

# MULTIPLE-INPUT MULTIPLE-OUTPUT SAS SIMULATION

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## 1 INTRODUCTION

In recent years, researchers have begun extending multiple-input multiple-output (MIMO) concepts to synthetic aperture radar (SAR) systems; however, research studying the use of MIMO techniques in synthetic aperture sonar (SAS) systems has been very limited<sup>1</sup>. Two significant differences between a SAR and a SAS system are the propagation speed and the physical nature of electromagnetic waves (which are polarized) versus sound waves (which are compressional). The speed of the carrier wave places constraints on the velocity of the sensor platform<sup>2</sup> and those are such that in order to obtain reasonable survey speeds, SAS systems generally use an array of receivers, whereas a single receiver is sufficient for SAR systems. Hence SAR is often a single-input single-output (SISO) system whereas SAS is a single-input multiple-output (SIMO) system. It is in the common interest of both to obtain images with a high azimuth (along-track) resolution without penalizing the unambiguous range or pulse repetition frequency (PRF). Many studies have shown that MIMO-SAR systems can outperform SISO-SAR system in terms of the azimuth resolution of the images that they produce<sup>3,4</sup>, suggesting potential benefits of using a MIMO configuration in a SAS system and motivating the present work.

This paper considers an extension of a SIMO-SAS system to a two-transmitter system in which the transmitters transmit mutually orthogonal waveforms simultaneously. Given the popularity of linear-frequency modulated (LFM) transmissions, here the MIMO transmissions are based on up- and down-chirps which are truly orthogonal only for zero relative delay. This choice illustrates the potential gains of using multiple transmissions. We also consider the use of a pair of up-chirps with a relative time shift, which are orthogonal for a chosen time period. This choice illustrates the importance of orthogonality. A third choice is phase-coded waveforms; phase-coded waveforms have excellent range resolution, but are not used in MIMO-radar systems because of their poor Doppler tolerance<sup>5</sup>. This requirement may not be as stringent in the case of SAS, so phase-coded waveforms may be a good choice.

The performance of the proposed MIMO system is evaluated using a previously developed SAS simulator<sup>6</sup> which allows us to illustrate the improved azimuth resolution using the point spread function as well as simulated target scenes.

## 2 SYSTEM MODEL

The SAS system at hand is assumed to comprise a platform carrying hydrophone array in uniform linear motion, with transmitting pings at a PRF of  $f_r$ . The system consists of  $M$  transmitters and  $N$  receivers. When a single transmitter is used, the return signal is modeled as a superposition of the returns of scatterers of the imaging scene. For simulation purposes, it is assumed that the echoes collected are free of propagation loss and motion errors, and that the array is stationary during each PRI and moves instantaneously to its next position at the following PRI, i.e., the *stop-and-hop* model is used. With  $N_T$  scatterers, the received signal at the  $n^{\text{th}}$  element and  $p^{\text{th}}$  ping is given by

(1)

where  $s(t)$  denotes the transmitted signal,  $P_e$  the two-way element beampattern,  $\tau_i$  the delay to the  $i^{\text{th}}$  scatterer corresponding to a distance  $R_i^{np}$  from scatterer  $i$  to the  $n^{\text{th}}$  element on the  $p^{\text{th}}$  ping. If two different transmitting signals are used, the returns for each signal are superposed. However, it is important to emphasize that the superposition must account for the relative positions of the transmitters.

A SAS simulator developed at the University of Toronto is used to simulate the behavior of a MIMO-SAS system. The parameters used in the simulation are given in Table 1. The parameters are chosen such that the array travels a fourth of the total array length between consecutive pings. Note that as long as the sampling points in cross-range are evenly spaced, the choice of system parameters is flexible.

Table 1. System parameters

Parameter	Value	Unit
Number of hydrophones	8	--
Hydrophone size	0.05	M
PRF	8	Hz
Speed	0.6	m/s
Center frequency ( $f_c$ )	100	kHz
Bandwidth	15	kHz

### 3 THE MIMO VIRTUAL ARRAY

Unless stated otherwise, the MIMO-SAS system in this paper refers specifically to a two-transmitter system with the following configuration: the transmitters are placed in the cross-range direction, with the two transmitters located at each end of the receiver array. A pair of orthogonal signals is emitted simultaneously from the two transmitters. As illustrated in Figure 1, this configuration introduces additional phase centers at each ping. The subfigure on the left illustrates the traditional SIMO setting with the associated phase centers. In the subfigure on the right, the presence of a second transmitter, Tx2, adds additional phase centers (the solid circles). This elongated virtual array allows improvement in azimuth resolution by introducing additional information at each ping. It is important to note that these phase centers are available due to the presumed orthogonality between the two transmitted waveforms.

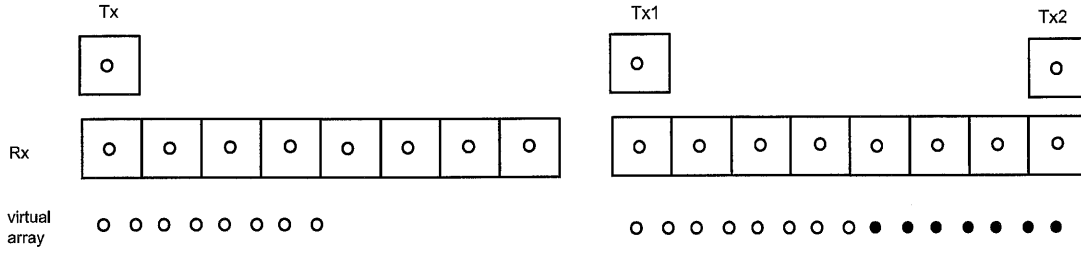


Figure 1. Additional phase centers introduced by the addition of one transmitter.

The signals emitted by the transmitter pairs need to be orthogonal to allow for a separation of the received signals into contributions from the individual transmissions. The transmitted signals have to occupy the same frequency band to allow for coherent integration of the received echoes, implying that signals that are orthogonal for all delays are not applicable for the system<sup>3</sup>. Several linear-frequency modulation (LFM) based waveforms have been proposed to fit these criteria. Two groups of orthogonal waveforms, the up and down chirps and the short-term shift-orthogonal chirps have been suggested and analysed in literature<sup>3,7</sup>. These sets of waveforms are

$$\text{Up-chirp: } , \text{ for } , \quad (2)$$

$$\text{Down-chirp: } , \text{ for } , \quad (3)$$

where  $f_c$  is the center frequency and  $T_p$  is the pulse duration and  $K$  is the chirp rate.

$$\text{Chirp without time shift: } , \text{ for } \quad (4)$$

$$\text{Chirp shifted in time: } , \text{ for and } \quad (5)$$

where  $\text{circshift}(\cdot)$  denotes a circular shift by the second index, here  $\Delta t$  stands for the time duration for which the chirp signal is shifted. We will also analyse a type of waveform that has been overlooked by MIMO-SAR studies, the phase-coded chirp. There are two ways of implementing the phase-coded chirp:

$$\text{Full-length chirp: } , \text{ for } \quad (6)$$

$$\text{Sectioned chirp: } , \text{ for } \quad (7)$$

where  $\text{mod}$  denotes the modulus function.  $c(t)$  is a polyphase sequence of length  $M$ ; each code bit takes a value of -1 or 1. The instantaneous frequency of a chirp signal described by Equation (6) increases linearly from time 0 to time  $T_p$ , with a phase change as dictated by the code sequence every  $T_p/M$  seconds. The chirp signal described by Equation (7) consists of  $M$  short-duration sub-chirps, each with duration  $T_p/M$  and chirp rate  $MK$ . The bandwidth of the transmitted signal described by (6) is slightly higher than that described by (7). However, as long as the choice of  $M$  is reasonable, there should not be an appreciable increase in the bandwidth of the transmitted pulse. Specifically, we would like  $M$  to be large, but it is also required that  $M/T_p$  be small compared to the bandwidth of the pulse. In a MIMO system with two transmitters, orthogonality of transmitted waveforms is introduced by using a pair of orthogonal codes.

## 4 SIMULATION RESULTS

Figure 2 (a) shows four point spread functions (PSF) in cross-range of a scene with one point target. The PSFs belong to one SIMO-SAS system and three MIMO-SAS systems with different waveforms in the order as indicated in the graph. For the presented simulation results, a time shift of a third of the pulse length is used to generate the short-term shift-orthogonal chirps, and Frank codes of length 50 are used to generate the phase-coded chirps. Other choices of time shifts and codes yield PSF with the same 3 dB width and almost identical shapes. The 3 dB width of the PSF of MIMO-SAS systems is narrower than that of the SIMO-SAS system because of the additional information collected by the additional transmitter in cross-range, as argued in Section 3. The PSFs of all three MIMO-SAS systems share the same 3 dB beamwidth, showing that azimuth resolution is very dependent on the system configuration, but not so much of the forms of transmitted waveforms.

The availability of a data simulator<sup>6</sup> allows us to investigate additional configurations that may not be strictly physically meaningful. To illustrate the importance of the transmitter locations, Figure 2(b) plots the PSF when the two transmitters are placed along the range direction. As is clear, the resulting azimuth resolution does not change from that of a SIMO-SAS system, as shown in Figure 2(b).

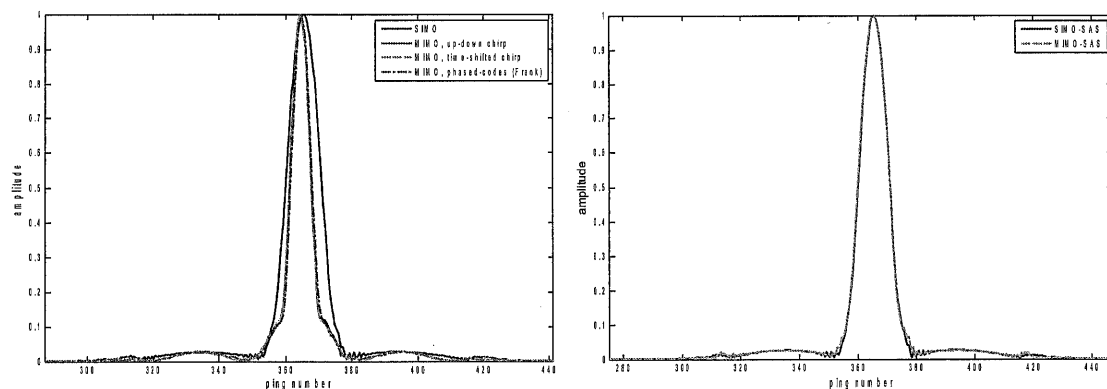


Figure 2 (a). Point spread function of a point target in cross-range. Transmitters are placed along cross-range direction.

Figure 2 (b). Point spread function of a point target in cross-range. Transmitters are placed along the range direction.

Figure 3. Target scene with four target points spaced 0.6 meters in cross-range

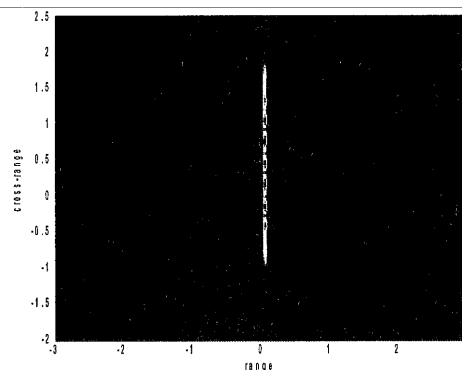


Figure 3 (a). SIMO-SAS system

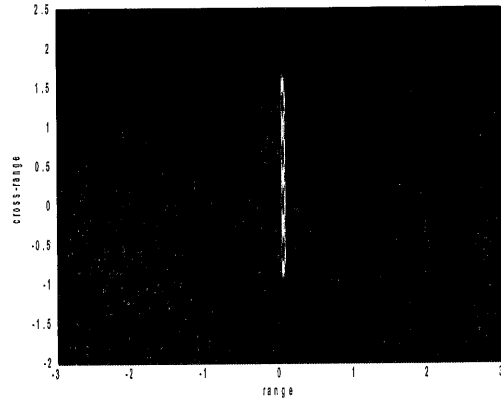
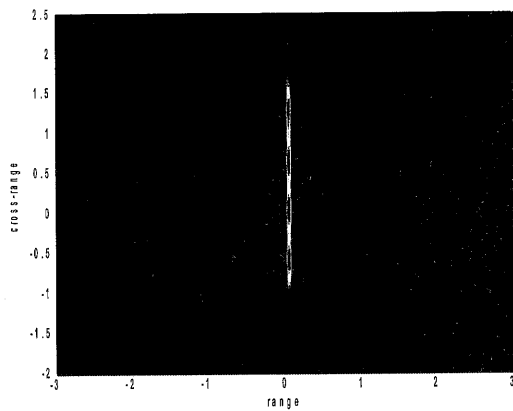


Figure 3 (b). MIMO-SAS system transmitting up and down chirps

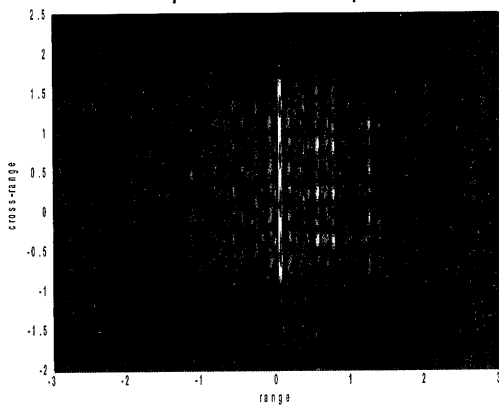


Figure 3 (c). MIMO-SAS system transmitting time-shifted chirps

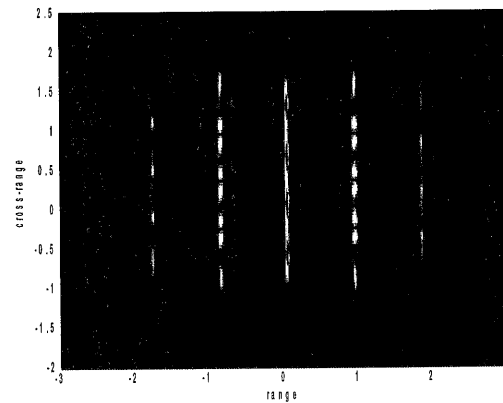


Figure 3 (d). MIMO-SAS system transmitting length-50 full-length Frank-coded chirps

Figure 3 (e). MIMO-SAS system transmitting length-50 sectioned Frank-coded chirps

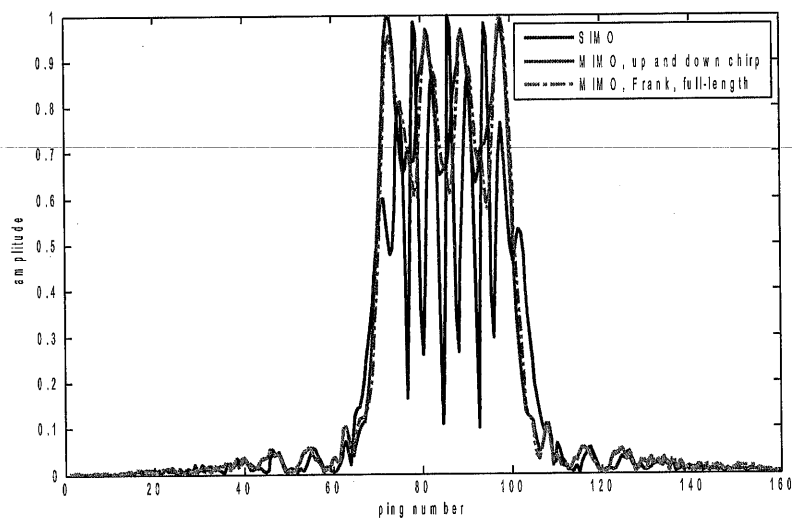


Figure 4. Cross-sections of the target scene in Figure 3 for some waveforms.

Figure 3 shows a comparison of the target scene with 4 closely spaced target points placed in cross-range. The figures illustrate the improved azimuth resolution – and the role of orthogonality amongst the transmitted waveforms. Up- and down- chirp signals are quasi-orthogonal<sup>8</sup>. The sidelobe of their cross-correlation function exists for a time shift as long as the pulse length, and the sidelobe level does not decrease with increasing time shift. Using long chirps on wide images would therefore suffer severely from smearing effect in range. Figure 3(c) shows that short-term shift-orthogonal chirps produce clean images in range direction because they are strictly orthogonal for a certain time period. Again this imposes a limit on the width of the imaging scene. Two targets that are separated by more than the shifting period cannot be resolved. Figure 3(c) and 3(d) are generated using two pairs of length-50 Frank codes, each generated using a different method as specified in Equations (6) and (7). Both figures show more or less the effect of the correlation between the signals.

Figure 4 is a combination of the cross-sections of the scenes in Figure 3 along the cross-range, across the target points. The cross-section view of short-term orthogonal-shifted chirp and sectioned phase-coded chirp are very similar to that of up- and down-chirps, and are therefore not displayed in the figure for clarity. The sets of waveforms used for MIMO-SAS system can be arranged from best to worst azimuth resolution in the following order: length-50 sectioned Frank coded chirps, up- and down-chirps, length-50 full-length Frank coded chirps and short-term orthogonal-shifted chirps. The azimuth resolution provided by full-length phase-coded chirps is significantly better compared to other waveforms, with the caveat of range spreading.

Figure 5. displays two simulated seafloor scenes as would be generated by a SIMO- and a MIMO-SAS system. The MIMO-SAS simulation uses two short-term shift-orthogonal signals. Both images are generated by simulating the combined effects of closely spaced (0.08 m in range and 0.3 m in cross-range) target points. The positions and number of target points are identical for both graphs. The difference between the two graphs is only due to resolution of the system. The non-reflective slits that can be seen in the right subfigure but not in the left subfigure are less than 1 m wide, showing that the MIMO-SAS system can discern features of underwater objects that cannot be identified via the SIMO-SAS system.

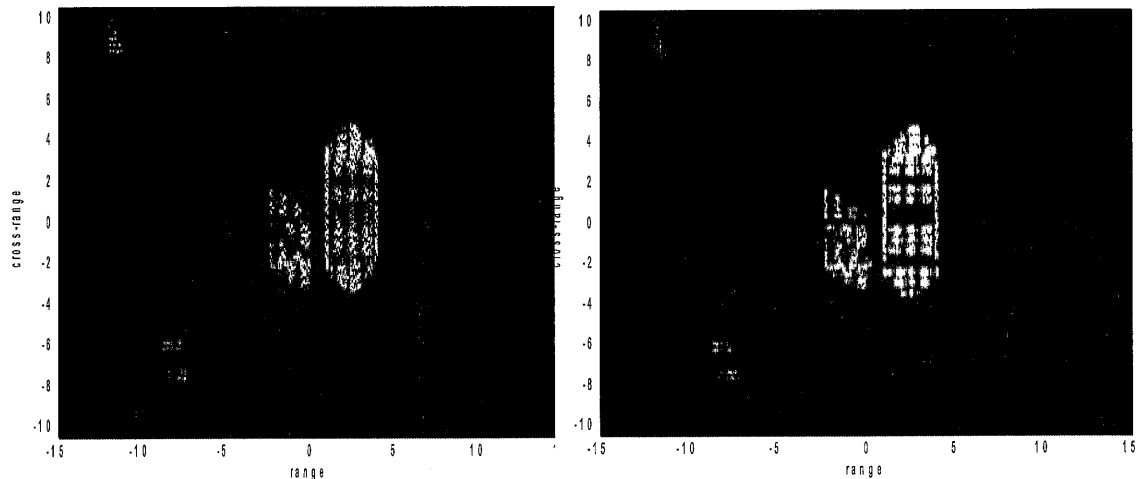


Figure 5 a). simulated SIMO-SAS scene

Figure 5 b). simulated MIMO-SAS scene

## 5 DISCUSSION AND CONCLUSIONS

This paper presents a preliminary investigation of the role of MIMO concepts could play in SAS systems. Our preliminary results show that there are potential gains to be had in terms of an improved azimuth resolution made possible using orthogonal waveforms.

Specifically, with the help of a simulation platform that we had built previously, we used several pairs of waveforms to verify that the azimuth resolution of image scenes formed by the MIMO-SAS system outperforms those formed by the SIMO-SAS system. We have shown that this improvement is consistent across all sets of orthogonal waveforms, showing that MIMO system would improve azimuth resolution regardless of the forms of transmitted signals. We have also demonstrated the importance in choosing transmitting waveforms. The sidelobes of cross-correlation of simultaneously transmitted waveforms can lead to deterioration of the image quality in the range direction.

It is anticipated that a MIMO-SAS system may have one of three advantages over SIMO-SAS system depending on system design, namely an improved azimuth resolution, a reduced PRF or an increased platform speed. We have shown that improved azimuth resolution can be achieved by holding the other two conditions the same. The other two advantages are to be verified in further works. More work in waveform design is also needed to produce high quality images. Finally, in our model, we have so far ignored nonlinear effects and dispersion, as well as some interference effects due to the compressional nature of sound waves. Some of these topics will be investigated in our subsequent work, but all require some attention for the practical development of MIMO techniques for SAS systems.

## 6 REFERENCES

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