).

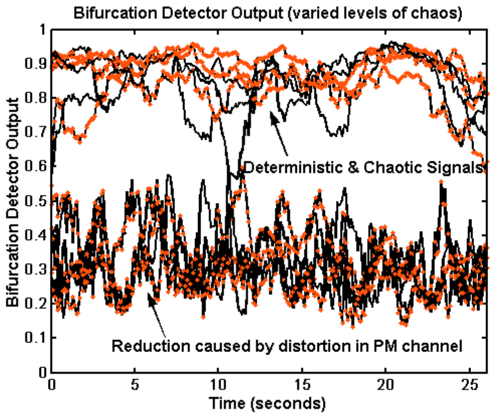

Figure 7 : Composite of six levels of chaotic behaviour of the transmitted signal

The upper four traces include both deterministic and chaotic transmissions and the high levels of coherence indicate that the system is working as expected. The lower eight traces represent chaotic transmissions with the lower levels of coherence being directly attributable to the multipath induced distortion in the phase modulated replica channel – these signals have a bandwidth in excess of the 9Hz limit determined by the multipath structure.

5. QUALITY MEASURES

The results presented in the previous section rely on the ability of the system to convey a faithful replica of the chaotic transmission signal. Sources of degradation are the varying range between the projector and the receiver, the wave-induced accelerations encountered by the transducers, differences in local sampling clocks and the multipath effects of the channel. Therefore, a suitable quality metric is required. The problem can be likened to that of a short-wave broadcaster who tape-records the demodulated signal at a distant point and compares with the source material in the studio. Here the tape stretches in a random manner and at a rate comparable with the source frequency. Measures such as carrier-to-noise ratio provide only a limited indication of the performance of the system. Results presented in this paper are from data having average carrier-to-noise ratios in excess of +20 dB.

A significant advantage of a chaotic source such as a Duffing oscillator is that the signal can be re-created at the receiver, provided the hardware and software are identical at both ends. This is an ideal. The sampling rates of identical digital audio sound cards typically vary by up to 5%. Thus, the quality measurements must use limited-length observation windows in order that the two nominally identical signals do not decorrelate. A suitable time lag must be inserted to account for the sampling drift.

In order to assess the ability of the replica channel to faithfully convey the transmitted data, the normalised covariance described in equation (4) was used to compare the local solution of the differential equation of the Duffing oscillator with that received by the phase modulated channel. This metric is transformed to the more familiar signal-to-self-noise ratio expressed in dB by:-

$$SSNR = 10\log_{10}\left(\frac{C_{xy}^2(m)}{1-C_{xy}^2(m)}\right) \quad \text{dB} \qquad (5)$$

The estimates are shown in figure 8a and 8b. The mean normalised covariance is 0.94 showing a good correlation between the transmitted and received modulation signals. This equates to a mean signal-to-self-noise ratio of 8.8 dB. The associated degradation is primarily due to the large multipath time spread as compared with the temporal correlation of the data source and the periodicity of transducer motion due to surface waves. The frequency content of these variations is of the same order-of-magnitude as the data source – thus the synchronous frequency and phase tracking algorithms within the receiver are presented with a difficult task.

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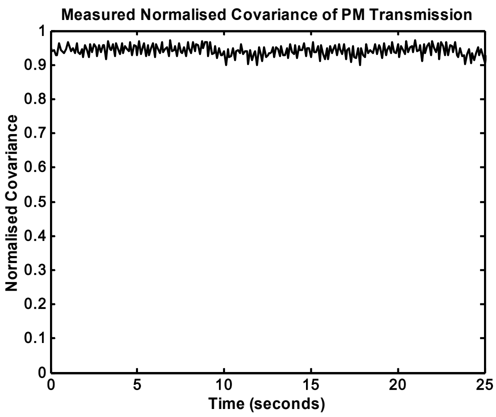

Figure 8a: Normalised covariance of TX and RX data using a phase modulated carrier

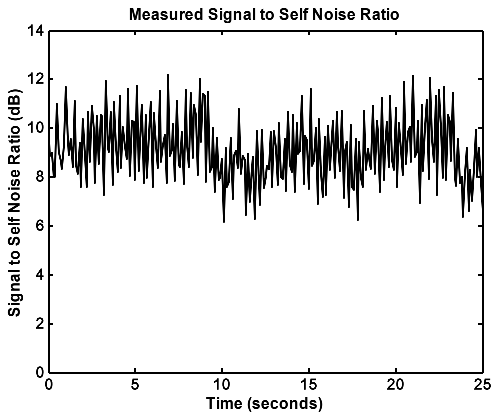

Figure 8b: Equivalent signal-to-self noise-ratio of the received signal

6. CONCLUSIONS

The results of transmitting a variety of time-compressed Duffing signals in a multipath dominated channel have been presented. The practical problems of using drifting projectors and receivers have been addressed by combining multiple carriers conveying both amplitude and phase modulated information. A synchronous, independent-sideband receiver is used to extract two versions of the frequency translated (and mirrored) Duffing source. A normalised covariance bifurcation detector has been shown to provide improved performance compared with a synchronous sampling detector in practical conditions.

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