

THE EFFECTS OF LOW-FREQUENCY ROOM MODES ON SOURCE AND RECEIVER RESPONSES

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1. INTRODUCTION.

The study of sound energy propagation within rooms has a long history. Because of the great complexity of an exact analytic solution, much of the historical work has concentrated on the treatment of the statistics of a hypothetical, uniform random distribution of sound energy. It has long been recognized that this simplification is invalid when the size of the enclosure is comparable in size to the wavelength. At low frequencies the wavelengths are so large that only the principle dimensions of the room are relevant. This paper is concerned with these 'low' frequencies. It assumes throughout that the sound energy receiver is sensitive to sound pressure, as are most single-element measuring systems and animal hearing systems (at low frequencies). For pressure-gradient sources and/or receivers, the analysis has an exact dual expressed in terms of velocity rather than pressure.

At low frequencies in relatively small rooms, the sound received by a microphone or listener is affected by the extents to which the source and the receiver are coupled to individual room modes.

2. LOW FREQUENCY ROOM MODES (EIGENTONES).

Analytically, any enclosure may be considered acoustically as a three-dimensional space bounded by surfaces of complex impedance. The acoustic wave-equation for such a bounded space may be solved to give solutions in the form of eigenfunctions, with characteristic time and spatial pressure distribution functions and damping factors. These functions and their associated spatial distributions (eigenmodes, or simply 'modes') describe the reverberant behaviour of the sound energy, that is, excluding the direct sound and low-order reflected sound.

In general, in a rectangular room, the modal frequencies are given by the Rayleigh Equation [1]:

$$f = 1/2\pi c \sqrt{(n_L/L)^2 + (n_W/W)^2 + (n_H/H)^2} \quad \dots(1)$$

where n_L , n_W , and n_H are the set of positive integers (including 0), L , W and H are the room dimensions and c is the sound wave velocity (≈ 340 m/s).

These modes behave like resonant systems, that is, they have characteristic natural resonance frequencies, bandwidths dependent on their individual loss (damping) factors and amplification factors ('Q'), also dependant on the damping. As in any other simple harmonic resonant system, the energy storage takes the form of cyclical interchange between kinetic and potential energies.

For example, in a room of 6m length, the lowest frequency mode will occur at about 28Hz and then at all integer multiples thereof. Similar patterns of standing waves will also be possible for the other two pairs of surfaces. Furthermore, this triple, infinite subset of 'axial' modes is only one of three types of mode, the other two being 'tangential', involving reflections from four surfaces in turn, and 'oblique', involving sequential reflections from all six surfaces.

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3. MODAL DENSITIES AND MODAL OVERLAP.

In a small room such as a control room, listening room or small studio, typical values for the frequencies of the three fundamental axial modes might be 28, 35 and 45 Hz. Thus, the lowest part of the important frequency range for audio is characterised by a relatively small number of discrete room modes. At or near to each of these frequencies, sound pressure will be enhanced (at least at some positions in the room). Above some critical lower limit there will be very many modes present and this amplification affects all of the sound. At lower frequencies, between the mode frequencies, the enhancement will not occur to the same extent. This will be perceived as a relative attenuation.

If the modal density is fairly high it may be approximated by [2]:

$$\delta N = (4\pi^2 V/c^3 + \pi f S/2c^2 + L/8c) \cdot \delta f \quad \dots (2)$$

where δN is the approximate number of modes with natural frequencies in the bandwidth δf , $V = LWH$ is the volume, $S = 2(LW + WH + HL)$ is the total surface area and $L' = 4(L + W + H)$ is the sum of the edge lengths.

At some frequency, the average modal density will become so high that the mode responses will overlap significantly. This frequency marks the upper limit of the low frequency region. One criterion for overlap is that the mean spacing between modes should be such that, on average, there are five modes in the frequency interval equal to the mode bandwidth, B . The bandwidth is related to the reverberation time, T_{60} , approximately by:

$$B = 2.2 / T_{60} \quad \dots (3)$$

By combining Eq. 2 and 3, an expression can be obtained for this limiting frequency, f_l , in any room [3]:

$$f_l = \sqrt{(Sc/16V)^2 - (L/8c - T_{60}) \cdot c^3/4\pi V} - Sc/16V \quad \dots (4)$$

For reasonable rooms, this can be approximated by:-

$$f_l \approx \sqrt{(T_{60}c^3/2\pi V) - Sc/16V} \quad \dots (5)$$

For a typical small room (6m x 5m x 3m) with a reverberation time of 0.3 s, Eq. 5 gives a limiting frequency of about 120Hz. The commonly-used 'Schroeder frequency', $f_s = 2000\sqrt{(T_{60}/V)}$ [4], is based on three modes per unit bandwidth, but actually gives nearly the same result for the example 'typical' room.

4. EFFECTS OF SOURCE AND LISTENER POSITIONS.

The overall frequency response from a source to a receiver in a small room is strongly influenced by the low-frequency modes - by the positions of both the source and receiver in relation to the spatial pressure distributions of all the modes within the effective mode bandwidth.

Pressure minima will exist at some places in the room. At such places, there can be no reverberant sound pressure at all (for an individual mode and for the theoretical case of zero damping factor). Thus, no pressure will be detected by a receiver sensitive to the pressure component.

For a source, the actual sound output may be modified by the modal pressure distribution. Most sound sources, including practically all musical instruments, the human voice and loudspeakers, are generators of volume velocity rather than sound power (that is, they have high specific acoustic impedance). The effective acoustic power output derives from the product of the volume velocity and the local sound pressure. If the source is located at or near a modal pressure node, then the acoustic pressure load (or radiation impedance) is low, at least for one direction of radiation, and the output power coupled to that

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mode is correspondingly low. This results in modifications of both the effective radiation pattern and the total sound output power.

In practice – with several modes, each with a finite damping value and excitation frequencies imperfectly matched to the eigenfrequencies – the overall response at any point will be the sum of the contributions of all significant modes, as functions of position relative to the modal distribution patterns and of frequency relative to the eigenfrequency. For non-continuous signals, the time response will also be modified by the time responses of the contributing components.

5. RESPONSE PREDICTION USING MODE SUMMATION [3,5,6].

The spatial pressure distribution and the coupling factors between a sound source or receiver in a rectangular room and a room mode may be represented by a three-dimensional sinusoidal function – a cosine if the coordinate origin is at a corner. At the positions on this function corresponding to $0, \pi, 2\pi, \dots$ radians, the coupling factor will be a maximum, alternating in phase between 0 and π at adjacent maxima. At the intermediate positions, at odd multiples of $\pi/2$, it will be theoretically zero; in practice, for damped systems, it will be very small. In a rectangular coordinate system:

$$c_i(x, y, z)_S(x, y, z)_R = \Psi(S) \cdot \Psi(R) \quad \dots(6)$$

where c_i is the combined coupling coefficient for the i th mode, S represents source quantities and R represents receiver quantities. The distribution functions can be represented by:

$$\Psi = \Psi(x, y, z) = \cos \frac{n_x \pi x}{l_x} \cos \frac{n_y \pi y}{l_y} \cos \frac{n_z \pi z}{l_z} \quad \dots(7)$$

where n is the mode order, l is the principal room dimension and x, y and z refer to the principle coordinate axes.

The maximum value of this coupling coefficient is inversely proportional to the damping coefficient of the mode, in accordance with the usual resonance response function. A third multiplication factor, related to the absolute difference between the two frequencies, is also required, if the forcing frequency is not identical to the mode natural frequency.

The nett contribution of each mode is given by:

$$|p| = \frac{\epsilon_{x,y,z} \Psi(S) \Psi(R)}{2\omega_N k_N / \omega + i(\omega_N^2 / \omega - \omega)} \quad \dots(8)$$

where ϵ is a scaling factor depending on the three-dimensional mode order, ω_N is the mode natural angular frequency, k_N is the damping factor of mode N and ω is the angular frequency at which the mode contribution is required. The absolute magnitude of the total resonant field is also governed by the strength of the source, the size of the room and a number of media constants.

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The total sound pressure at the receiving position due to all of the room modes is given by the summation [7]:

$$|p| = \frac{Q_0 \rho c^2}{\omega V} \sum_N \frac{e_{x,y,z} \Psi(S) \Psi(R)}{2\omega_N k_N / \omega + i(\omega_N^2 / \omega - \omega)} \quad \dots(9)$$

where V is the room volume, Q_0 is the volume velocity of the source, ρ is the density of the medium and c is the velocity of sound for the medium.

In addition to these mode components, the direct sound may be significant. An expression for the direct sound field at a distance from a simple source is:

$$p = \frac{-i\omega\rho}{4\pi r} Q_0 e^{i\omega(\frac{r}{c} - t)} \quad \dots(10)$$

where r is the radial distance from the source.

Equations 9 and 10 include the absolute strength of the source. Real loudspeaker sources have an output volume velocity which is inversely proportional to frequency.

Figures 1 and 2 show an example of the results of such a calculation for an acoustically treated listening room with a volume of about 70m^3 . The overall response shown in Fig. 1a is the vector summation of about 200 modes, some of which are shown in Fig. 2. For comparison, Fig. 1b shows a measured response for the same conditions. (In fact, the detail of the responses are so dependant on positions, even at low frequencies, that the measurements themselves were not wholly repeatable.) There are clear indications that the overall characteristics of the response were predicted, even if some of the frequencies and amplitudes are in error. Even the major characteristics at frequencies near to 200Hz show significant similarities.

6. LIMITATIONS OF PREDICTIONS.

Whatever method of prediction is employed, it will be subject to uncertainty arising from lack of knowledge of the acoustic properties of the materials. No method of calculation can overcome this limitation. The effective properties of intentional acoustic treatment are difficult enough to obtain to any degree of accuracy. Data for the acoustic properties of other structural or decorative materials is essentially impossible to obtain. At low frequencies many structures exhibit panel or Helmholtz resonances, with associated large changes in impedance, particularly in the reactive component, over small frequency ranges.

Such a lack of accurate data prompts the question of how far it is worth pursuing any method of prediction.

7. POSSIBLE METHODS OF IMPROVING LOW-FREQUENCY RESPONSES.

7.1 Increasing the modal damping factors.

One way to increase the damping factors of all of the modes, and thus reduce the low-frequency response irregularities, is by adding acoustic absorption. This is generally done to some extent as part of the normal acoustic design, to control the average 'reverberation time'. The maximum amplitude of the mode resonance is reduced and the range of frequencies over which it acts is widened. However, it is likely that a

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room so treated, to the extent of significantly reducing the subjective irregularities (in comparison with a conventionally treated room), would sound 'lifeless' and severely lacking in bass response.

7.2 Golden ratios.

It is often suggested that a pair of 'golden ratios' be used to select the room proportions. The use of such non-simple ratios is helpful, but the essential problem remains. Some aspects of room proportions are discussed in References 8,9 and 10.

7.3 Non-rectangular rooms.

It is also frequently suggested that a possible solution is to make the room non-rectangular. Many modern rooms are significantly non-rectangular, generally for reasons other than the low-frequency modal behaviour. In such cases, the mode structure cannot be derived by simple equations or by inspection (although approximate methods, based on mean dimensions, may give sufficiently accurate results for many purposes). Mathematical modelling techniques, for example Finite Element Analysis, can in principle give arbitrarily accurate answers. *However, the modal behaviour is no less pronounced than in an equivalent rectangular room - just different!*

It may even be argued that a room which approximates to rectangular at low frequencies gives the widest possible scope for even distribution of the eigenfrequencies. Rooms which approximate to significantly non-rectangular shapes at low frequencies (for example, cylinders or spheres), are likely to be more degenerate than a rectangular room.

7.4 Relocation of sound sources.

Another factor often employed in control rooms, again usually for other reasons, is the incorporation of loudspeakers into the walls. Theoretically, all modes have pressure maxima at their reflection points. Thus, a hard wall surface is a pressure anti-node for one third of the axial modes, two thirds of the tangential modes and all of the oblique modes. Placing the sound source there ensures that it is better coupled to more of the room modes. However, such an approach does nothing whatever about the irregularities resulting from the listener's position with respect to the room modes. It may well be that an excess response at one frequency, as a result of the listener's (or microphone) position, requires a degree of partial cancellation by locating the loudspeaker away from the position giving the most effective source coupling.

One way in which the loudspeakers can be located at other than the normal stereo listening positions, without detriment to the higher-frequency image localisation, is the use of 'sub-woofers'. Because the problem is one of low-frequency response and there is little, if any, image-localising information at frequencies up to about 200Hz, the lowest frequency part of the spectrum can be reproduced by loudspeakers specially located to achieve a more even low-frequency response. This arrangement may also have benefits in loudspeaker design and power-handling capabilities, not to mention acceptability in the domestic environment.

Much of this paper has been related to the reproduction of sound in a control room or listening room. All of it is equally relevant to studios and live performance spaces - they also have sources and listeners (or microphones) subject to the same considerations. However, many of these spaces are larger, some very much larger, so that the critical low-frequency limit may be very low indeed.

7.5 Electronic and electro-acoustic equalisation.

Many proposals for response correction by electronic systems have been made. Many attempt to address the problems over the entire frequency range. Sometimes, equalisation is included in the reproduction chain as a standard procedure, in order that the measured frequency response, *at a predefined listening*

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location, can be measured and then made to match some predetermined 'ideal'. However, this is usually implemented using 1/3rd-octave, or similar, equalisers and measurements. Alternatively, it may be implemented through some form of digital filter which might be adjusted by an objective measurement technique. The results bear little relationship either to the human perception of sound or the fine detail of the response irregularities. The latter method especially is very sensitive to changes in receiver location.

At low frequencies in control rooms and listening rooms there may be valid reasons for introducing electronic correction in order to compensate for irregularities introduced as a result of the loudspeaker interaction with widely-spaced room modes. The effect of an isolated room mode on the effective output power of the loudspeaker is essentially indistinguishable from a real departure of the loudspeaker from a level response. Equalisation could remove such effects. However, it would not necessarily create a more uniform response at any particular point within a room because the effects of the listener's position within the room mode structure may as likely as not be partially cancelling any adverse effects of the source location.

7.6 Low-frequency diffusion.

Reductions in low-frequency mode clustering and discrete mode amplitudes through the use of diffusing structures have been reported [11]. In general, existing designs for low-frequency acoustic diffusers based on phase changes on reflection are rather bulky. Diffusers based on differential absorption characteristics may be more practicable. In that context, it has always been the practice in the acoustic design of professional rooms to distribute the acoustic treatment in a more-or-less irregular pattern. That may have (accidentally and unwittingly) helped to make low-frequency responses more uniform to a small extent.

8. SUB-WOOFER LOCATION.

One degree of freedom which is available for retrospective correction of low-frequency room response faults is the provision of separate low-frequency loudspeakers. Although colloquially known as 'sub-woofers', the intention in these cases is not to extend the low-frequency response below the normal limits. The main purpose is to permit relocation of the low-frequency sources to positions which result in a more uniform response, without sacrificing the higher-frequency, stereophonic performance.

At the present state of knowledge, it is not possible to predict the optimum sub-woofer location reliably. It is usually necessary to resolve conflicting balances between several different frequency ranges, in spaces where possible additional loudspeaker locations might be very few.

9. CONCLUSIONS.

A discussion of the acoustic parameters governing the behaviour of low-frequency sound energy inside relatively small enclosures has been presented. The limitations imposed by these physical restraints on the achievable subjective sound quality have also been discussed.

A method for predicting objective frequency responses has been outlined. All prediction methods require detailed information about material properties and behaviour which is generally unobtainable. Some are analytically intractable or computationally extravagant. Others bear little resemblance to real acoustics. One method, that of mode summation, is practicable, at least for rooms which can be approximated acoustically to the simple rectangular form, and gives results which at least resemble measured responses.

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Some guidance on the design of small rooms to optimise the low-frequency response has also been given. However, it has to be acknowledged that, in general, the problem is insoluble. Even if perfect predictions could be made, the final result will inevitably contain large low-frequency irregularities. The optimisation of such responses for different positions within the room, for the relative importance of different parts of the low-frequency spectrum and for different listeners' preferences is a matter for subjective assessment rather than for any theoretical dogmatism.

Additional degrees of freedom can be derived from the use of separate, low-frequency loudspeakers. This permits the optimisation of low-frequency responses without detriment to the higher-frequency stereophonic imaging.

It has to be accepted that in rooms there will be a low-frequency limit, below which the perceived frequency response will be irregular or very irregular. For practical sizes of studios, control rooms and listening rooms, this frequency limit will be well within the range normally considered to be the important audio spectrum.

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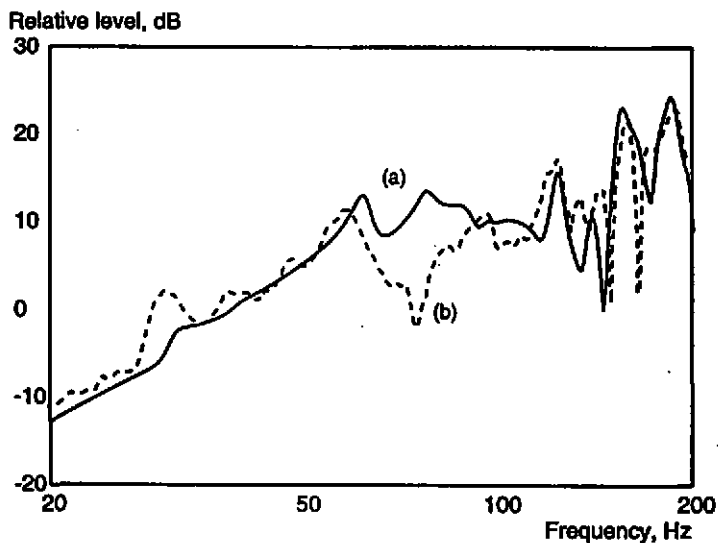


Fig. 1 Calculated and measured low-frequency responses in a listening room.

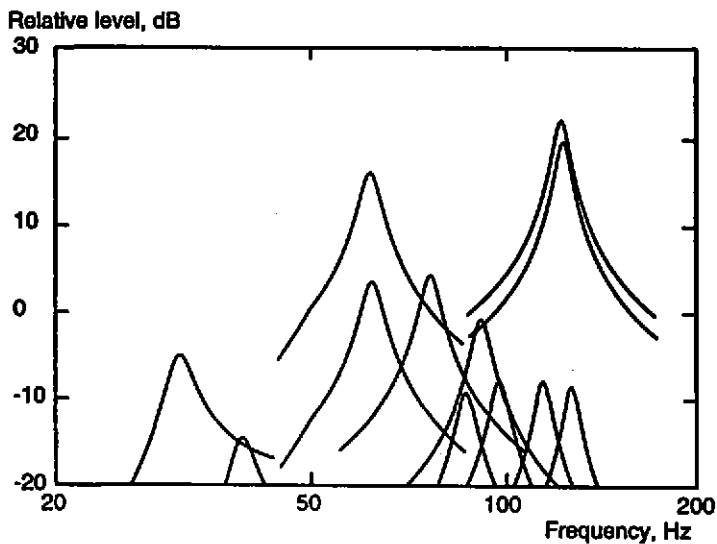


Fig. 2 Contributions from some individual modes to the response of Fig. 1.