THE NORMAL MODES OF CYMBALS

R.Perrin Institute of Fundamental Sciences, Massey University, Palmerston North, NZ,
G.M.Swallowe Physics Department, Loughborough University, Loughborough, LE11 3TU, UK
S.A.Zietlow and
T.R.Moore Rollins College, Winter Park, FL32789, USA

1 INTRODUCTION

In a previous report[1] we described an investigation into the normal modes of an 18 inch crash cymbal. Various experimental methods were used in turn including electronic speckle pattern interferometry (ESPI), laser vibrometry and Chladni sand patterns as well as more conventional approaches involving transducers and accelerometers. Over 100 modes, plus many split "degenerate" partners, were found and identified. The results were compared with the predictions of a finite-element model and the requirements of group representation theory. Although overall agreement was good it was found that, contrary to prediction, the vast majority of patterns produced were of mixed symmetry types. This could not be explained by conventional mode coupling because all the modes involved had high Q values, of the order of 10^3 as measured by the decay method. We have further investigated some of these mixed modes by using ESPI and laser vibrometry simultaneously. Additionally the combination of ESPI and laser vibrometry has been used to study the modes of a 12" ridged cymbal. This is stiffer than the cymbal previously investigated and its profile does not include the 'lip' formed by a sharp change in slope close to the outside of the 18" cymbal. Profiles are illustrated in Figure 1. After reviewing some background theory and the previous results, we shall discuss some of the findings of these new experiments and their interpretation.

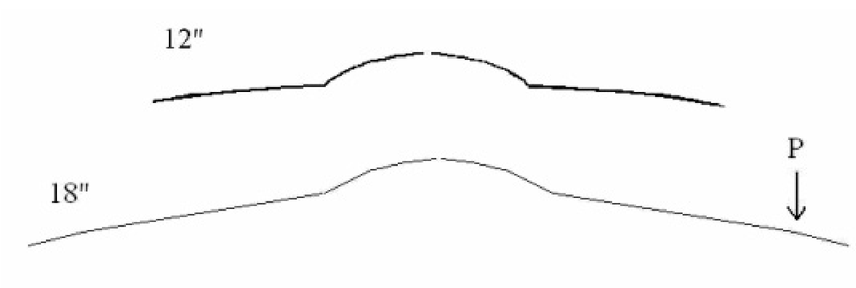


Figure 1: Profiles of the two cymbals investigated. The 18" cymbal has a sharp change in slope at the radius indicated P.

1.1 SOME BACKGROUND THEORY

Cymbals can be regarded as perturbed flat circular plates with retained axial symmetry. The consequences are, as for any axially symmetric system[2] that the normal modes occur in degenerate pairs whose modal functions vary like sin(mθ) and cos(mθ) where θ is the polar angle and m = 0,1,2,... The nodal patterns therefore consist of m equally spaced "diameters" and n circles concentric with the rim. The value of m determines the symmetry type of a mode. Those with m = 0 (axisymmetric) are exceptional in being singlets, the other modes all being doublets. Any degeneracies beyond these doublets are "accidental", having no connection with the symmetries, and are expected to be rare. In practice the axial symmetry will always be broken, at least slightly, so the doublet frequencies are non-degenerate and the locations of the nodal diameters become fixed with those of one member being exactly midway between those of its partner. For the flat circular plate the label (m,n) identifies a doublet uniquely. While the label is also useful for cymbals it transpires that, as for bells[3], the value of n can lead to ambiguities; for instance, two modes can have equal numbers of circles but located in different places. In the present report, when needed, we use subscripted labels A and B to differentiate between doublet members.

2 REVIEW OF RESULTS

2.1 MODAL FREQUENCIES

The finite element model we used was fully described in the previous paper[1], as were the experimental methods employed and the detailed process of mode identification. In Figure 2 we show an overall summary of the experimental data up to about 3000Hz for the 18" cymbal. A similar data set for the 12" cymbal is illustrated in Figure 3.

We emphasise that, in line with the symmetry requirements, the model predicted all modes to be of pure symmetry types with those having m > 0 in exactly degenerate orthogonal pairs. The experiments confirmed that most of these modes were actually in the form of split pairs. The convention of always using the higher of the two, usually close, frequencies has been adopted in Figure 2 and other comparisons with theory presented later. Many of the gaps in the experimental data for the 12" cymbal are due to the presence of mixed modes which make the identification of modal frequencies very difficult as described below.

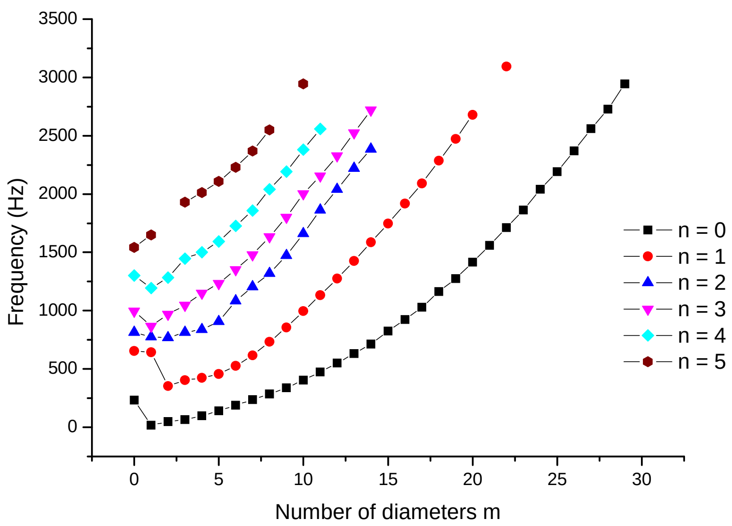


Figure 2: Summary of experimental data for 18" cymbal

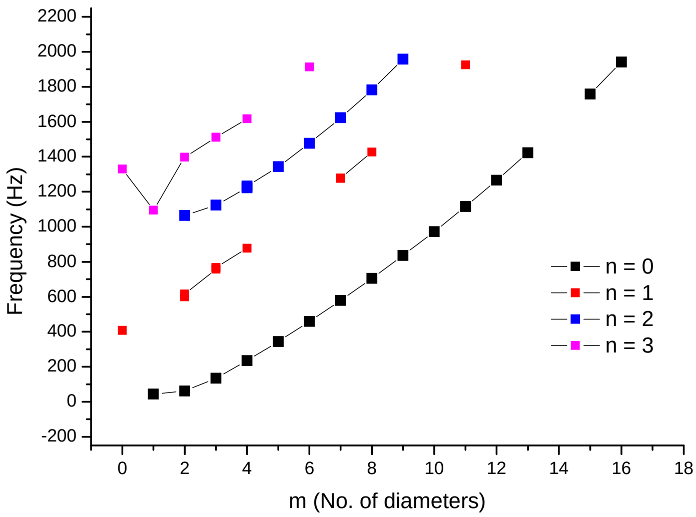
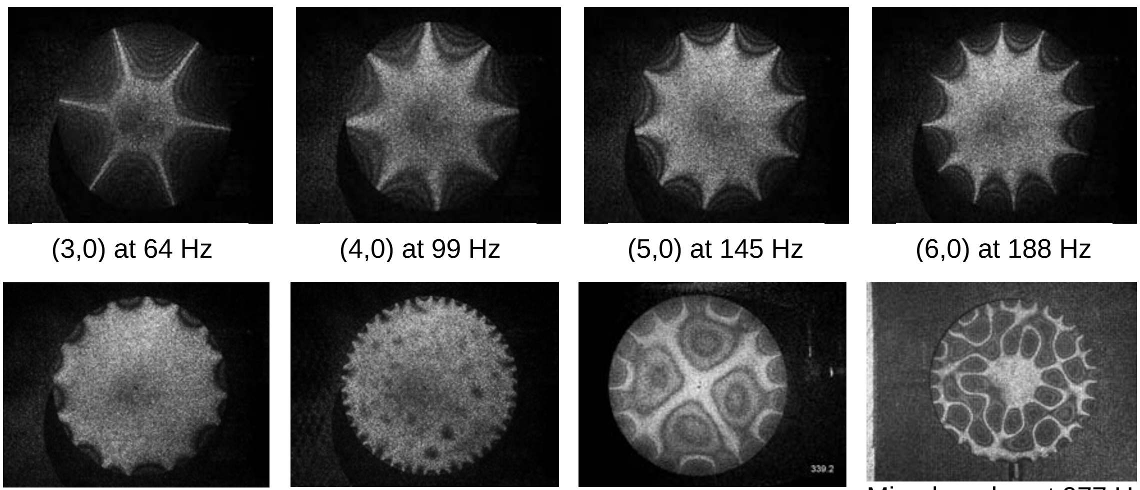


Figure 3. Summary of experimental data for 12” cymbal.

2.2 EXPERIMENTAL NODAL PATTERNS

AN ESPI system[4] was used to obtain nodal patterns using acoustic drive at low amplitude. Typical maximum amplitudes of oscillation for the strongest modes were of the order of 20μm with most modes having maximum displacements of < 5μm. Selections of modal patterns for the 18” cymbal are shown in Figure 4 and the 12” cymbal in Figure 5.



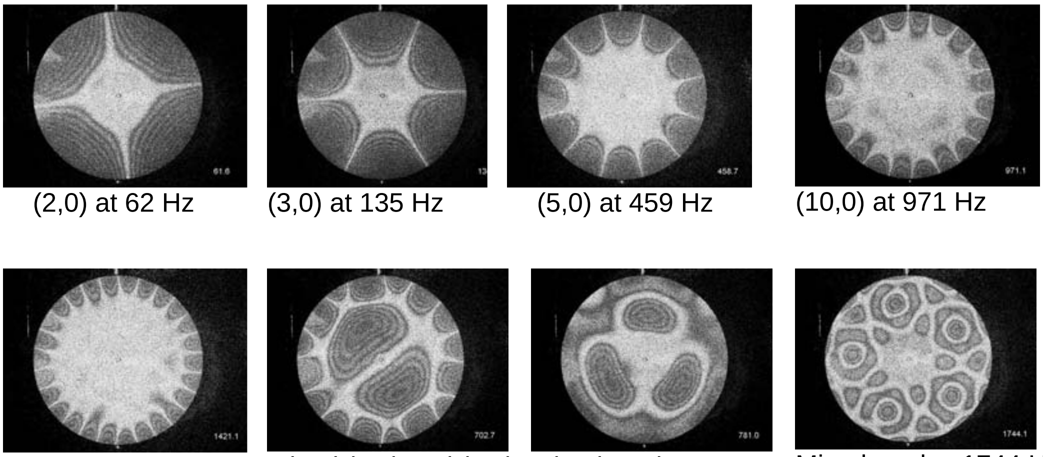
(3,0) at 64 Hz | (4,0) at 99 Hz | (5,0) at 145 Hz | (6,0) at 188 Hz

(8,0) at 285 Hz | (23,0) at 1864 Hz | Mixed (9,0) and (2,1) | Mixed modes at 977 Hz

Figure 4: Modes of the 18” cymbal

It is clear from Figure 2 that the n = 0 modes are the lowest frequency family. They are also the easiest to excite and differ from those with higher values of n in that they are subject to a more extreme form of inextensibility[5]. In Figure 4 we show the patterns of the n = 0 modes for m = 3-6, 8 and 23 for the 18” cymbal. The lower modes are clearly all of pure symmetry types. As m increases the evanescent region grows until, by about m = 8, the pattern has become restricted to the outer rim of the cymbal. This is just a reflection of the analytical forms of the $J_m(kr)$ and is why they are sometimes called “rim modes”. For m > 8 the rim modes still appear but are usually accompanied by a pattern of lower symmetry type in the main body of the cymbal. Thus, for example, the (9,0) appears mixed with the (2,1). For m = 9–14 the mixing is always with a lone (m,1) mode. For m > 14 mixing almost always occurs, sometimes with a mode of higher n or even with some indecipherable combination of higher modes. Only rarely are pure modes observed and the most common patterns are complicated mixed modes as in the 977 Hz example.

The 12” cymbal displays a similar pattern but the mode spacing is greater, see Figure 3, leading to a greater number of observable pure modes. The same type of behaviour of the n = 0 modes is observed but, in this case mixing involving (m,0) modes is rare until m > 13. It is however observed for the (8,0) mode, see Figure 5. In the 12” cymbal many unusual, and theoretically impossible if pure, modes with 3-fold, 5-fold symmetry etc. are observed. Two of these are illustrated in Figure 5.



(2,0) at 62 Hz | (3,0) at 135 Hz | (5,0) at 459 Hz | (10,0) at 971 Hz

(13,0) at 1421 Hz | Mixed (8,0) and (1,1) | Mixed modes 781Hz | Mixed modes 1744 Hz

Figure 5: Modes of 12” cymbal

2.3 THE MIXED MODES

A study of all the nodal patterns obtained by ESPI shows that, especially for the 18” cymbal where the mode spacing is closer, there are actually very few pure symmetry type modes present. Careful examination of the data shows that modes only couple with others that are close in frequency, and usually only with their nearest neighbours. The pure modes are always ones that have no near neighbours at all. Figures 4 and 5 show that the low (m,0) modes are all of this type. Looking at the n = 0 curve in Figure 2 we see that, as m increases beyond its lowest values, new modes appear at regular small frequency intervals. The density of modes is quite high and, as the frequency reaches points where higher n values occur, further similar sets of contributions become apparent. The fact that most modes are split doublets will make the density higher still. The chance of any mode having a near neighbour is therefore high once the frequency exceeds that of the lowest (m,0) modes. It seems reasonable that this high density might cause coupling because the neighbouring spectral curves could overlap.

Figure 6 shows part of the spectrum of the 12” cymbal obtained by driving it acoustically and detecting the output with the laser vibrometer. Clearly most of the peaks are extremely sharp, meaning high Q values and little likelihood of resonance curves overlapping. The results for the 18” cymbal are very similar. To confirm this, the Qs of the peaks for both cymbals were measured using the decay method. They were almost all of the order of 10^3 or greater for all modes, including the mixed ones.

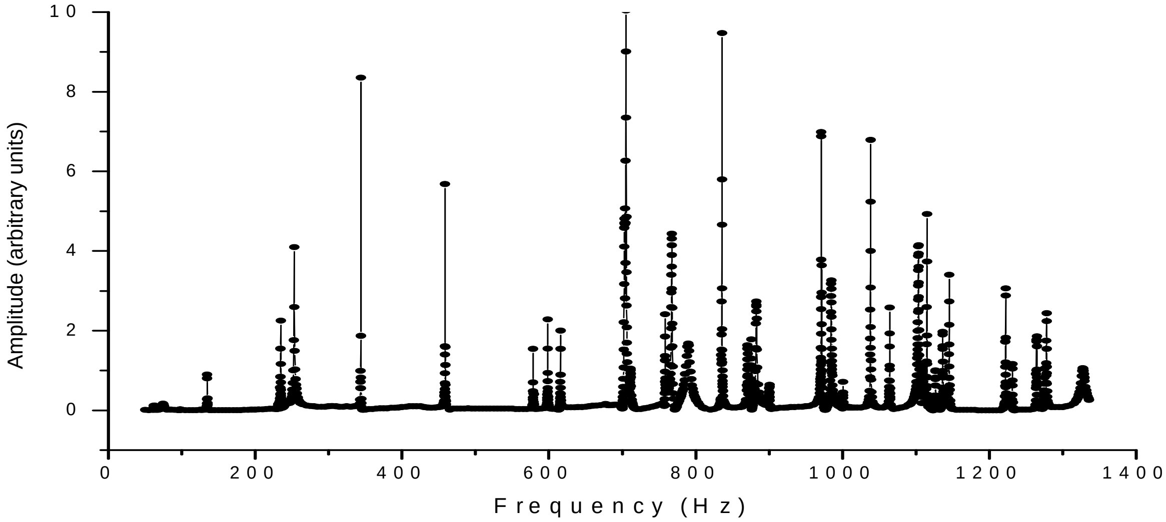


Figure 6: Resonances of the 12" cymbal

2.4 A CASE STUDY: {(2,1) + (9,0)}

As an example we consider the lowest frequency case of mode mixing in the 18" cymbal. This is between the (9,0) and the (2,1) modes. These nearest neighbours are separated by no more than 15Hz. Looking at the appropriate region of the frequency spectrum we would expect to see four peaks, two from each split pair. Figure 7, which is a detailed section of the 18" cymbal spectrum, shows that four is exactly what is seen, and the extreme peak sharpness is again evident. Table1 lists details of the four peaks, including their Q values as measured by the decay method.

Table 1: Details of the {(9,0) + (2,1)} peaks in the 18" cymbal

Frequency (Hz)	Pattern observed	Q (±10%)
339.1	$(2,1)_A$ + $(9,0)_A$	4300
343.1	$(2,1)_B$ + $(9,0)_B$	3600
346.9	$(2,1)_B$ + $(9,0)_B$	4500
354.3	$(2,1)_A$ + $(9,0)_A$	5600

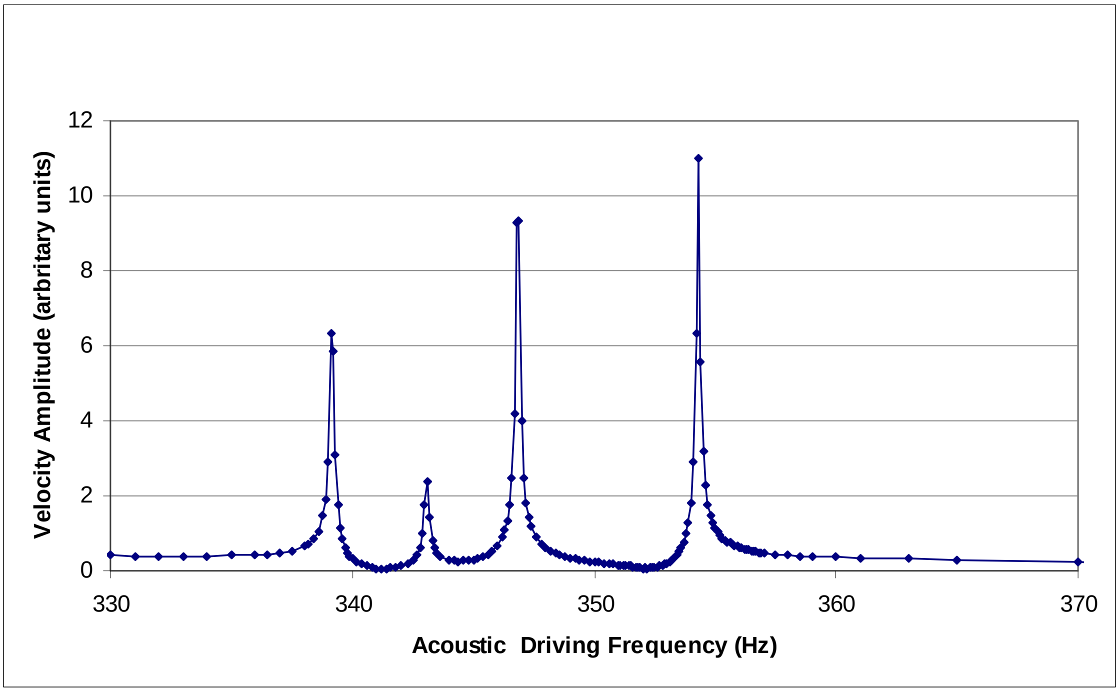


Figure 7: Details of resonances between 330 and 370 Hz

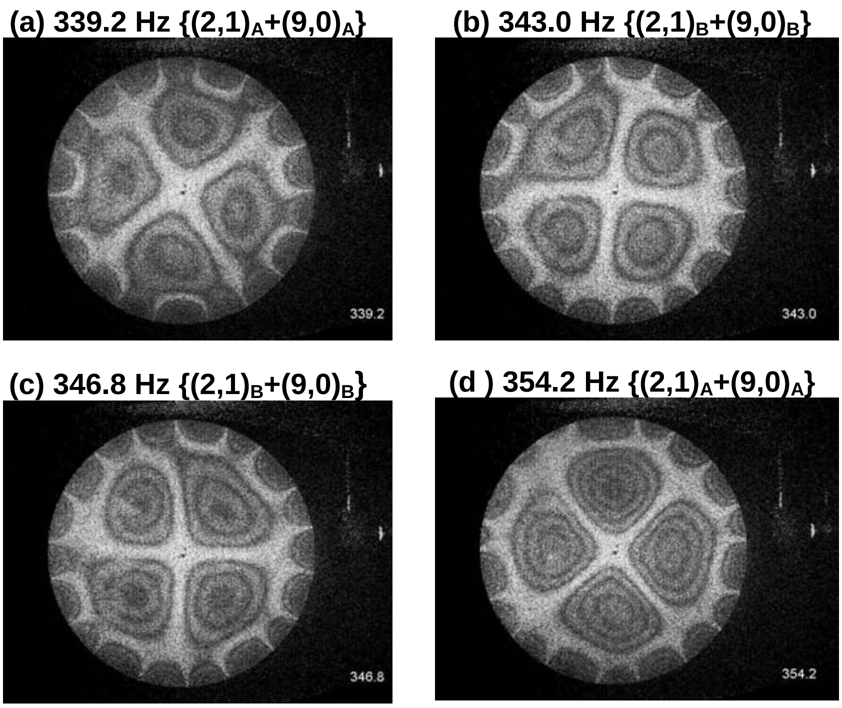


Figure 8: [(9,0) + (2,1)] mixed modes

Driving the cymbal acoustically with low amplitude at the frequency of the lowest of the four peaks gave the mixed mode shown in Figure 8(a). At resonance the vibrometer was used to detect the surface velocity of the cymbal at a point along a radius half way between the centre and the edge. A

Fourier transform of the vibrometer output was used to obtain a power spectrum and search for the frequency components of the oscillation. Despite the pattern being mixed, only one significant peak was observed in the spectrum and this was at the driving frequency. Harmonics of the driving frequency were also observed, but these were typically at least 30 dB lower in amplitude than the driving frequency response. Repeating this procedure with the other three peaks shown in Figure 7, and described in Table 1, produced very similar power spectra and their patterns are shown in Figure 8. The fact that all four observed patterns are mixed underlines the difficulty of uniquely identifying the modes; the question being which two peaks should be taken as corresponding to which two split partners? Fortunately, they are close enough in frequency for the choices made not to have much influence on the appearance of the family curves in Figures 2 and 3. It should be noted that, at or near a peak, the patterns all remained mixed no matter how low the driving amplitude.

A detailed look at the four mixed patterns is revealing. Cases (a) and (d) both contain the same (2,1) component while the other two contain its partner, with the diameters rotated through 45°. Identifying the (9,0) components is more difficult because a rotation of only 10° is involved and there is some distortion. However, it appears that (a) and (d) both contain the same component while (b) and (c) both contain the orthogonal one. This is counterintuitive as one might expect each of the four to uniquely couple with each of the other pair. However, higher frequency clusters seem to follow a similar pattern, although the components of the rim modes become increasingly difficult to distinguish. In particular {(6,1) + (12,0)} seems to behave in exactly the same way. As one goes to yet higher frequencies the patterns become, in general, ever more difficult to decipher making it difficult to be certain that the coupling always follows this pattern. Similar mixing is observed with the (8,0) and (1,1) modes in the 12" cymbal: an example of one the four patterns from this set is shown in Figure 5. The same difficulty in deciphering the modes involved in the mixing also arises at higher frequencies in the 12" cymbal.

3 DISCUSSION

The question remains as to what is the mechanism for coupling the various split doublet components together. The original experiments were carried out with the 18" cymbal and it was initially considered possible that the sharp slope change near the outer edge might contribute to the generation of the mixed modes. However, similar behaviour is observed in the 12" cymbal which has a continuous slowly curving profile so the effect must be common to cymbals in general. The high Q values seem to rule out the normal linear explanation since, when looked at in detail (see Figure 7), there is virtually no overlap of the resonance peaks.

The cymbal is very well known as displaying non-linear and even chaotic behaviour[6,7]. The vibrations of the cymbal have been recorded using the laser vibrometer at each of the observed modes and Fourier Transforms of the data made. These showed no evidence of significant amplitude harmonics or sub-harmonics being involved in the coupling, as were observed by Legge and Fletcher[8] who reported amplitudes of vibration of up to 2mm. Indeed it is clear from the FTs that only one significant frequency is present in any of the modes when acoustically excited at low amplitudes. However, these non-linear phenomena were easy to provoke and generation of significant harmonics was observed when the amplitude of cymbal vibration exceeded 20 µm. Thus it is clear that the cymbal displays non-linearity down to very small amplitudes, but not at amplitudes as low as those reported in this work.

Modes with $m>0$ always occur in 'degenerate' pairs and it is observed that mixing only occurs between near neighbours. Since the 'degenerate' pairs are mainly split there is a significant perturbation present in the cymbal. The precise nature of this is unknown. First-order perturbation theory predicts not only the splitting of the frequencies of the pairs but also the perturbed modal functions as linear combinations of the unperturbed ones[9]. The magnitude of the contribution of each unperturbed mode is inversely proportional to the frequency difference between the unperturbed mode frequencies. Thus near neighbours tend to couple far more strongly than remoter ones. Therefore the mode coupling we are seeing is almost certainly via the perturbation mechanism rather than non-linear effects.

The odd symmetries observed in some of the patterns can also be explained via perturbation theory. These also occur in closely spaced groups of similar symmetry. For example the 5-fold symmetry example shown in Figure 5 has three other 5-fold symmetry neighbours at 1745, 1755 and 1758 Hz. These can be explained as mixing between the (10,1) and (5,3) modes. Other odd symmetry patterns can be similarly explained.

4 CONCLUSIONS

The normal modes of both crash and normal cymbals are well understood in a general way; however, large numbers of unexpected mixed symmetry patterns occur. The mixing cannot be explained by conventional theory because Q values are very high and the resonances do not overlap. The modes which do mix together are always near neighbours in terms of frequency. The observed mixed mode patterns can all be explained in terms of first order perturbation theory.

5 REFERENCES

1 R. Perrin, G. M. Swallowe, T. R. Moore, S. A. Zietlow, “Normal Modes of an 18 inch Crash Cymbal”, Proc. Inst. Acoust. 28, pp. 653-662 (2006).

2 R. Perrin, T. Charnley, “Group Theory and the Bell”, J. Sound Vib., 31, pp.411-418 (1973).

3 R. Perrin, T.Charnley, J. dePont, “Normal Modes of the Modern English Church Bell”, J. Sound Vib., 90, pp 22-49 (1983).

4 T.R. Moore, A Simple Design for an Electronic Speckle Pattern Interferometer, Am.J.Phys., 72, 1380-1384 (2004). Erratum, Am.J.Phys., 73, 189 (2005).

5 R. Perrin, G. M. Swallowe, “Rayleigh’s Bell Model Revisited”, Proc. Stockholm Acoust. Conf., (SMAC 03) pp. 347-350 (2003).

6 N. H. Fletcher, R. Perrin, K. A. Legge, “Non-linearity and Chaos in Acoustics”, Acoustics Australia, 18, pp. 9-13 (1990).

7 N. H. Fletcher, “The nonlinear physics of musical instruments”, Rep. Prog. Phys., 62, pp. 723-76 (1999).

8 K. A. Legge, N. H. Fletcher, “Non-linearity, chaos and the sound of shallow gongs” J. Acoust. Soc. Am. 86, pp2439-2443 . (1989)

9 S. Gasiorowicz, “Quantum Physics” Chapter 16, pp266-270, Wiley (2003)