MODELLING DISSIPATIVE SILENCERS IN HVAC DUCTS

R Kirby Brunel University, Department of Mechanical Engineering, Uxbridge, Middlesex. UB8 3PH
JB Lawrie Brunel University, Department of Mathematical Sciences, Uxbridge, Middlesex. UB8 3PH

1. INTRODUCTION

Modelling the acoustic performance of dissipative silencers in HVAC ducts is far from straightforward. Traditional modelling techniques assume the silencer to be infinite in length and compute the attenuation of the fundamental mode only. It has long been known however that the fundamental mode is not necessarily the least attenuated mode [1] and that higher order modes may significantly affect silencer performance; Ramakrishnan and Watson [2] proposed that only higher order modes whose attenuation is within 20 dB of the least attenuated mode affect silencer performance. Furthermore Ramakrishnan and Watson recognised that the inlet and outlet planes of the silencer may also affect silencer performance. A traditional theoretical analysis, based upon a silencer of infinite length, cannot however quantify these so-called "end-effects"; Ramakrishnan and Watson instead postulated an additional sound attenuation ranging from 0 to 10 dB was directly attributable to end effects.

To model accurately the acoustic performance of a dissipative silencer one should, ideally, include both higher order modes and end-effects. This may be achieved only by modelling a finite length silencer. Theoretical models for finite length, bulk reacting, dissipative silencers are available in the literature (see for example [3] and [4]), however they tend to focus on relatively small automotive dissipative silencers. HVAC ducts are, typically, much larger and modelling techniques appropriate for small silencers are not necessarily suitable, especially if optimising CPU expenditure is important. One computationally efficient approach is first to model only the duct cross-section and then to enforce continuity of acoustic pressure and velocity on either side of the inlet/outlet planes of the silencer, assuming of course that the duct has a uniform cross-section. This approach was adopted by Mechel [5], who enforced continuity conditions by using an exact analytic mode matching formulation, although, crucially, the analysis was restricted to locally reacting materials. A more general approach is to match continuity conditions numerically, for example the finite element method (FEM) is ideally suited to matching at discrete nodal points over a transverse finite element mesh. This technique, which we shall refer to as point collocation, was successfully applied by Astley *et al* [6] in matching continuity conditions between an empty rectangular hard walled duct, and a rectangular duct with a single, acoustically lined, flexible wall.

Point collocation appears to be the most promising method with which to model large finite length dissipative silencers since it incorporates the generality of the FEM whilst avoiding a three dimensional mesh and the associated high CPU expenditure. In principle point collocation may be applied to complex HVAC systems which contain any number of silencer sections (provided they are uniform in cross-section), and may readily include other phenomena such as structural flanking transmission and the effects of a perforate and mean flow. The accuracy of point collocation is however uncertain, particularly for large HVAC ducts, and so before applying this technique to complex problems it is necessary first to investigate prediction accuracy. The aim of this paper is to investigate both the accuracy of the point collocation method and to quantify silencer "end effects". Thus transmission loss predictions are presented here for a finite length bulk reacting splitter silencer, of a type commonly used in HVAC applications, and the point collocation method is validated by comparison with analytic mode matching predictions.

2. THEORY

The geometry of the splitter silencer is shown in Figure 1 below.

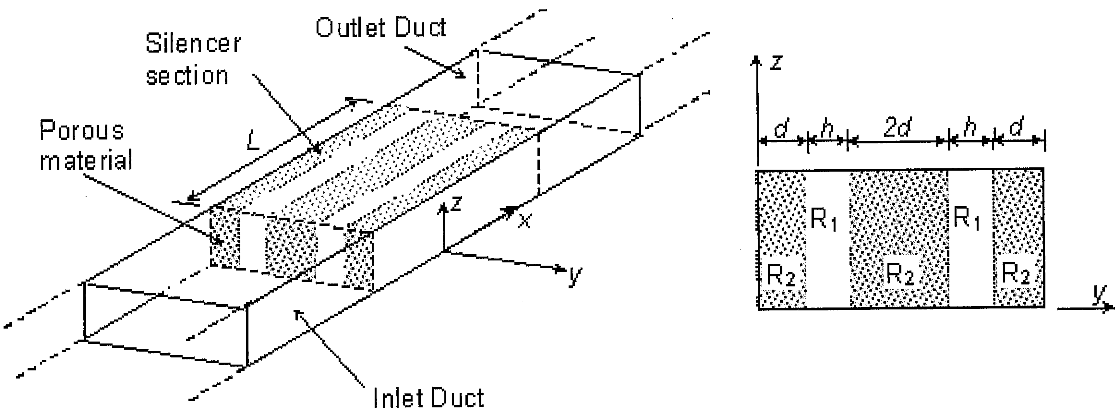

Figure 1. Geometry of splitter silencer.

The silencer is placed inside a rectangular duct whose walls are assumed to be rigid and impervious, it is assumed also that there is no mean gas flow in the airway, that no perforate is present, and the porous material is bulk reacting. For a harmonic time dependence of $e^{i\omega t}$ ($i = \sqrt{-1}$, ω is radian frequency, *t* is time), the Helmholtz equation gives

$$(\nabla^2 + 1)p = 0, \qquad \text{and} \qquad (\nabla^2 - \Gamma^2)p = 0, \qquad \text{(1a, b)}$$

for the airway and the absorbent respectively. Here *p* is the acoustic pressure and Γ is the (non-dimensional) propagation constant for the porous material. The acoustic field in each duct section is expanded as an infinite sum to give

$$p_{inlet}(x,y,z) = e^{-ix} + \sum_{l=0}^{\infty} A_l \Psi_l(y,z) e^{\eta_l x}, \qquad (2)$$

$$p_{silencer}(x,y,z) = \sum_{m=0}^{\infty} B_m Y_m(y,z) e^{-S_m x} + \sum_{m=0}^{\infty} C_m Y_m(y,z) e^{S_m x}, \qquad (3)$$

$$p_{outlet}(x,y,z) = e^{-ix} + \sum_{n=0}^{\infty} D_n \Psi_n(y,z) e^{-\eta_n x}, \qquad (4)$$

assuming only an incident plane wave propagates in the inlet duct and the outlet duct is terminated anechoically. Here η and *S* are the axial wavenumbers, Ψ and *Y* the transverse modal eigenfunctions and *A*, *B*, *C* and *D* are the modal amplitudes.

2.1. Boundary Conditions

At the inlet and outlet planes, continuity of acoustic pressure gives:

$$p_{\text{inlet}}(\text{Plane A}, y, z) = p_{\text{silencer}}(\text{Plane A}, y, z) \tag{5}$$

$$p_{\text{outlet}}(\text{Plane B}, y, z) = p_{\text{silencer}}(\text{Plane B}, y, z). \tag{6}$$

Continuity of axial particle velocity for region R_1 (see Figure 1) yields,

$$\frac{\partial p_{\text{inlet}}(\text{Plane A}, y, z)}{\partial x} = \frac{\partial p_{\text{silencer}}(\text{Plane A}, y, z)}{\partial x} \qquad (y, z \in R_1), \tag{7}$$

$$\frac{\partial p_{\text{outlet}}(\text{Plane B}, y, z)}{\partial x} = \frac{\partial p_{\text{silencer}}(\text{Plane B}, y, z)}{\partial x} \qquad (y, z \in R_1), \tag{8}$$

and for region R_2,

$$\frac{\partial p_{\text{inlet}}(\text{Plane A}, y, z)}{\partial x} = \frac{\rho}{\rho(\omega)} \frac{\partial p_{\text{silencer}}(\text{Plane A}, y, z)}{\partial x} \qquad (y, z \in R_2), \tag{9}$$

$$\frac{\partial p_{\text{outlet}}(\text{Plane B}, y, z)}{\partial x} = \frac{\rho}{\rho(\omega)} \frac{\partial p_{\text{silencer}}(\text{Plane B}, y, z)}{\partial x} \qquad (y, z \in R_2), \tag{10}$$

where ρ is the density of air and $\rho(\omega)$ is the equivalent complex density of the porous material. Application of the above continuity conditions delivers, after suitable truncation of the modal sums, the modal amplitudes and subsequently the sound pressure field in each duct section. To enforce the above conditions one must first compute the axial wavenumbers and transverse duct eigenfunctions present in the original modal expansions. The theoretical analysis therefore splits into two parts: an eigenvalue analysis for each duct section followed by the matching procedure.

An eigenvalue analysis for the empty inlet/outlet ducts is straightforward and well documented in the literature. For the silencer section, the eigenvalue analysis has formed the basis of traditional silencer design techniques, although usually only the fundamental mode is computed, or at least only a small additional number of modes. The eigenvalue analysis for the silencer section is generally well understood (see, for example, the analytic approach of Cummings and Sormaz [7] and the finite element approach of Ramakrishnan and Watson [2]) and the details do not need repeating here. It should be noted however that when using an eigenvalue analysis as the basis for enforcing continuity conditions, different demands are placed on the results of the analysis. It is not sufficient here merely to compute a small number of higher order modes, sufficient modes must be taken to ensure that the solution to the truncated system closely approximates that of the infinite system. For an automotive dissipative silencer, Cummings and Chang [4] achieved this with only 6 modes, however for much larger HVAC systems it is likely that many more higher order modes will be necessary to achieve convergence. Computing a large number of higher order modes provides little difficulty when implementing the FEM (the number of modes found is equal to the number of degrees of freedom in the finite element mesh) and so is ideally suited to the subsequent implementation of point collocation. However for an analytic mode matching scheme, accurately locating a relatively large number of higher order modes presents many problems, primarily in avoiding missing relevant roots of the dispersion relation. Cummings and

Sormaz [7] employed Muller's method to locate a large number of higher order modes for a splitter silencer and devised an elaborate algorithm to avoid missing roots, however they were unable to demonstrate, unambiguously, that all required modes had successfully been located. Consequently, although a fully analytic approach provides an exact solution, it may not be suitable for use as an iterative design tool. Nevertheless exact methods provide an ideal means of validating numerical tools that may be used in the design of HVAC ducts, such as the point collocation method developed here.

2.2. Point Collocation

The application of point collocation to the inlet and outlet planes of the silencer is facilitated by using an identical finite element mesh to compute the eigenvalues and the accompanying eigenvectors in each duct section. Matching proceeds by substituting the expanded acoustic pressure fields (equations (2) to (4)) into the continuity boundary conditions given by equations (5) to (10) to give

$$\sum_{j=0}^{N} A_j \mathbf{\Psi}_j - \sum_{m=0}^{N} B_m \mathbf{Y}_m - \sum_{m=0}^{N} C_m \mathbf{Y}_m = -1, \tag{11}$$

$$\sum_{j=0}^{N} A_j \eta_j \mathbf{\Psi}_j + R\sum_{m=0}^{N} B_m S_m \mathbf{Y}_m - R\sum_{m=0}^{N} C_m S_m \mathbf{Y}_m = i, \tag{12}$$

$$\sum_{m=0}^{N} B_m \mathbf{Y}_m e^{-S_m L} + \sum_{m=0}^{N} C_m \mathbf{Y}_m e^{S_m L} - \sum_{n=0}^{N} D_n \mathbf{\Psi}_n = 0, \tag{13}$$

$$-R\sum_{m=0}^{N} B_m S_m \mathbf{Y}_m e^{-S_m L} + R\sum_{m=0}^{N} C_m S_m \mathbf{Y}_m e^{S_m L} + \sum_{n=0}^{N} D_n \eta_n \mathbf{\Psi}_n = 0, \tag{14}$$

where

$$R = \begin{cases} 1 & (y,z) \in R_1 \\ \rho/\rho(\omega) & (y,z) \in R_2 \end{cases},$$

and $\mathbf{\Psi}_j$ is a vector of length N (the number of nodes, or collocation points, in the finite element mesh), containing the transverse eigenfunction of mode j in the inlet/outlet duct; $\mathbf{Y}_m$ is the equivalent for the silencer section. Equations (11) to (14) form a complete set of 4N linear equations (the collocation points) and may be solved simultaneously for the 4N unknown modal amplitudes.

The conventional method for quantifying silencer performance is to compute the silencer transmission loss since this is independent of the source and radiation impedance. Silencer transmission loss (*TL*) is defined as the difference between the incident and transmitted sound power and, for the modal expansions adopted here, this gives

$$TL = -10\log_{10} \mathrm{Re}\left[\sum_{n=0}^{N} \frac{\varepsilon_n}{2}\eta_n |D_n|^2\right] \tag{15}$$

where $\varepsilon_n = \begin{cases} 2, & n = 0 \\ 1, & n > 0 \end{cases}$.

3. NUMERICAL RESULTS AND DISCUSSION

Theoretical transmission loss predictions are presented here for a range of duct sizes and these are given in Table 1 below. The sizes are chosen to provide a rigorous validation of point collocation as well as illustrating the influence both of end-effects and higher order modes on silencer performance. The absorbent material is assumed to be fibrous and the empirical expressions of Delany and Bazley [8] are used here to specify the propagation constant and characteristic impedance of the porous material; flow resistivities are specified in Table 1. It is well known that Delany and Bazley's empirical expressions may be invalid at low frequencies and so the semi-empirical low frequency corrections described by Kirby and Cummings [9] are further employed to define the material properties.

The theoretical development in Section 2 assumes an plane incident wave is present in the inlet duct. This is likely to be true only for silencers placed at a relatively large distance from duct discontinuities and/or fans. Higher order incident modes can readily be accommodated in the analysis although this requires a detailed knowledge of the characteristics of the incident sound field and makes any ensuing observations source dependent. The assumption of plane wave excitation does however reduce the theoretical analysis to 2 dimensions (the *x* and *y* plane) and hence only a one dimensional eigenvalue analysis is necessary for each duct section. For the finite element eigenvalue analysis, the adoption of one dimensional finite elements greatly reduces the number of degrees of freedom in the overall problem and also makes mesh generation very straightforward. For the silencers studied here, three noded quadratic elements were used and the number of degrees of freedom are listed below in Table 1.

Table 1. Silencer data.

Silencer	Width of splitter $2d$ (m)	Percentage open area Δ (%)	Air Gap h (m)	Length L (m)	Material Flow Resistivity (Rayls/m)	Degrees of freedom in mesh
1	1.2	25	0.4	1	8000	121
2	0.4	25 40	0.13333 0.26667	1	8000	61
3	0.6	25 40	0.2 0.4	1	8000	61
4	0.3	25	0.1	1	3000	61

Analytic transmission loss predictions are derived first by solving the governing dispersion relation (see for example Cummings and Sormaz [7]) using the Newton Raphson method and then by employing the orthogonality relation reported by Cummings [10] to mode match over each discontinuity. Truncation at 25 modes was necessary to obtain a good approximation of the actual solution. On obtaining 25 roots to the dispersion relation, the following integral relation was used in order to check that no modes were missing:

$$\frac{1}{2\pi i}\int_C \frac{f'(S)}{f(S)}\,dS = N - P \qquad (16)$$

Here $f(S)$ represents the dispersion relation and the contour C does not pass through any zeros or poles of $f(S)$. The integers N and P represent the number of zeros and poles interior to C. The above relation does not, or course, guarantee the location of all relevant modes, it merely indicates when modes are missing. Consequently, only those transmission loss predictions in which the above integral has first been satisfied are compared with finite element point collocation predictions.

Transmission loss predictions computed using numerical point collocation and analytic mode matching are compared for silencer 1 (see Table 1) in Figure 2. Excellent agreement between the two schemes is observed - the two curves are virtually indistinguishable from one another - up to approximately 1400Hz. A relatively large silencer (overall width of 3.2m) was chosen in order to test rigorously the accuracy of the two techniques. The close agreement between the two predictions appears to validate the accuracy of the point collocation technique and confirms the suitability of using point collocation to analyse dissipative silencers in large HVAC ducts. Furthermore, transmission loss predictions computed using both techniques for silencers 2, 3 and 4 were also virtually indistinguishable from one another and so only the point collocation predictions are presented in Figures 3 to 5. Problems are however evident with the analytic predictions in Figure 2 (although this is difficult to visualise), since transmission loss predictions were unobtainable after 1400 Hz. This was caused by failure to locate all the required roots of the dispersion relation, a task which becomes ever more difficult for large ducts and serves to illustrates the difficulties inherent with the analytic approach. An interesting feature of Figure 2 is however the peak and sudden reduction in transmission loss at approximately 200 Hz. This effect is caused by a higher order mode cutting on in the outlet duct section; the effect is repeated at approximately 420 Hz.

Transmission loss predictions computed using point collocation automatically account for higher order modes and so-called end effects. It is instructive here to compare transmission loss predictions with the traditional modal attenuation predictions which generally ignore these effects. In Figures 3 and 4 point collocation transmission loss predictions are compared with fundamental mode attenuation predictions for two splitter silencers (see Table 1) both 1m long. It is evident here that the attenuation curves under-predict the overall sound power loss by as much as 10 dB, although this varies for silencer and excitation frequency. This difference appears to correlate with the heuristic figure attributed to end effects by Ramakrishnan and Watson [2]. The biggest discrepancies between point collocation and fundamental mode attenuation appear to occur just prior to the cut-on frequency of higher order modes in the outlet duct, a phenomena which the attenuation curves cannot, of course, account for. The above observations for silencers 2 and 3 may not of course be valid for all silencers, or for different types of material. For example, the attenuation of the fundamental mode will not necessarily always closely agree with transmission loss predictions. In Figure 5, the fundamental mode for silencer 4 is evidently not always the least attenuated mode and one can see that the transmission loss of the silencer does in fact closely follow the least attenuated mode, but not the fundamental mode, although as before the attenuation predictions generally underestimate silencer transmission loss. Inevitably, only a small number of silencers have been studied here - silencer 4 was also contrived (a very low flow resistivity was used) to demonstrate a desired effect - and it remains possible therefore that much larger discrepancies between the modal attenuation and point collocation predictions may arise for different silencers.

4. CONCLUSIONS

A numerical point collocation scheme has been validated here for finite length dissipative silencers placed in large HVAC ducts and thus point collocation appears to be a suitable technique with which to model more complex HVAC systems. Point collocation also offers a reliable, robust and computationally efficient scheme which may readily be applied as an iterative design tool.

For the silencers studied here the traditional, least attenuated mode, predictions have been shown to under-predict the silencer transmission loss by up to 10dB. This difference is probably attributable to the so-called silencer end-effects. Whilst the least attenuated mode predictions generally perform well here, one cannot be certain that this is universally true and so to fully characterise the acoustic performance of dissipative silencers in HVAC ducts one should, ideally, model accurately the silencer end effects.

5. REFERENCES

[1] C. Wassilieff 1987 *Journal of Sound and Vibration* **114**, 239-251. Experimental verification of duct attenuation models with bulk reacting linings.

[2] R. Ramakrishnan and W.R. Watson 1992 *Applied Acoustics* **35**, 1-24. Design curves for rectangular splitter silencers.

[3] K.S. Peat and K.L. Rathi 1995 *Journal of Sound and Vibration* **184**, 529-545. A finite element analysis of the convected acoustic wave motion in dissipative silencers.

[4] A. Cummings and I.-J. Chang 1988 *Journal of Sound and Vibration* **127**, 1-17. Sound attenuation in a finite length dissipative flow duct silencer with internal mean flow in the absorbent.

[5] F.P. Mechel 1990 *Acustica* **70**, 93-111. Theory of baffle-type silencers.

[6] R.J. Astley, A. Cummings and N. Sormaz 1991 *Journal of Sound and Vibration* **150**, 119-138. A finite element scheme for acoustic propagation in flexible-walled ducts with bulk-reacting liners, and comparison with experiment.

[7] A. Cummings and N. Sormaz 1993 *Journal of Sound and Vibration* **168**, 209-227. Acoustic attenuation in dissipative splitter silencers containing mean fluid flow.

[8] M.E. Delany and E.N. Bazley 1970 *Applied Acoustics* **3**, 105-116. Acoustical properties of fibrous materials.

[9] R. Kirby and A. Cummings 1999 *Applied Acoustics* **56**, 101-125. Prediction of the bulk acoustic properties of fibrous materials at low frequencies.

[10] A. Cummings 1989 *Proceeding of the Institute of Acoustics* **11**, 643-650. Sound generation in a duct with a bulk-reacting liner.

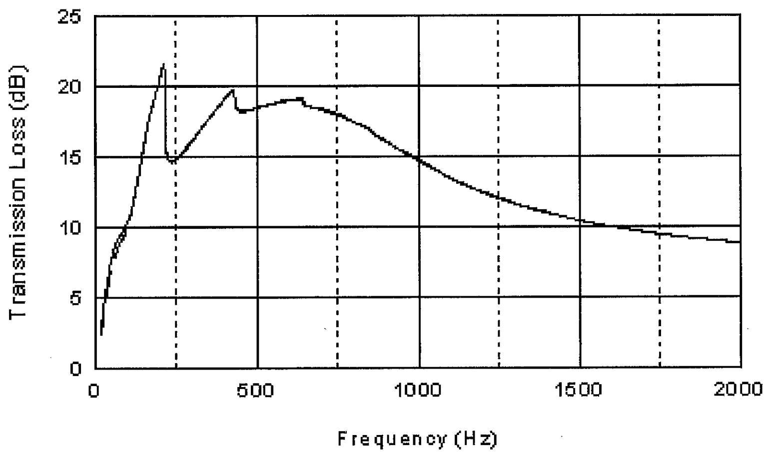

Figure 2. Transmission loss for Silencer 1. ——, Point collocation; — — —, Analytic Solution

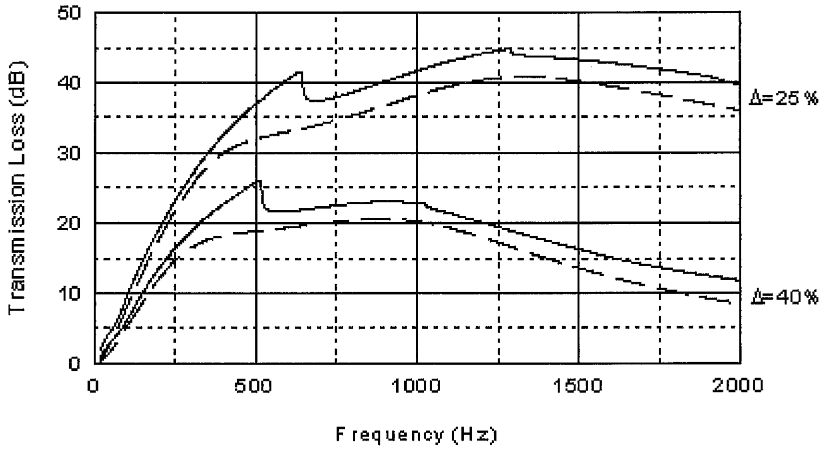

Figure 3. Transmission loss for Silencer 2. ——, Point collocation/analytic; — — —, Fundamental mode.

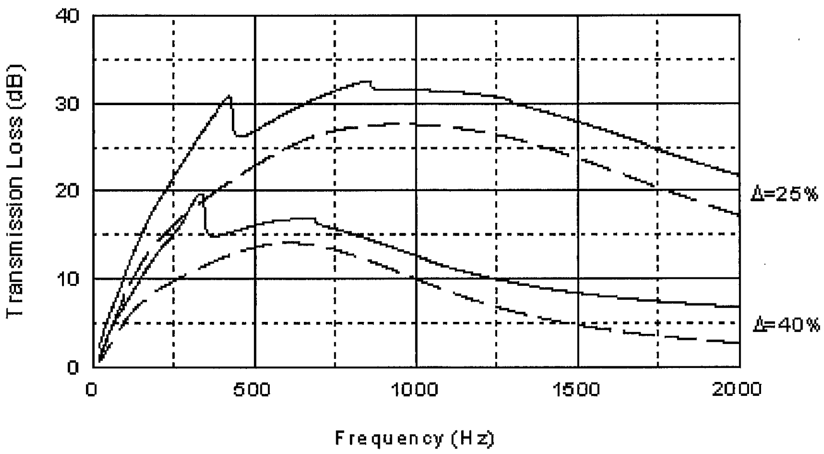

Figure 4. Transmission loss for Silencer 3. ——, Point collocation/analytic; — — —, Fundamental mode.

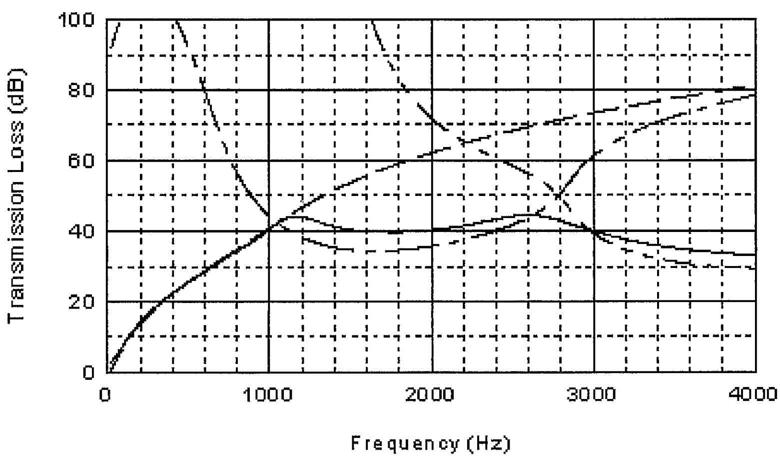

Figure 5. Transmission loss for Silencer 4. ——, Point collocation/analytic; — — —, Fundamental mode; — - —, Mode 1; — - - —, Mode 2.