# SACRED GEOMETRY AND THE WESTERN BELL

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## 1 INTRODUCTION

The idea that there is a connection between visual beauty or, more often, "visual aesthetic satisfaction" and various aspects of "sacred" geometry is far from new. It goes back at least to Classical Greece and has attracted the attention of various savants down the centuries. It has also been well rehearsed in the popular mathematical/scientific literature over recent decades[1-4]. Among topics considered have been paintings (especially the Old Masters), architecture (including the Parthenon and the Great Pyramids) and natural history (including the human body). Of special interest to the authors has been the use of sacred geometry to analyse the violins of the Cremona school and other stringed musical instruments[5].

While we do not suggest that the Modern Western Bell can compete with the Stradivarius violin in the visual beauty stakes, its form is certainly aesthetically pleasing. Since the bell has hitherto escaped the attention of golden geometers, the present authors decided it would be interesting to see whether it actually contains any of the features associated with sacred geometry. Since only the outer surface of the bell is "on view" we have restricted our analysis to the outer profile: founders would have no reason to make inner profiles look beautiful. In this preliminary study we have, in the main, limited ourselves to church bells, large carillon bells and handbells of relatively recent dates.


Figure 1.

"Sacred" geometry is a generic term applied to the golden ratio and various geometrical figures based upon it. Sometimes it is called the "divine proportion" or the "extreme and mean ratio" and is usually given the symbol φ. It can be defined by reference to Figure 1 where the straight line AB is divided internally at a point C chosen such that the ratio of the whole length to the larger part is equal to the ratio of the larger part to the smaller part.

$$AB/AC = AC/CB \equiv \varphi$$

It is a simple matter to show that[1] $\varphi = \frac{1}{2}(1 + \sqrt{5}) = 1.6180...$

Obviously this is an irrational number. It is equal (coincidentally?) to the asymptotic limit of the ratio of successive terms in the Fibonacci series. Of the many remarkable properties of φ the following two are especially "unusual" and can easily be proved by direct substitution:

$$\varphi^2 = \varphi + 1 \quad \text{and} \quad \varphi^{-1} = \varphi - 1$$

The geometrical figures with which the golden ratio is particularly associated are the golden rectangle (whose long to short sides are in the ratio of φ:1), the golden triangle (an isosceles whose equal sides are in a ratio of φ:1 with the base), the pentagram and the two highest Platonic solids (dodecahedron and icosahedron). In the present work we are concerned only with the first two in this list plus the so-called “golden angle”. Further details will be presented in the text as required.

Hard scientific evidence for a connection between visual aesthetic satisfaction and sacred geometry being “hard-wired” into the human brain came first from the 19th century physicist/psychologist Fechner[6]. He asked a group of subjects to look at a series of rectangles with varying aspect ratios and select the one they found most pleasing. The results showed a significant preference for golden and near-golden cases. Similar results were found for ellipses using ratios of major to minor axes. More recent workers have taken various views but have broadly confirmed Fechner’s findings[7].

# 2 MODERN CHURCH BELLS

## 2.1 $D_5$ Taylor Church Bell

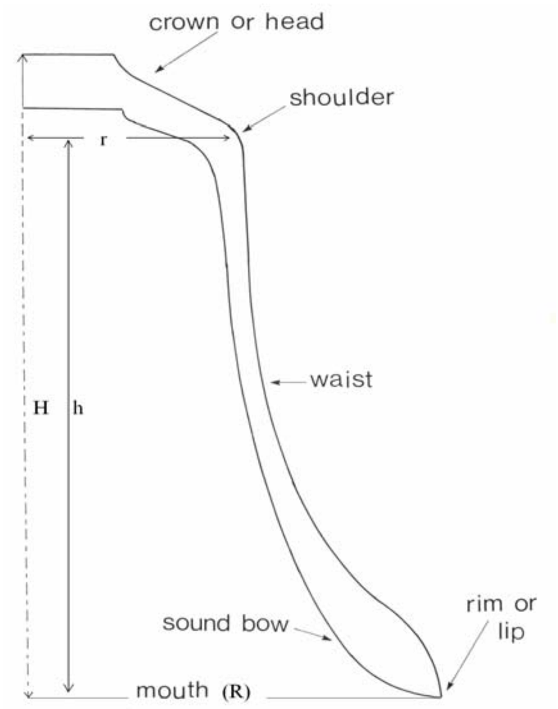


Figure 2. Half cross-section of $D_5$ Taylor church bell

In Figure 2 we have taken a modern 214 Kg Taylor church bell, stood it on a horizontal surface, taken a vertical cross-section containing its symmetry axis and then discarded the left half. Included is some terminology for readers unfamiliar with campanological jargon. The geometry of this particular bell had been measured previously with considerable accuracy for use in a finite-element model[8]. It should be noted that the inner and outer profiles are rather different although, in this paper, we are only concerned with the outer one. Note also that, being a modern bell, there are no “cannons” cast into the crown for hanging purposes. Parameters of particular importance in describing the outer profile are the mouth radius R, the shoulder radius r, the vertical heights above the mouth of the crown H and of the shoulder h. There is some small ambiguity over precisely where the shoulder is to be considered as located. This problem, which does not arise in handbells, will be discussed later.

## 2.2 A Golden Rectangle

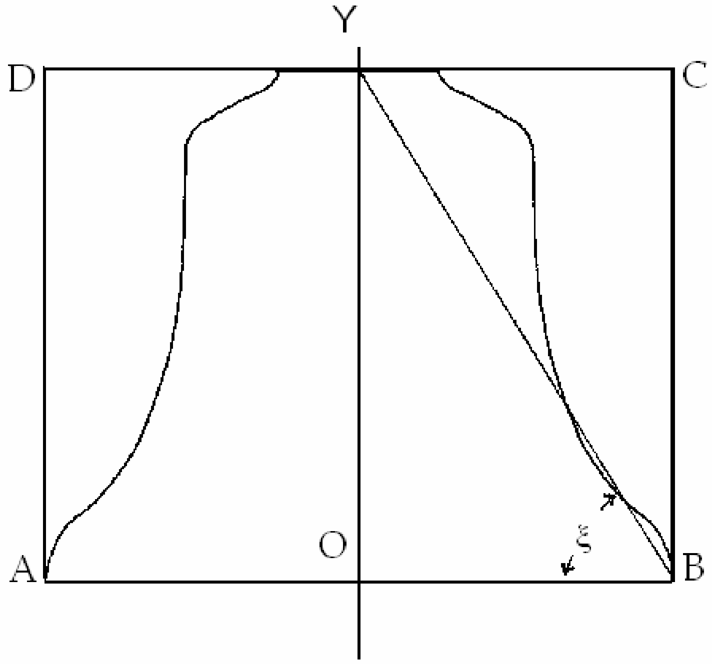


Figure 3. Outer Profile of Taylor Church Bell Showing Golden Rectangles.

Figure 3 shows the outer profile of the Taylor church bell. For this bell R = 350 mm and H = 568 mm so that H/R = 1.623 which is not far removed from the golden ratio of 1.618 so the rectangle OBCY is very close indeed to being golden. As a convenient measure of its “goldenness” we join the rim at B to the centre of the crown at Y and measure the angle $\xi$ which this makes with the mouth. Clearly $\xi = \tan^{-1}(H/R)$ which gives $\xi = (58.4 \pm 0.1)°$ in this case. This compares very favourably with the golden value of $\xi_G = \tan^{-1}(\varphi) = 58.28°$. The difference of only 0.1° clearly shows OBCY to be a golden rectangle. The larger rectangle ABCD containing the entire bell is built up of two identical golden rectangles laid side by side. This particular feature is not one which we are aware of as arising elsewhere in the applications of sacred geometry.

## 2.3 A Golden Triangle

The question of where to take the shoulder location now needs to be addressed. There are several possibilities but we find the most satisfactory is to draw the common tangent to the shoulder and the sound bow regions and take the point S where it touches the shoulder region as defining it. In Figure 4 we again show the Taylor bell but now with the common tangent on the right extrapolated to cut the symmetry axis at E and the plane of the mouth at B’ just beyond B. For this Taylor bell the difference between B and B’ is extremely small. For some bells, such as the Malmark handbell shown in Figure 5 the two points actually coincide. The common tangent B’E plus its mirror image in the symmetry axis A’E and the baseline A’OB’ form an isosceles triangle A’B’E. We are interested in the angle OB’E $\equiv \psi$. Using values of R = 350 mm, r = 192 mm and h = 484 mm we find

$$\psi = \tan^{-1} (h / (R-r)) = (71.9 \pm 0.2)°$$

A golden triangle is an isosceles triangle in which the length of the equal sides is φ times that of the base. This gives a golden value for ψ of

$$\psi_G = \cos^{-1} (1/2\varphi) = 72° \text{ exactly.}$$

The triangle defined by the common tangents and base is thus an almost perfect golden triangle.

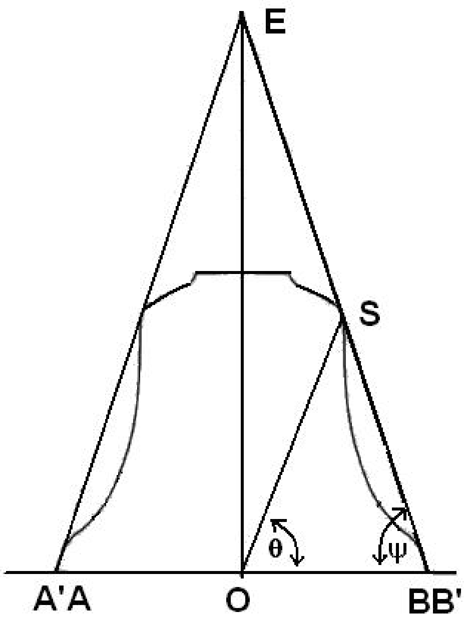


Figure 4. Outer Profile of Taylor Church Bell Showing Golden Triangle and Golden Angle

## 2.4 A Golden Angle

What seems to be happening is that a golden rectangle fixes the overall height of the bell relative to its mouth radius. A golden triangle then constrains the shoulder to lie along a particular straight line as locus. Fixing the absolute location of the shoulder along this line requires a third criterion.

It is usual to define “the golden angle” as half a complete rotation divided by φ. This is 111.25°. However, it is more convenient for our purpose to work with the complementary angle of 68.75° which we shall designate as $\theta_G$.

In Figure 4 we also draw a straight line from the centre of the mouth O to the shoulder S as defined above. The angle θ that OS makes with the mouth line AB fixes the location of the shoulder on the tangent line. Clearly

$$\theta = \tan^{-1} (h/r) = (68.4 \pm 0.1)°.$$

This result agrees remarkably well with $\theta_G$.

## 2.5 Other Large Bells

The identification of three aspects of sacred geometry within the outer profile of this Taylor church bell is very interesting but might be unique to this particular founder or even to this particular bell. Clearly it needs to be checked for other Taylor bells and those of other founders. This would be a huge task but, as a first step, we have looked at a selection of English and Dutch bells whose profiles we happened to have to hand. They were all church bells or large carillon bells and were all reasonably modern except a 17th century (Dutch) Hemmony bell. The results are listed in Table 1 expressed in terms of the angles ξ, ψ and θ.

| Table 1. Fits to outer profiles of church and large carillon bells | | | |
|---|---|---|---|
| | | | |
| Founder | ξ | ψ | θ |
| | ( ± 0.1° ) | ( ± 0.2° ) | ( ± 0.1° ) |
| | | | |
| Sacred theory | 58.28 | 72.00 | 68.75 |
| | | | |
| Taylor | 58.4 | 71.9 | 68.4 |
| Whitechapel | 58.1 | 71.6 | 69.3 |
| Royal Eijbouts | 57.8 | 72.0 | 68.1 |
| Hemmony | 58.0 | 72.8 | 68.5 |
| Gillet & Johnson | 58.7 | 74.2 | 70.5 |
| Petit & Fritzen | 57.9 | 74.8 | 70.1 |

The measured values of ξ are all to within ±½° of the golden rectangle value. We conclude that all these bells have their aspect ratios fixed by the golden ratio. The values of ψ are not quite so good but 3 cases are to within ±½° and another to within ±1°. The last two in the list are out by ± (2 - 3)° which is larger than we had expected. However, the bells of Petit and Fritzen are well-known for being “different” so their case does not worry us too much. Overall the involvement of the golden triangle seems to be confirmed. The results for θ are similar to those for ψ. The first 4 cases agree to within ±1°.The last two are out by ± (1 - 2)° so the situation is slightly better than for ψ. Overall it seems we can say that the golden rectangle works well for all the bells, while the golden triangle and golden angle work well for some and approximately for the others.

# 3 HANDBELLS

In Figure 5 we show the half profile of a $D_3^{\#}$ Malmark handbell. This is very typical of modern handbells, which are generally much simpler in form than church and carillon bells. In particular there is no doubt where the shoulder is located and there is no complication of structure at the sound bow, so the lower vertices of any golden triangle will be located exactly at the rim, coinciding with those of any golden rectangle. In this particular case R = 130.0 mm, r = 72.5 mm, H = 206.0 mm and h = 182.0 mm, giving ξ = 57.7°, ψ = 72.5° and θ = 68.3°. These all match the golden values to within about ±½°.

In Figure 6 the outer profile of the same bell is shown complete with golden rectangle OBCY, golden triangle ABE and golden angle SOB. The analysis was repeated with all the handbells for which we had details to hand. The results are given in Table 2 and are in even better agreement with sacred theory than were those for the church and carillon bells. The ξ and θ values are all to within ±1° of golden values and the ψ are to within ±½°.

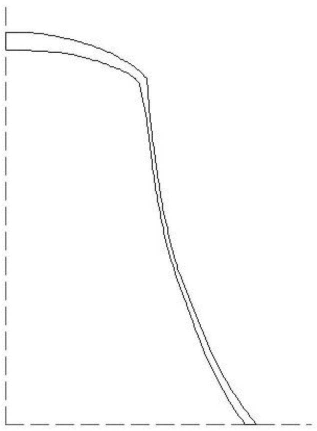

Figure 5. Half Profile of a Malmark $D_3^{\#}$ Handbell

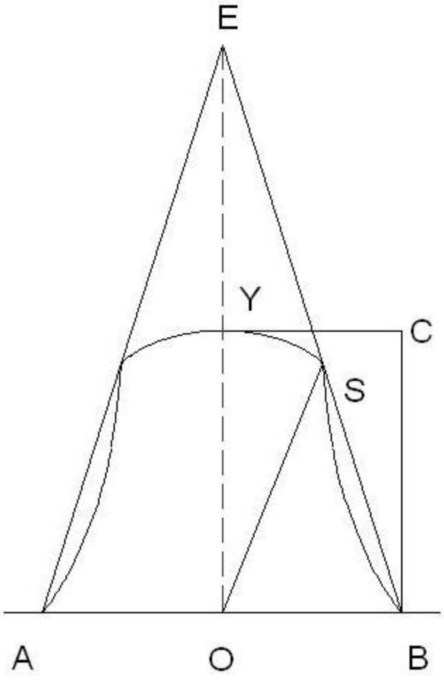


Figure 6. Outer Profile of Malmark $D_3^{\#}$ Showing Golden Features

| Table 2. Fits to outer profiles of handbells. | | | |
|---|---|---|---|
| | | | |
| Maker | ξ | ψ | θ |
| | ( ± 0.1° ) | ( ± 0.2° ) | ( ± 0.1° ) |
| | | | |
| Sacred Theory | 58.28 | 72.00 | 68.75 |
| | | | |
| Malmark | 57.7 | 72.5 | 68.3 |
| Taylor | 57.8 | 71.8 | 68.7 |
| Whitechapel | 58.7 | 72.5 | 68.2 |
| Schulmerich | 58.3 | 72.2 | 69.0 |
| Shaw | 57.4 | 72.4 | 68.2 |

# 4 DISCUSSION

All the handbells considered in this study show very strong evidence of golden rectangles, triangles and angles underpinning their outer profiles. Apart from the Shaw bell, which was perhaps 50 years old, all the bells measured were very modern and either English or American. Clearly it is desirable to extend the study to older bells and to bells of other makers.

The church/carillon bells were all reasonably modern apart from the 17$^{th}$ century Hemmony. The evidence for golden rectangles is very strong. That for golden triangles and angles is less strong but still good. Only English and Dutch bells were used so clearly it would be of interest to consider bells from other Western countries. It is also important to cover a greater range of ages as it is well established[9] that the shapes of English bells have changed over the centuries: 12$^{th}$ century bells, for example, tended to be far more conical than modern ones. It would be interesting to see the extent to which these early bells were "golden", how this may have changed as centuries passed and how it varied from founder to founder.

Although some founders are secretive about the geometrical constructions underlying the strickle boards from which the profiles in the moulds are generated, quite a lot is known about it. Some information has been published[9,10] and the present writers have had the opportunity to study a few of the original geometrical drawings from which the strickle boards of Messrs Taylors were first produced. The drawings show a high degree of technical skill including, for example, the use of mechanical ellipsographs to generate the region from shoulder to soundbow[11]. However, there is no evidence whatsoever for their having used the kind of methods employed by a golden geometer. The founders have used a complicated procedure to produce the "desired" result by other means. In other words they have chosen profiles that look good, not because they come out that way automatically by using the golden geometers' types of construction. Precisely how the founders managed to produce profiles with golden characteristics without using standard golden methods is a question we will discuss in a future article.

# 5 CONCLUSIONS

All the bells considered in this study showed strong evidence for their crown heights being related to their mouth radii by the golden ratio. Thus their half profiles fitted exactly inside golden rectangles.

All the bells had their shoulder positions constrained to lie, at least approximately, on the equal sides of golden triangles with the bell mouths as baselines. All the handbells and most of the larger bells obeyed this condition to a high degree of accuracy.

The actual shoulder locations on these golden triangles' sides were found to be determined by a golden angle measured from the centre of the mouth relative to its plane. Again all the handbells and most of the larger bells satisfied this condition with a high degree of accuracy.

As all the bells used in the study were English, Dutch or American and were mostly less than 50 years old, it is desirable to repeat these measurements on older bells and Western-type bells of other nations. It is not suggested that founders have been deliberately using sacred geometry in drawing up their strickle boards' patterns but that they probably used their own methods to produce the same aesthetically pleasing results.

# 6 REFERENCES

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