ACCURATE LOW FREQUENCY IMPEDANCE TUBE MEASUREMENTS

R Oldfield University of Salford, Salford, UK
F Bechwati University of Salford, Salford, UK

1 INTRODUCTION

For many years impedance tubes have been used to measure the normal incidence acoustic absorption coefficient of materials. Two principle techniques are used, namely the standing wave method[1] and the transfer function method[2]. The standing wave method measures pressure maxima and minima of standing waves set up in a tube (with the sample at the end) and calculates the reflection factor from the resulting standing wave ratio and the complex acoustic impedance can also be determined with phase data from the position of the first pressure minimum from the sample surface. This technique is very reliable but can be time consuming as it must be done frequency by frequency; a more time efficient method was therefore introduced that provides easy determination of the acoustic impedance and hence absorption coefficient of samples over a continuous frequency range. This method is often called the transfer function method as it makes use of multiple frequency response measurements at discrete points in the tube and uses the transfer function between these measurements to calculate the complex reflection factor of the sample from equation 1 from which the surface impedance and absorption coefficient of the sample can be determined. The theory of impedance tube measurements is well known and an overview covering both of these measurement techniques can be found in *Acoustic Absorbers and Diffusers* by Cox and D'Antonio[3].

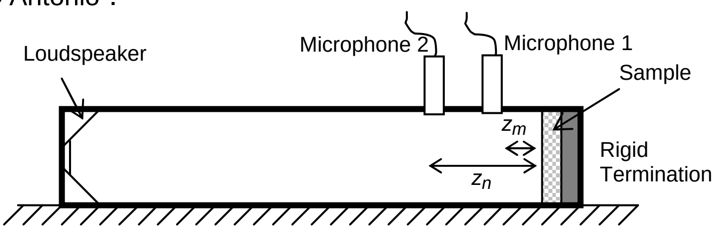

Figure 1: *A schematic representation of a typical transfer function impedance tube*

$$R = \frac{H_{12}e^{jkz_1} - e^{jkz_2}}{e^{-jkz_2} - H_{12}e^{-jkz_1}} \qquad (1)$$

The transfer function method is well used for mid and high frequency testing but for low frequency determination of surface impedance there are a number of practical difficulties that need to be overcome for accurate and stable measurements. This paper presents the design, build and use of a specially constructed impedance tube that allows the accurate measurement of acoustic impedance even down to very low frequencies (17Hz). Low frequency measurements are often characterised by high noise levels and instability; the tube presented here exhibits several features that help to alleviate these problems including the use of phase-matched microphones, thick walls of the tube and four microphones for an increased measurement range. The paper also includes an overview of the equations used for accurate analysis of the results using a least mean squares approach first introduced by Cho[4] and an assessment of the required sound pressure level used for the test signal for increased low frequency stability.

2 LOW FREQUENCY TUBE

When building an impedance tube suitable for the low frequency measurement of normal incidence acoustic impedance, several design issues should be addressed: The considerable size required for large microphone separation, losses through the tube walls at low frequencies as well as the problematically large background noise levels often present at low frequencies. This paper will cover these important issues and some practical points for impedance tube testing. Section 2.1 outlines the design and construction of the tube, section 2.2 presents an overview of a least mean squares technique for processing the results such that they have greater accuracy at low frequencies and section 2.3 presents some practical points of impedance tube testing including an analysis of the test signals used, the use of phase-matched microphones and the rigid termination of the tube.

2.1 Design and construction

In order to perform measurements at low frequencies, the tube has to be long enough to allow for large microphone spacing, with the microphone spacing needing to be around 1/20th of the wavelength at the lowest required frequency of measurement. The microphone positions and the diameter of the tube were chosen in such a way as to satisfy the following equations for the upper and lower frequency limits of impedance tubes[1, 2].

$$f_u < \frac{c}{2d} \qquad (2)$$

$$f_u < \frac{0.45c}{|z_m - z_n|} \qquad (3)$$

$$f_l > \frac{0.05c}{|z_m - z_n|} \qquad (4)$$

f_U and f_L are the upper and lower frequency limits; z_m and z_n are the distances from the sample to microphones as shown in Figure 1. The lowest frequency value found from either equation 2 or 3 is taken as the upper frequency limit of the tube. The diameter of tube is d, and the speed of sound is denoted by c. Four microphone positions were chosen to give an operating frequency range between 17Hz and 530Hz, however an upper frequency of 500Hz was chosen for subsequent measurements to ensure that the deformations in the assumed plane pressure waves that might start at frequencies slightly lower than the upper limit do not affect the results.

The tube was constructed from 22mm thick mild steel, thus providing enough mass to significantly reduce losses from the walls of the tube even at the low frequency end of the measurement bandwidth. The inner diameter of the tube was chosen as 324mm, to be sufficient for the measurement of large samples; the tube thus allows even the measurement of some extended reactors such as resonant membrane absorbers which are usually designed for use in the low frequency region and are relatively large in size. The tube presented here was designed to be standing upright to conserve space and to allow the testing of granular materials. A schematic diagram of the tube is shown in Figure 2.

The impedance tube consists of four sections of steel, two making up the main chamber and a further two sections to be used as sample holders (a large one and a smaller one) allowing for different testing conditions i.e. absorbers with different backing volumes. The sample holder is placed on the trolley and then jacked into place using the bottle jack to allow for a very tight fit, ensuring no small cracks for leaks between the sections of tubing. A rigid backing of 50mm thick MDF was bolted on to the end of each sample holder providing a very large termination impedance. The chosen loudspeaker was a 12” Eminence driver within a standard MDF cabinet; the rim of the cabinet was routed so it would fit tightly on the top of the tube and could not fall off. Plugs were made to block the microphone holes not in use during testing so there would be no leaks in the tube. Mineral wool was placed in the top section of the tube in order to reduce any cross modes that might be generated by the loudspeaker especially at higher levels and at higher frequencies when pistonic motion of the driver can no longer be guaranteed, allowing only plane wave propagation down the tube.

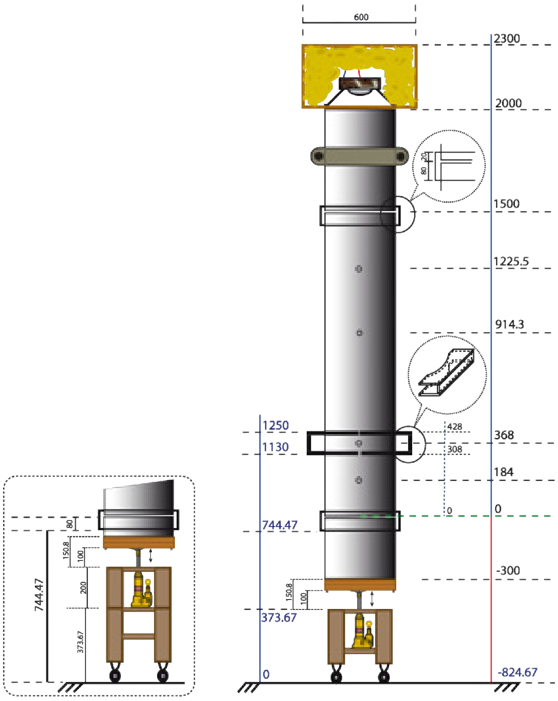

Figure 2: *A schematic representation of the low frequency impedance tube*

2.2 Processing the results

Using the transfer function method it is possible to plot six separate curves pertaining to the transfer functions between each combination of microphone pairs. There will however be substantial overlap in frequency between these six transfer functions so for increased accuracy a more elegant method of processing results that combines the data from all the transfer functions from multi microphone position is used, allowing the results over the full frequency range to be plotted with a single line. The improved method proposed by Cho[4] (outlined in section 2.2.1) uses least squares curve fitting to optimize the response from all of the microphone positions to produce a result with minimum error.

2.2.1 Least Squares Optimisation Technique

The least squares method uses an idea of an imaginary source equidistant from the sample but in the negative *x* direction as shown in Figure 3. Initially this is considered for a two microphone case although the theory can be extended to work for multiple microphone measurements.

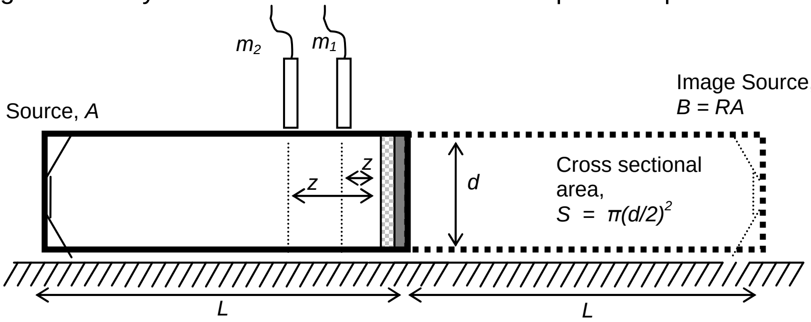

Figure 3: *Image source technique for multiple microphone positions*

Assuming the signal strength from the actual source is of amplitude, *A* and the amplitude from the image source is:

$$B = RA \tag{5}$$

Where *R* is the complex reflection factor, the pressure at each microphone can be deduced from a Green's function relating the output from each microphone to the input of the each source (both actual and image):

$$p_1 = Ag_{1A} + Bg_{1B} = A\left(g_{1A} + Rg_{1B}\right)$$
$$p_2 = Ag_{2A} + Bg_{2B} = A\left(g_{2A} + Rg_{2B}\right) \tag{6}$$

The Green's Function g_{1A}, relates the output of microphone 1 to the input source *A*, g_{1B} relates the output of microphone 1 with the input source of the imaginary source *B*. g_{2A} and g_{2B} are the same but relate to microphone 2. In an impedance tube these Green's functions can be approximated by equation 7 as there is plane wave propagation in the tube:

$$g_{1A} = \frac{\rho_0 c}{2s} e^{-jk(L-z_1)}, \qquad g_{1B} = \frac{\rho_0 c}{2s} e^{-jk(L+z_1)}$$
$$g_{2A} = \frac{\rho_0 c}{2s} e^{-jk(L-z2)}, \qquad g_{2B} = \frac{\rho_0 c}{2s} e^{-jk(L+z2)} \tag{7}$$

where *s* is the cross sectional area of the tube. Taking the transfer function of the pressures from each microphone as given in equation 6 gives:

$$H_{12} = \frac{p_2}{p_1} = \frac{g_{2A} + g_{2B}R}{g_{1A} + g_{1B}R} \tag{8}$$

Rearranging this equation allows the complex reflection factor to be given as:

$$R = \frac{g_{2A} - g_{1A}H_{12}}{g_{1B}H_{12} - g_{2B}} \tag{9}$$

Cho defines both a theoretical and a measured transfer function given as H_{12} and $\hat{H}_{12}$ respectively. He uses a least squares solution to obtain an optimally estimated reflection factor given as:

$$R_{opt} = \frac{g_{2A} - g_{1A}\hat{H}_{12}}{g_{1B}\hat{H}_{12} - g_{2B}} \tag{10}$$

This is the same as the theoretical reflection factor given by (9) but the theoretical transfer function is replaced with the measured transfer function. The theory is initially applied to a two microphone case but is later extended to work for tubes with multiple microphone positions such as the one presented here. Using the same method as above an optimised reflection factor from the least squares solution for multiple microphone positions can be given as:

$$R_{opt} = -\frac{\sum_{m=2}^{M}\left(g_{1A}\hat{H}_{1m} - g_{mA}\right)\left(g_{1B}\hat{H}_{1m} - g_{mB}\right)^*}{\sum_{m=2}^{M}\left|g_{1B}\hat{H}_{1m} - g_{mB}\right|^2} \tag{11}$$

This result is obtained by minimising the sum of the errors between the measured and the analytically derived pressures. It can be used for any number of microphone positions, *M* where *m* is the number of each microphone in the tube. This reflection factor can then be used to determine the acoustic impedance and absorption coefficient in the normal way (13). Cho's original method references all of the microphone positions to microphone *A* and calculated the least squares results from that. An extended version of this method can also be formulated that references all of the possible permutations of microphone combinations rather than just the three that related to microphone *A*. As a result a new equation based on all five of the microphone transfer functions can be given as:

$$R_{opt} = -\frac{\sum_{n=1}^{M-1}\sum_{m=n+1}^{M}\left(g_{nA}\hat{H}_{nm} - g_{mA}\right)\left(g_{nB}\hat{H}_{nm} - g_{mB}\right)^{*}}{\sum_{n=1}^{M-1}\sum_{m=n+1}^{M}\left|g_{nB}\hat{H}_{nm} - g_{mB}\right|^{2}} \qquad (12)$$

where *n* is the number of the reference microphone and *m* is the number of the second microphone. The surface impedance and absorption coefficient are then obtained from the following two expressions:

$$Z_s = \rho_0 c\left(\frac{1+R_{opt}}{1-R_{opt}}\right)$$
$$\alpha = 1+\left|R_{opt}\right|^2 \qquad (13)$$

Using the extended method increases the number of transfer functions on which the least squares routine can be applied to and therefore is deemed to be a more accurate method for processing impedance tube results. In order to check the validity of the generalised expression for the reflection coefficient, presented in (12), measurements were performed on a sample of mineral wool, 12cm thick. Results from the raw data versus Cho's 4-microphone version of the optimised absorption coefficient and its generalised counterpart are shown in Figure 4 below.

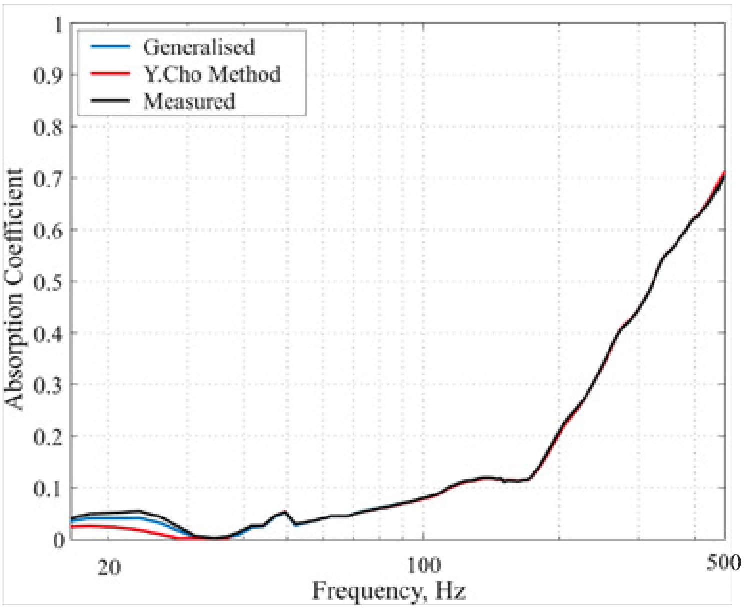

Figure 4: *Plot showing the validity of the generalised version of Cho's least-square optimisation method, showing a better agreement at low frequencies with the measured data for a 12cm thick mineral wool sample. Note that the measured plot was obtained by manually overlapping the measured data for each microphone pair according to the corresponding frequency range each best describes.*

It is clear from the above plot that the generalised version of Cho's estimation method, although very similar to the standard four-microphone version above 40Hz, does achieve a slightly better prediction at the very low frequencies.

In order to check the validity of the results with other impedance tubes, the same mineral wool sample used above is then tested in a smaller tube, capable of a measurement bandwidth between 200Hz and 1700Hz. An overlap of 300Hz is then covered between the two tubes. Being an uncommon piece of equipment, it was difficult to fully commission the low frequency impedance tube, where usually samples would be tested in other similar devices. Measurement results, presented in Figure 5 below, do however show good correlation between the two impedance tubes used and also with a simple Delany and Bazley[5] prediction model. Results from the large tube, although not exactly similar at the higher frequencies to the low frequency results of the small impedance tube, do show a comparable trend in the absorption coefficient of mineral wool between the two tubes.

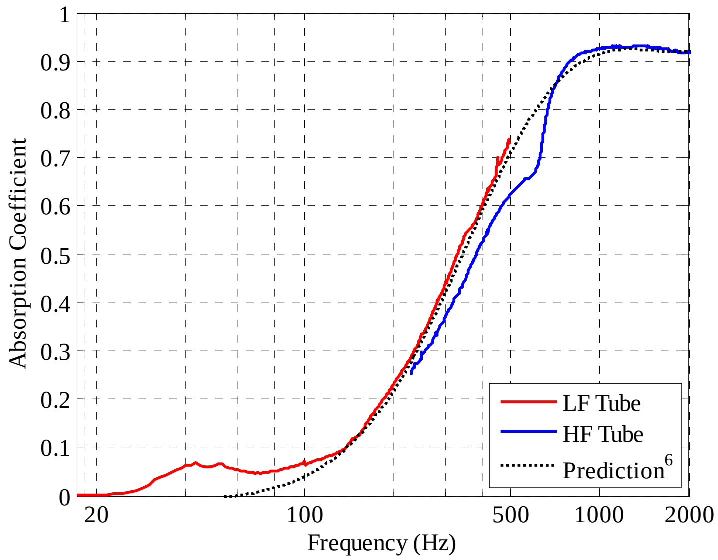

Figure 5: *Results for absorption coefficient of a 12cm thick sample of mineral wool as measured in the large low frequency impedance tube and a smaller high frequency tube.*

2.3 Practical Points

This section outlines a number of practical considerations for low frequency impedance tube testing including the type and level of the test signal, microphones used and the rigid back of the tube.

2.3.1 Test Signal

Signal Type

An issue that requires appropriate consideration when building an impedance tube is the type of the input signal used and its amplitude inside the tube. In addressing the first issue, short reference to the two most common types of input signals will be mentioned. The first is a broadband white noise signal which, whilst still enjoying wide practice due to the well-established methods and relative ease in processing results, is not suitable for low frequency impedance tube measurements. The main problem from using non-deterministic time-variant signals such as white noise is the time needed to perform several averages, especially important if accurate measurements are required at the very low frequencies. A deterministic noise function such as an MLS[6] signal is therefore often preferable, this has the advantage that the impulse response can be measured at several locations using the same microphones. Another signal type is a swept sine wave[7], this is a deterministic, linear and time-invariant signal; it is characterised by a sweep in the frequency domain between a pre-determined start and end frequency, these are usually chosen to correspond to the high and low frequency limits of the measurement system (17Hz and 500Hz respectively in the case of this low frequency impedance tube). The use of such a signal type has many benefits concerned with time, harmonic distortion and measurement errors. Less time is required to perform each measurement and errors in the results are reduced due to the time invariance of the signal. Distortion is eliminated from the measurement by the matched filter deconvolution process, where the harmonic components are shifted forward in time to precede the impulse peak such that they may be removed by temporal gating.

Signal Strength

Regarding the appropriate signal strengths that can be used in an impedance tube, care must first be taken so that enough energy is supplied at all frequencies of measurement and that the signal levels measured at all microphone positions are significantly larger than background noise levels at those frequencies. Conversely, care needs to be taken so that not too much energy is passed into the loudspeaker such that it is driven into regions of nonlinear excursion. Consequently a study was done on the measurements performed in the low frequency tube with varying input signal amplitudes. A set of figures is presented below to illustrate the effect on surface impedance and absorption coefficient results for the same swept sine input signal at several amplitudes.

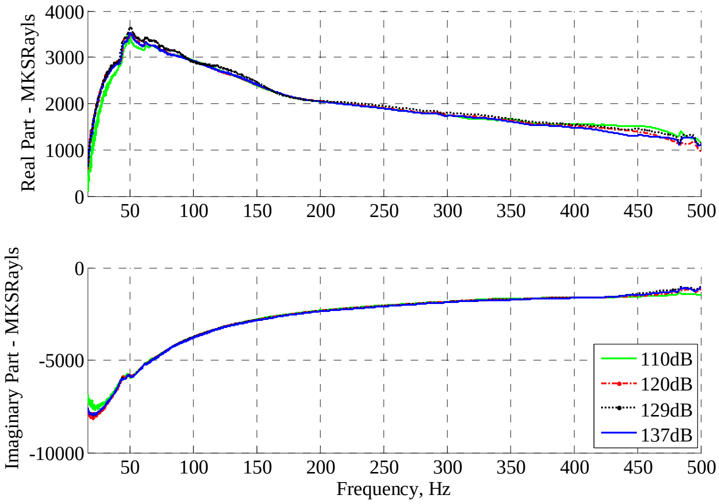

Figure 6: *Surface impedance plot for a granular material under various input signal amplitudes.*

Figure 7 shows a magnification of the above impedance plots around the low frequency region where the issues of linearity are of greatest importance for the use of this tube.

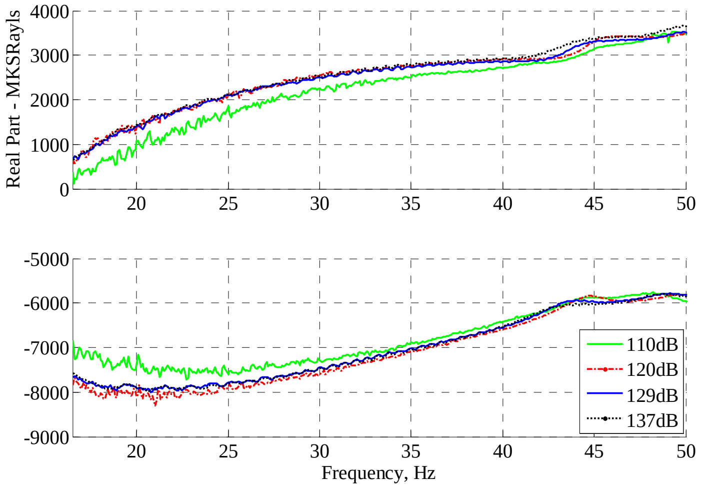

Figure 7: *Magnification of the surface impedance measurement at low frequencies.*

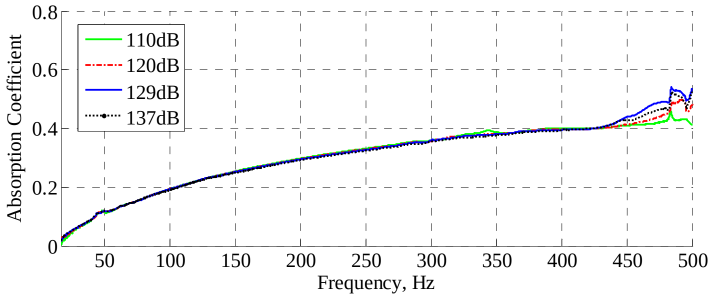

Figure 8: *Absorption coefficient plot for a granular material under various input signal amplitudes.*

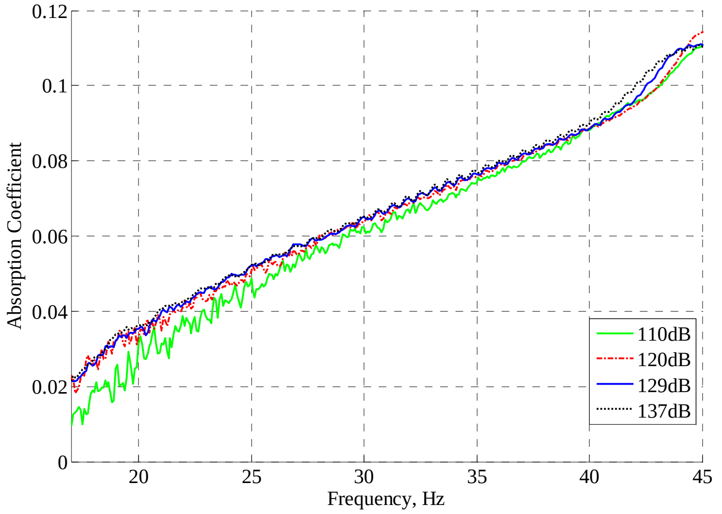

Figure 9: *Magnification of the above absorption coefficient plot.*

The amplitude values listed in the legends above refer to maximum sound pressure levels measured inside the tube during the measurement. As the amplitude of the input signal is increased, noise in the low frequency response of the measured surface impedance and absorption coefficient is reduced, as can be seen from Figure 7 and Figure 9. Results from measurements performed at amplitudes above 129dB seem to be almost noise-free at low frequencies, whereas below 110dB, measurement results for the real and imaginary parts of the surface impedance fall slightly short of predicting the correct values. This could be due to a relatively small signal to noise ratio, considering that the actual sound pressure levels at the very low frequencies would be slightly smaller than the maximum values listed in the legends of the above figures. A plot of the measured sound pressure level at the four microphone positions is shown in Figure 10 below for the loudest case. In Figure 8, the differences in results seen in the high frequency response are due to the resonance of the granular sample that was tested; as the amplitude is increased, the sample resonance is enhanced in relation to the energy in the input signal.

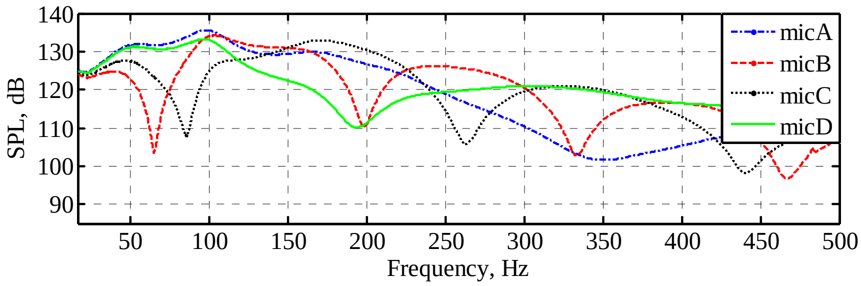

Figure 10: *Plot of sound pressure levels at the various microphone positions, referring to an input signal with the largest amplitude.*

2.3.2 Microphones

Traditionally, when the transfer function method is used for impedance tube measurements, pressures are recorded at the various fixed microphone positions using the same microphone to ensure appropriate phase matching between the measurements, given a deterministic test signal with background noise levels low enough to not have any effect and losses in the tube being negligible. However, when results are required at very low frequencies (as low as 17Hz for example); this method becomes unsuitable due to the variance in the background noise spectrum with time. For that reason, a sound-intensity microphone pair is used to measure pressures simultaneously at two microphone locations, ensuring appropriate phase matching across the whole frequency range of operation. One microphone is fixed at the location nearest to the sample and the second changed over between the three other microphone positions for each measurement, to increase accuracy still further 4 phase matched microphones should be used to measure at the four microphone positions simultaneously, however this presents both calibration and cost issues. Figure 11 below illustrates the coherence between the two microphones for a typical measurement. It is worth noting that in all the measurements performed using the sound-intensity pair, the coherence between the two microphones was always very close to unity across the measurement frequency range, thus the results taken using this method have minimal error and maximal repeatability.

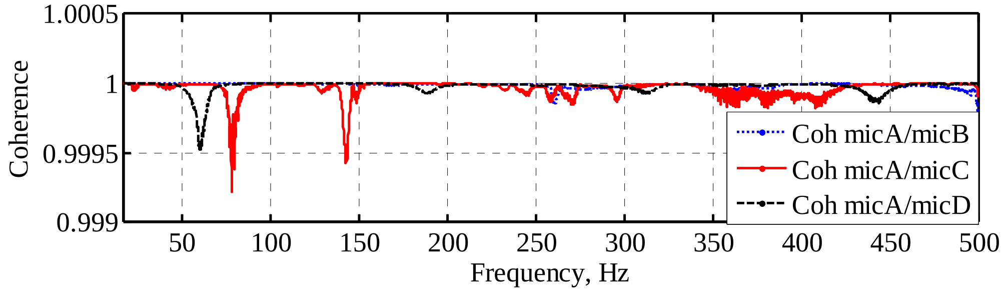

Figure 11: *Coherence plot for measured pressures at the various microphone positions.*

2.3.3 Rigid Backing

Another important issue that should be considered when constructing a low frequency impedance tube is the rigid backing. The rigid backing is designed such that the termination impedance of the empty tube is as high as possible. The reason for this is so it is only the sample that is being measured and the backing has no influence on the results at all. This is generally the case with higher frequency impedance tubes; however at lower frequencies it is important to put extra effort in when thinking about the rigid backing as any vibrations of the backing or resonances will cause spurious results. It was initially found that there was an undesirable resonance of the rigid backing so the mounting condition was improved by exerting a greater force with the jack and over a wider surface area such that the effects of this resonance were diminished and absorption results were no longer affected. To more closely analyse the effects of the rigid backing on the results, an accelerometer was used to measure the velocity response as shown in *Figure 12*.

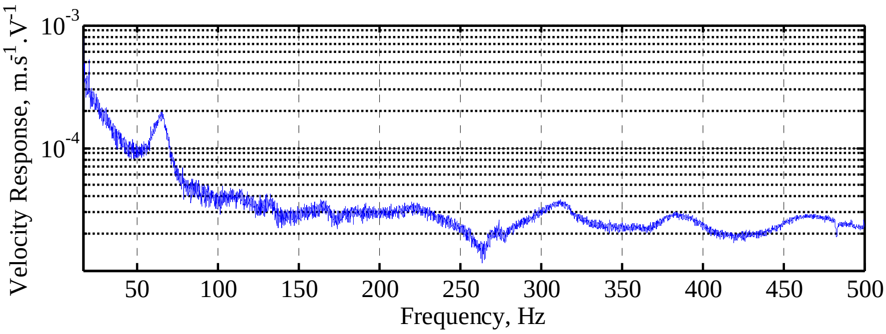

Figure 12 *Velocity response of the rigid backing*

At low frequencies, the velocity increases exponentially and the assumption of a rigid backing condition starts to break down. The effects of this low frequency velocity response on measurements can be seen in Figure 13 and in subsequent absorption coefficient results as very small increases at the very low frequencies in some samples tested. Even though the backing condition cannot be considered rigid at such low frequencies, the effects observed are quite small since the maximum velocity amplitude at the lowest measurement frequency is only 0.002m.s^{-1}. A test was also performed in the empty impedance tube to see how the velocity response affected the absorption of the rigid backing; the results are seen in *Figure 13*.

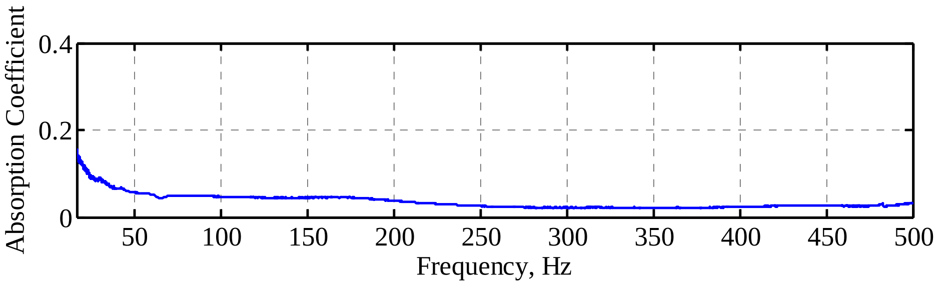

Figure 13 *Absorption coefficient of the rigid backing as measured in the empty tube*

There is an increase in absorption of the rigid backing as a result of its increased velocity response below 50Hz. Depending on the sample being tested this can cause an increase in the measured absorption in the very low frequency range, however these affects are only appreciable in samples with very low absorption coefficients in this region.

3 CONCLUSION

This paper has presented the design, build and commissioning of a low frequency impedance tube to be used for the evaluation of the complex surface impedance of different materials and absorber types. It has been built in an upright configuration to allow the testing of granular materials and has a large cross sectional area such that some extended reacting materials can also be tested. The tube has been built to operate over the widest possible frequency range by using four microphones with the results being processed using a least squares optimisation technique to increase accuracy by combining the measurements from all of the microphones into a single curve across the frequency range of the measurement (17Hz-500Hz in this case). Problems due to exterior noise are minimised by the thick walled construction of the tube and a deterministic test signal with suitably large amplitude even in the lowest frequency region. The effects of background noise, most prominent at low frequencies, are circumvented still further by utilising a pair of phase-matched microphones to measure pressures at two locations simultaneously hence making the transfer function between the two microphone signals independent of background noise. A brief analysis of the suitable levels for the test signal is given such that a level of 137dB is used for the lowest frequency measurements (providing the loudspeaker is not operating non-linearly such that it no longer produces plane waves). Attention is also briefly given to the topic of the rigid backing of the tube and the affects that this can have on measurements.

4 REFERENCES

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