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RADIATION DAMPING IN PLATES INDUCED BY POROUS MEDIA

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1. INTRODUCTION

Lightweight double leaf construction is used by a wide range of different industries. Often, as in the case of aerospace structures, it is used out of necessity because there is no recourse to using mass to achieve the necessary sound insulation. In the case of the building industry, where it may be found under various guises (e.g. plasterboard stud partitions, timber floors) its choice may be economic, for increased flexibility or for speed of construction etc.

Irrespective of its primary function, if a lightweight double wall is to attain a reasonable level of sound insulation, it is important that some form of absorption, usually a mineral quilt, is incorporated into the cavity in order to damp out resonances that would otherwise occur in the airspace. While the importance of the quilt has been understood for some time, a detailed theoretical understanding of the mechanisms involved in transmission has lagged behind the design of walls which has generally proceeded on an empirical basis. Part of the reason for this is because the inclusion of a quilt dramatically increases the complexity of the theoretical model required to predict transmission.

The approach adopted by Price and Crocker [1] for tackling the problem was only partially to fill the cavity with quilt, confining it to the perimeter. They were then able to treat it as an area of absorption at the boundary of the space and calculate how the cavity reverberation time was affected. Whilst this technique appeared to work quite well, it limits the scope of the model for tackling practical structures. It is more usual for the cavity in a lightweight double wall to be largely or completely filled with quilt so that the sound waves spend a significant amount of their time propagating through porous material. In this situation the approach adopted by Price and Crocker is no longer applicable and if the system is to be modelled using statistical energy analysis, some other treatment is required.

This paper investigates an alternative technique for modelling the behaviour of porous material: the "equivalent fluid" representation described by Delany and Bazley [2]. Its adoption has implications on how the remainder of the double wall system is modelled, affecting radiation and excitation of the leaves in addition to the behaviour of the sound in the cavity. This paper only investigates part of the problem, namely the radiation of sound from a plate into a porous medium. A theoretical model is presented and the results are compared with data obtained from measurements performed on a simply supported aluminium plate in a baffle radiating sound into a slab of highly-reticulated polyester foam.

2. EQUIVALENT FLUID MODELS OF POROUS MATERIALS

When sound propagates through an acoustic quilt it is subject to viscous and thermal losses resulting from interactions with the structural skeleton of the material. These losses result in an attenuation of the wave as it progresses through the material, the effect being more noticeable with increasing frequency. Theoretical models for predicting these effects can be very complicated, relying on an understanding of the microstructure of the material, however, for many practical purposes an empirical "equivalent fluid" representation of porous materials is perfectly adequate. This was first proposed by Delany and Bazley [2] for representing the acoustical properties of fibrous materials and it has been applied successfully to numerous materials by other investigators.

If the porous material is assumed to have a rigid frame (i.e. there is no interaction between the fluid and solid phases of the porous material) the wave propagation through the voids may be described using two complex, frequency dependant parameters, usually the characteristic impedance, $Z_a = r + ix$, and propagation coefficient, $\Gamma_a = ik_a = \alpha + i\beta$ (although the related parameters, density and wavespeed, are often used too). If it is assumed that a plane wave is travelling in the x-direction, the sound pressure can be described using $e^{-ik_a x}$, from which it may be seen that α is the spatial decay coefficient and β is the phase coefficient.

Delany and Bazley measured these parameters for a variety of different fibrous materials and discovered that when plotted against $\xi = \rho_o f/\sigma$, which is a function of their steady viscous flow resistivity, σ, the data collapsed and could be described through curve fitting formulae. The characteristic impedance, normalised to the impedance of air, $\rho_o c_o$, can be described using,

$$Z_a / \rho_o c_o = 1 + c_1 \xi^{c_2} - ic_3 \xi^{c_4} \tag{1}$$

and the wavenumber k_a, normalised to the wavenumber for air, k_o, by,

$$ik_a / k_o = 1 + c_5 \xi^{c_6} + ic_7 \xi^{c_8} \tag{2}$$

Where the constants c_1-c_8 are determined by the curve fitting procedure.

The material used in this study was a heavily reticulated polyester foam with a flow resistivity of 6992 SI rayls/m. Its characteristic impedance and wavenumber were measured in an impedance tube and plotted against ξ to find c_1-c_8 which were 0.1590, -0.5517, 0.0632, -0.8730, 0.1442, -0.7323, 0.2779 and -0.3699 respectively.

3. THEORETICAL MODEL FOR RADIATION FROM A SIMPLY SUPPORTED PLATE INTO A SEMI-INFINITE POROUS MEDIUM

The model presented in this paper is an exact solution for the acoustic power radiated by a baffled simply-supported plate into a semi-infinite layer of porous dissipative material. As the resulting expression is not amenable to analytical integration, numerically-determined predictions are presented. The model of the simply supported plate in a baffle is shown in Figure 1. It is assumed that it radiates into air on one side and into a semi-infinite dissipative medium on the other. This is modelled as an equivalent fluid and is assumed to have a frequency dependant complex wavenumber, k_a, and a frequency dependant complex density, ρ_a.

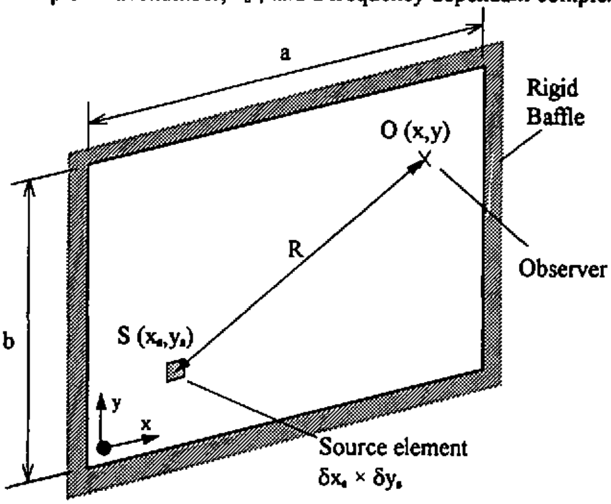

Figure 1 A simply supported plate mounted in a rigid baffle

It is customary to find the sound power radiated by a vibrating structure from the acoustic far field, however this is not possible in the present case because sound power flow is not conserved between the plate surface and the far field. A different strategy must therefore be adopted and the one chosen here is to represent the plate as a number of monopoles and to integrate the acoustic intensity for each over the plate surface.

If a harmonic acoustic monopole with volume velocity amplitude, Q_o, is assumed to be present on the side of the plate radiating into the dissipative medium the sound pressure at time t, and distance r, can be written as,

$$p(r,t) = \frac{i\omega\rho_a Q_0}{2\pi r}\exp[i(\omega t - k_a r)] \tag{3}$$

If the plate is simply supported, its vibrational velocity, u, may be given as

$$u(x,y;t) = U_0 e^{i\omega t}\sin\left(\frac{m\pi x_s}{a}\right)\sin\left(\frac{n\pi y_s}{b}\right) \tag{4}$$

where a and b are its height and width. The volume velocity of the plate element at $S(x_s, y_s)$ can is given by the product of its velocity (eqn 4) and its area $\delta x_s \delta y_s$, which permits the sound pressure at $O(x,y)$, radiated by the plate element at $S(x_s, y_s)$ to be written as,

$$\delta p(x,y;t) = \frac{i\omega\rho_a}{2\pi R} e^{i\omega t} U_0 \delta x_s \delta y_s \sin\left(\frac{m\pi x_s}{a}\right)\sin\left(\frac{n\pi y_s}{b}\right)\exp(-ik_a R), \tag{5}$$

In this equation R is the distance between the source and the receiver given by

$$R = [(x-x_s)^2 + (y-y_s)^2]^{1/2}. \tag{6}$$

The total sound pressure at $O(x,y)$, from the entire panel, is therefore

$$p(x,y;t) = \sum \delta p(x,y;t) = \frac{i\omega\rho_a e^{i\omega t} U_0}{2\pi}\int_0^b\int_0^a \sin\left(\frac{m\pi x_s}{a}\right)\sin\left(\frac{n\pi y_s}{b}\right)\frac{\exp(-ik_a R)}{R}dx_s dy_s. \tag{7}$$

If the density and wavenumber for the dissipative medium are written in the form $\rho_a = |\rho_a|e^{i\phi}$ and $ik_a = \alpha + i\beta$; then,

$$\mathrm{Re}[p(x,y;t)] = -\frac{\omega|\rho_a|U_0}{2\pi}\int_0^b\int_0^a \sin\left(\frac{m\pi x_s}{a}\right)\sin\left(\frac{n\pi y_s}{b}\right)\frac{e^{-\alpha R}}{R}\sin(\omega t + \phi - \beta R)dx_s dy_s. \tag{8}$$

Also,

$$\mathrm{Re}[u(x,y;t)] = U_0\cos(\omega t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{9}$$

and since the acoustic intensity radiated normally outward from the plate surface is

$I(x,y) = \overline{\mathrm{Re}[(p(x,y;t)]\mathrm{Re}[(u(x,y;t)]}$, the total radiated sound power is

$$W_{rad} = \int_0^b\int_0^a I(x,y)dxdy = -\frac{\omega|\rho_a|U_0^2}{4\pi}\int_0^b\int_0^a \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \times\int_0^b\int_0^a \sin\left(\frac{m\pi x_s}{a}\right)\sin\left(\frac{n\pi y_s}{b}\right)\frac{e^{-\alpha R}}{R}\sin(\phi-\beta R)dx_s dy_s dxdy. \quad (10)$$

This integration cannot apparently be carried out analytically, and so Gauss-Legendre quadrature was employed, piecewise over each "phase cell" of the plate vibration pattern. In order to avoid singularities in the computation (at points where $x = x_s$ and $y = y_s$), six-point quadrature was used for the inner two integrals and five-point quadrature for the outer two. The integration scheme was checked by comparing predictions of the radiation efficiency for a series of plate modes radiating into air with data published by Wallace [3]. Excellent agreement between the two sets of data was observed at frequencies up to and beyond the critical frequency of the plate.

In order to provide data for comparison with the measured results (presented in the next section), eqn(10) was evaluated at the natural frequencies of the system under study. This was a simply supported panel, the modal frequencies of which are given by

$$f_{mn} = (1/2\pi)[(m\pi/a)^2 + (n\pi/b)^2][Eh^2/12\rho_p(1-\mu^2)]^{1/2}, \quad (11)$$

where (m,n) are mode numbers and E, h, ρ_p and μ are the Young's modulus, thickness, density and Poisson's ratio of the plate material. The radiated power was then converted into the radiation loss factor η, using,

$$W_{rad} = M_p U_o^2 \omega\eta, \quad (12)$$

where M_p is the total mass of the plate.

4. MEASURED RESULTS

Measured data for comparison with the results from the theoretical model were obtained using the experimental set-up shown below.

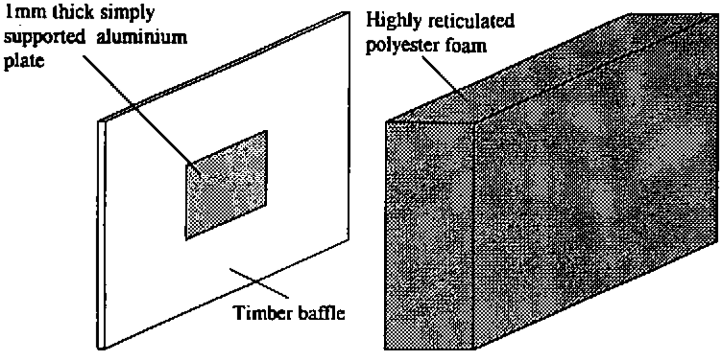

Figure 2 Experimental set-up

An aluminium plate, 0.458×0.250×0.001 m was mounted in a timber baffle on a simple support. The plate was excited using a shaker fed with random noise and the transfer function between the force at the point of excitation and the plate response was measured. The measured data were used to determine the plate mobility from which the plate modal frequencies could be identified. A curve fitting procedure was then used to determine the damping of each mode.

The initial measurement was performed with the plate radiating into air on both sides. In this case the damping values obtained from the curve fits represent the total loss factor for the plate, comprising the loss factor for radiation into air, the boundary losses and the internal material losses. A comparison between these data and the predicted loss factor for radiation into air revealed that they were dominated by boundary losses.

The measurements were then repeated in the presence of a large slab of highly reticulated polyester foam which measured 1.45×1.24×0.34 m. Initially this was positioned a considerable distance from the plate and was then moved closer after each set of measurements. Figure 3 shows how the mobility spectra changed as the size of the gap between the foam and the plate was varied.

It can be seen that as the foam is moved closer to the plate the loss factors for each of the modes increases, this being most dramatic for the narrower separations. There is also a drop in the modal frequency which is probably due to the attached mass of the air in the foam. These spectra were used to obtain new total loss factors for the plate in the presence of the foam.

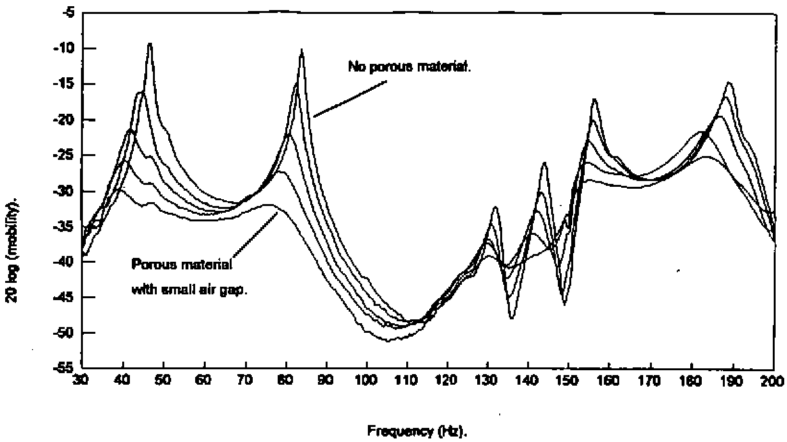

Figure 3 **Variation of mobility spectra with changing separation between the plate and the porous material**

The damping data for the plate with no foam present was then subtracted from the data for the plate radiating into foam to obtain the radiation loss factor. The results are shown in Figure 4.

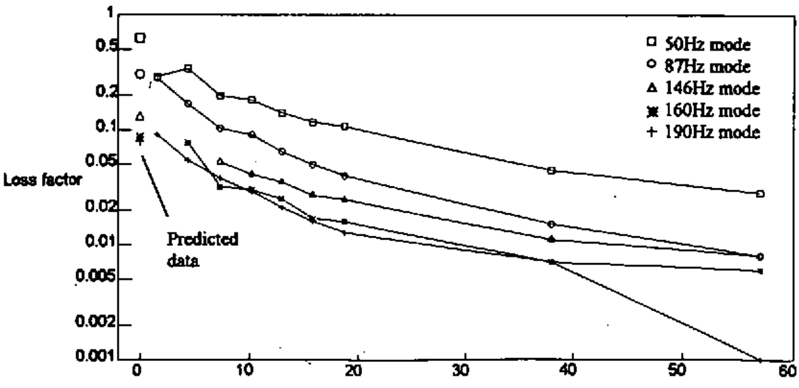

Figure 4 **Comparison of measured and predicted radiation loss factors**

It can be seen that for each mode the radiation loss factor grows rapidly as the separation between the plate and the porous material decreases. No measured data for a gap of zero width was obtained as it would not have been possible to excite the fluid phase of the porous material without also exciting the skeletal frame. This would have resulted in the presence of an additional damping mechanism which is not accounted for by the theory. The results from the theoretical model are also plotted on the figure and it may be seen that the measured results tend towards the predictions as the gap size approaches zero.

5. DISCUSSION

The theoretical model presented in this paper permits the acoustic power radiated from a finite plate into a semi-infinite region of porous material to be predicted. Despite the rather idealised nature of the system, the results agree tolerably well with measured data obtained from a baffled plate radiating into a slab of highly reticulated foamed plastic. Work is currently in progress to include the air gap in the theory and to permit predictions to be made for porous layers of finite thickness.

The results from this study indicate that radiation from a plate into a porous medium can be significantly stronger than radiation into air. For lightly damped structures, such as aircraft, this increased radiation damping can significantly increase the total loss factors of some components. In buildings, where the materials used tend to have relatively high internal damping, these effects may be less noticeable.

The "equivalent fluid" representation of a porous material offers a means of predicting sound transmission through lightweight cavity walls. The model presented in this paper may be viewed as an initial step towards achieving this, however further work, especially in modelling the behaviour of sound in the cavity and the excitation of the second leaf, is required before the analysis is complete.

6. REFERENCES

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7. ACKNOWLEDGEMENT

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