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MULTICHANNEL NORMALIZED X-BLOCK LMS ALGORITHM FOR ACTIVE NOISE AND VIBRATION CONTROL

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1. INTRODUCTION

The filtered-X block least mean square (X-BLMS) algorithm has been presented for active control of sound and vibration [1,2]. Such an algorithm is an extension of the standard block LMS algorithm [3] to the filtered-X structure used when dynamics exist between the error sensors and the actuators [4]. The LMS (as well as X-BLMS) based algorithm has inherently slow convergence characteristics, particularly with a highly correlated input signal. One way of improving the convergence speed of the algorithm is to utilize a time-varying convergence factor. Various time-varying convergence factors have been employed for the X-BLMS algorithm. For example, an optimal time-varying convergence factor has been applied to both single channel and multichannel X-BLMS algorithms [5,6]. In these algorithms, the convergence factor attempts to minimize the cost at each iteration. Normalized convergence factors have also been proposed for (single channel) active noise cancellation [7]. These convergence factors not only improve the convergence and tracking capability of the algorithm, but also eliminate the trial and error process in determining an appropriate convergence factor.

In this paper, we describe normalized X-BLMS algorithm for an active noise and vibration control system with multiple sensors and actuators. The convergence factors described here are of matrix form and there may exist an infinite number of convergence matrices which provide an equivalent optimal solution. The proposed algorithm is referred to as the multichannel normalized X-BLMS algorithm because one of these matrices has a form that is analogous to the normalized one [8]. The proposed algorithm exhibits superior convergence properties and is particularly well-suited for the case where the autocorrelation function of the filtered reference signal has a large eigenvalue spread.

2. THE MULTICHANNEL NORMALIZED X-BLOCK LMS ALGORITHM

Figure 1 describes an active control system using the multichannel normalized X-BLMS algorithm. In the diagram, $\mathbf{H}^p$ represents the physical paths between the excitation signal and error microphones and $\mathbf{H}^s$ represents the paths between the actuators and error microphones. The model of $\mathbf{H}^s$ has been denoted as $\hat{\mathbf{H}}^s$ and S/P and P/S represent serial-to-parallel and parallel-to-serial converters which may be realized by either software or hardware. For a system consisting of M sensors (input channels) and N actuators (output channels), the error and the disturbance signals over all the channels can be expressed in a vector form as

$$\boldsymbol{e}_j = \boldsymbol{d}_j + \boldsymbol{r}_j \boldsymbol{w}_j \tag{1}$$

where

$$\boldsymbol{e}_j = [\boldsymbol{e}_{1,j}{}^T \quad \boldsymbol{e}_{2,j}{}^T \quad \dots \quad \boldsymbol{e}_{m,j}{}^T \dots \quad \boldsymbol{e}_{M,j}{}^T]^T \tag{2}$$

$$d_j = [d_{1,j}{}^T \quad d_{2,j}{}^T \quad \ldots \quad d_{m,j}{}^T \ldots \quad d_{M,j}{}^T]^T \tag{3}$$

and each sub-vector is formed as

$$e_{m,j} = [e_m(js) \quad e_m(js+1) \quad \ldots \quad e_m(js+L-1)]^T \tag{4}$$

$$d_{m,j} = [d_m(js) \quad d_m(js+1) \quad \ldots \quad d_m(js+L-1)]^T \tag{5}$$

Equations (4) and (5) are the error and disturbance block signal at the m^{th} error microphone where L is the block length, j is the block index and s is a block shift which specifies the extent to which successive blocks overlap ($1<s<L$). The feedforward adaptive filters are expressed by

$$w_j = [w_{1,j}{}^T w_{2,j}{}^T \quad \ldots \quad w_{n,j}{}^T \quad \ldots \quad w_{N,j}{}^T]^T \tag{6}$$

where each sub-vector in (6) is an I-tap FIR filter. The r_j in (1) is an MLxNI filtered reference matrix with a block Toeplitz structure. The $(m,n)^{\text{th}}$ Toeptlitz sub-matrix is of the form [2]

$$r_{m,n,j} = \begin{bmatrix} r_{m,n}(js) & r_{m,n}(js-1) & \cdots & r_{m,n}(js-I+1) \\ r_{m,n}(js+1) & r_{m,n}(js) & \cdots & r_{m,n}(js-I+2) \\ \vdots & \vdots & \vdots & \vdots \\ r_{m,n}(js+L-1) & r_{m,n}(js+L-2) & \cdots & r_{m,n}(js+L-I) \end{bmatrix} \tag{7}$$

where each component is the reference signal filtered by the transfer function between the m^{th} actuator and n^{th} sensor at a different time index [2].

A cost function at the j^{th} iteration is defined as

$$J_j = \frac{1}{L} e_j^T e_j \tag{8}$$

Assuming that input is stationary and that the filter adaptation is slow compared to the system dynamics, the stochastic gradient decent algorithm can be applied to obtain a global optimal solution that minimizes the cost function (8). The gradient of the cost function with respect to the filter coefficients is

$$\nabla_j = \frac{\partial J_j}{\partial w_j} = \frac{2}{L} r_j{}^T e_j \tag{9}$$

and a gradient descent algorithm is given by

$$w_{j+1} = w_j - M_j \nabla_j = w_j - \frac{2}{L} M_j r_j^T e_j \tag{10}$$

where M_j is analogous to the conventional scalar convergence factor μ_j except that M_j is now an NIxNI matrix that is configured to minimize the cost function J_{j+1}.

To obtain the convergence factor matrix M_j, a Taylor series expansion is applied to the error signal at the $(j+1)^{\text{th}}$ iteration as a function of the error signal at the j^{th} iteration. Since second and higher order terms are zero for a linear system, this expansion may be expressed as,

$$e_{j+1} = e_j + \frac{\partial e_j}{\partial w_j}(w_{j+1} - w_j) = (I_{ML} - \frac{2}{L} r_j M_j r_j^T) e_j \tag{11}$$

where I_{ML} is an ML^{th} dimensional identity matrix. To find the optimal convergence matrix M_j, we take the derivative of the cost function (8) at the $(j+1)^{\text{th}}$ iteration with respect to M_j and set it to zero, i.e., $\partial J_{j+1}/\partial M_j = 0$, which, using matrix calculus, and (8), (11) yields

$$\frac{2}{L} r_j{}^T r_j M_j r_j{}^T e_j e_j^T r_j - r_j{}^T e_j e_j^T r_j = 0 \tag{12}$$

Equation (12) represents a necessary condition for optimal M_j. In general, there exists an infinite number of solutions for an optimum M_j when $1<ML<NI$. One of these solutions takes a form analogous to that of conventional normalized LMS [8],

$$M_j = \frac{\alpha\ L}{2}(r_j^T r_j)^+ \tag{13}$$

where superscript (+) denotes the pseudo-inverse and α is a scaling factor ($0<\alpha<1$). Note that if $ML \geq NI$ and $r_j^T r_j$ is of full rank, the pseudo-inverse reduces to the conventional inverse. More generally, a class of solutions to (12) can be expressed as [9].

$$M_j = \frac{\alpha\ L}{2}(r_j^T r_j)^+ [G_j G_j^+ + Y(I_{NI} - G_j G_j^+)] \tag{14}$$

where $G_j = r_j^T e_j e_j^T r_j$ and Y is an arbitrary NIxNI matrix. For example, (14) reduces to (13) when $Y = I_{NI}$. In addition, it may be shown that setting $Y=0$ results in a minimal $\|M_j\|_F$ where $\|\cdot\|_F$ is the Frobenius norm. Any convergence factor matrix M_j as parametrized by (14) will result in identical performance when used in algorithm (10). However, the additional degree of freedom provided by the arbitrary matrix Y may be possibly used to provide a benefit in a practical implementation of the algorithm. The convergence factor matrix tends to equalize the convergence rates associated with different eigenvalues of the filtered-reference autocorrelation matrix. The algorithm achieves fast convergence rate compared to traditional LMS algorithm as suggested in the simulation. However, such improvement is obtained at the expense of additional computational burden.

3. SIMULATION RESULTS

A computer simulation was conducted to evaluate the performance of the multichannel normalized X-block LMS algorithm. The simulation is based on the application of a structural/acoustic noise control system in a commuter turboprop aircraft. Experimental transfer functions were collected within a fuselage of the aircraft. Figure 2 shows a cross-section of this fuselage instrumented with two structural inertial actuators and four microphones located within the same cross-sectional plane. A disturbance speaker (the primary source) is outside of the fuselage within the same plane, and is intended to emulate prop wash impinging on the fuselage. The structural actuators are attached directly to one of the fuselage frames. The microphones are attached along the same frame.

The excitation signal is a synthesis of typical turboprop noise which comprises a number of harmonics related to the blade passage frequency (100 Hz) and some bandlimited random noise (~300 Hz) coming from different disturbance sources. Sampling rate is chosen to be 1000 Hz. Considering the frequency bandwidth of the external noise signal, 32-tap truncated impulse responses of both primary and secondary acoustic paths are used in the simulation.

Figure 3 shows the convergence history of various block adaptive algorithms (with s=1) in 2000 iterations. As indicated in Fig.3, the dashed line represents the convergence curve for the multichannel X-block LMS algorithm with a fast convergence rate (μ_B= 0.000005) and the solid line represents the convergence history of the multichannel normalized X-block LMS algorithm (α=0.05). From the simulation, it is seen that the normalized X-block LMS algorithm demonstrates faster convergence rate.

4. CONCLUDING REMARKS

A multichannel block normalized adaptive algorithm was presented for active noise cancellation. The algorithm utilizes a matrix of optimal convergence factors to minimize the cost function of the algorithm from one iteration to the next. The algorithm improves the convergence rate of the adaptive system. Superior convergence performance was demonstrated by a computer simulation using a synthetic turboprop aircraft noise and transfer functions obtained from a commuter turboprop fuselage.

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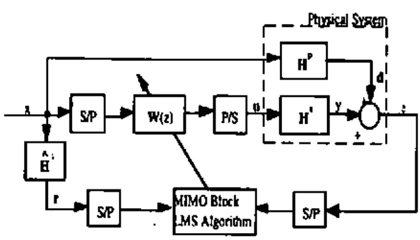

Fig. 1 Block Diagram of Algorithm Simulation

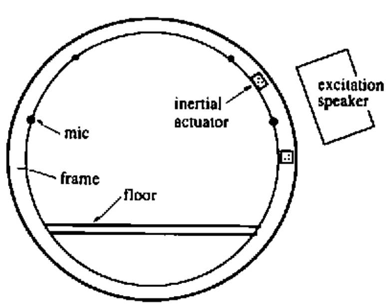

Fig. 2 Cross-section of an aircraft fuselage instrumented with two inertial actuators and four microphones.

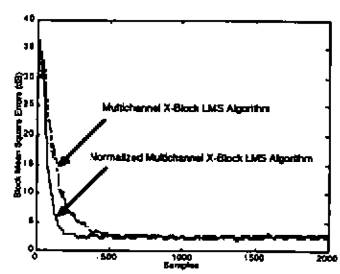

Fig. 3 Convergence Curves of Multichannel X-BLMS Algorithms with Different Convergence Factors