APPLICATION OF PHASE CONJUGATE ARRAYS TO SONAR

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ABSTRACT

A phase conjugate array (time reversal mirror) is a method of focusing acoustic energy spatially and temporally in a complex ocean propagation environment. Signals from a probe source are received on an array, time-reversed and re-transmitted. Multipath components are directed back along the paths over which they arrived, in the reverse order they were received. Numerical models have been developed to predict the focusing achievable with different array configurations in different types of environment. The effect of a time dependent rough surface has also been calculated. The models are described and examples of results obtained presented. An alternative focusing method based on inverse filters has been developed which may be an important enhancement to the basic reversal method. The relationship between the two methods and the advantages of each are explained. The models of focusing performance need validation using data taken in real environments. An experimental system being constructed to provide this validation is described. The focusing techniques can be used in a number of ways to improve the performance of sonar and underwater communication systems. Examples of new types of sonar that exploit the focusing techniques to deliver improved performance are described.

1. INTRODUCTION

A number of our potential adversaries already operate extremely capable modern conventional submarines which could be a significant threat to allied shipping in a future military conflict. Acoustic systems are likely to remain the principal surveillance method for detecting such threats on account of the relatively good propagation characteristics of acoustic signals in the underwater environment (compared to other signals). Nevertheless, the shallow and littoral water environments in which future conflicts are likely to occur are extremely challenging environments for acoustic systems. Passive sonars are faced with high levels of ambient noise against which the low signatures of threat submarines have to be detected. Active sonars are affected by high levels of reverberation and high numbers of false alarms, which makes the detection of clad slow-moving submarines extremely difficult. There is a need for research on new types of sonar and new methods of processing which have the potential to lead to major improvements in performance.

A phase conjugate array (time-reversal mirror) is a concept that may deliver the needed improvements in performance. The idea behind the technique is that an acoustic transmission can be focused in range and depth, and temporal dispersion reversed, if a probe signal received on an array is time-reversed and retransmitted. Multipath components are directed back along the paths over which they arrived, in the reverse order they were received. The components recombine at the focus, the point the probe signal was generated. The probe signal received on the array contains exactly the information required to achieve the focusing. There are a number of ways (discussed later) in which the technique could be used in an acoustic detection system. Environments that are difficult for conventional acoustic systems (environments where there is a significant multipath structure) are precisely those where the technique has the potential for producing the largest gains.

As well as the underwater surveillance requirement, acoustic communication systems have an increasingly important role in both military and civil applications. In military systems, acoustic communication is required, for example, to enable a surface force to communicate with a submarine operating in conjunction with it. In the off-shore industry, acoustic communication systems are used for transmitting data to, and from, deployed systems on the sea-bed or in the water. Inter-symbol interference caused by dispersive multipath encountered in shallow water environments limits the capacity of these systems. There is a need for systems to be higher data rate (with the same or lower error rate), and for power consumption to be reduced. For the military requirement, low probability of intercept is often also desired.

The idea of obtaining spatial and temporal focusing using phase conjugation is not new. It was first demonstrated in optics where the non-linear properties of the medium were used to produce a time-reversed, retro-directed replica of an incident field. In acoustics, frequencies are considerably lower and the time-reversal (or phase conjugation) can be accomplished directly (electronically). The first experimental study dates back to the 1960s when Parvulescu and Clay used a time-reversed transmission to compensate for multi-path effects [1]. They used an omnidirectional transmitter and they were consequently able to produce temporal but not spatial focusing. There has been renewed interest in the phase conjugate technique in recent years. Theoretical analysis of the phenomenon has been developed [2]. An experimental demonstration of focusing in an ocean environment has been successfully achieved by the Marine Physics Laboratory (MPL) and the SACLANT Undersea Research Centre (SACLANTCEN) [3]. Experimental and theoretical work on focusing in inhomogeneous media at ultrasonic frequencies in the laboratory has also been undertaken [4].

The work on focusing in the ocean environment has concentrated on the focusing achievable over a two-way transmission path (i.e. the generation of a probe signal and then the return transmission from the array). This is important for both the underwater surveillance and communication applications. However, focusing over a four-way transmission path, i.e., the use of an initial insonification of a volume of water to produce an echo which can be used as the input to the process, has received less attention. Here, we have set out to develop a modelling capability that will enable an assessment of the gains likely to be obtained in both the two- and four-way path cases to be made. Variability of the environment, particularly of the sea surface, is expected to lead to a degradation of the focusing. The numerical models have been extended to allow these effects to be assessed.

Methods based on inverse filters for generating a desired field at a certain point, or over a given volume, have been developed in room acoustics [5]. Here, the application of these techniques to underwater acoustics has been considered. This has resulted in a new focusing method that may prove to be an important enhancement of the basic time-reversal technique. The performance of the new method has been modelled and comparisons made to the performance of time-reversal.

The models enable the focusing obtainable from an array to be predicted. However, focusing depends on a number of subtle environmental effects. The models need validation using data taken in real environments in order to confirm that the important effects have been included and that they have been represented accurately. An experimental system is being constructed to allow validation experiments to be conducted. Design issues that need to be considered in such a system are discussed. Finally, a number of possible new concepts for sonars that exploit the focusing technique to deliver improved performance are described.

2. MODELLING

A primary objective of the work reported here was the development of a simulation capability so that new concepts exploiting focusing techniques could be studied and the performance of different array options in these concepts evaluated. To simulate the performance of a phase conjugate array in a realistic environment, a high fidelity model of the propagation of acoustic pulses was required. The model used was the SPUR model which was developed as part of a MoD programme on Synthetic Acoustic Environments. SPUR uses as its kernel a wide-angle parabolic equation solver, RAM. The dispersion of each spectral component of a pulse over a propagation path is calculated and the final pulse evaluated by transforming back into the time domain. Time-reversal at the array in the model is achieved by taking the complex conjugate of each spectral component. More details of the numerical simulations are given in [6]

Performance was studied in a number of shallow water environments, with different sound speed profiles and bottom reflectivities. Calculations have been carried out for systems with a two-way and a four-way transmission path. The configuration used for each was the same but the sequence the sources and receivers were operated was different. The configuration is shown in Fig. 1.

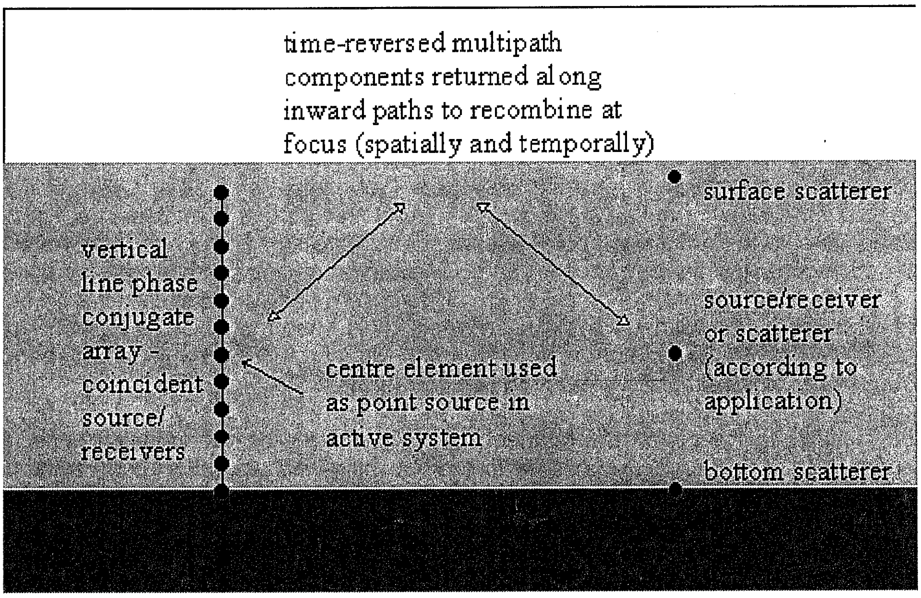

Fig.1: Phase conjugate array and source/scatterer configuration for simulations

The phase conjugate array is a source/receiver array, a line of equally spaced sources and coincident receivers. For the two-way transmission path case, a pulse is generated at an omnidirectional probe source (at the right of the figure). The signals received at each of the elements of the array are time-reversed, amplified and retransmitted. In the simulations a normalisation was applied so that the total energy transmitted by the array was the same as that transmitted by the point source. The field in the focal region is of interest hence a receiver is shown coincident with the probe source. The field near the surface and at the bottom (at the points shown) is also calculated to assess the extent to which the field is focused away from the boundaries.

For the four-way transmission path case, a pulse is generated at the centre element of the array. The echo from the target is received on the array, time-reversed and retransmitted. In practice the energy from the target is received within the reverberation field from surface, seabed and volume scatterers in the environment. It was not practical in the present study to undertake a full simulation of reverberation. Instead, discrete scatterers at the target range were modelled. The energy from the phase conjugate transmission at the three scatterers (the target and the scatterers at the boundaries), and the echoes from these at the centre element of the array, was calculated.

Array performance was studied in three shallow water environments. A baseline environment had a sound speed profile representative of summer conditions and a poorly reflective seabed. A winter environment had a sound speed profile representative of well-mixed, isothermal conditions. A third environment had a more reflective seabed but the same summer SSP as the baseline environment. The depth assumed was 91m. The range of the source from the array was 10km. The array is shown in Fig. 1 as vertical and extending over the full water column, but various lengths of array, various numbers of elements, and various orientations (vertical, and horizontal with the probe source or the scatterer at either broadside or endfire) have been considered. Simulations were performed for an initial 2kHz CW pulse with a 0.1s Hanning window.

Focusing is expected to produce benefits via three routes: to increase the energy at the focus (for a given power output); to increase the ratio of energy at the focus to that at other points (particularly the ocean boundaries which are sources of reverberation in an active system); and to produce signals at the final receiver which are less dispersed in time. Measures of performance in each of these areas have been developed to allow the benefit of different array configurations to be quantified. The first of these measures, focusing gain, was defined as the ratio of the energy at the focus from the time-reversed transmission, relative to the energy from a transmission from an omnidirectional source. Surface (and bottom) gain was defined as the increase in the ratio of the energy at the focus compared to the energy at a point near the surface (or the bottom). Time compression was defined as the reduction in the length of a pulse at the focus (measured by the interval between when the intensity is half its maximum value).

Time series of the pressure generated in the two-way transmission path case with a 31 element vertical array are shown in Fig. 2. On the left are the time series of the probe transmission at the

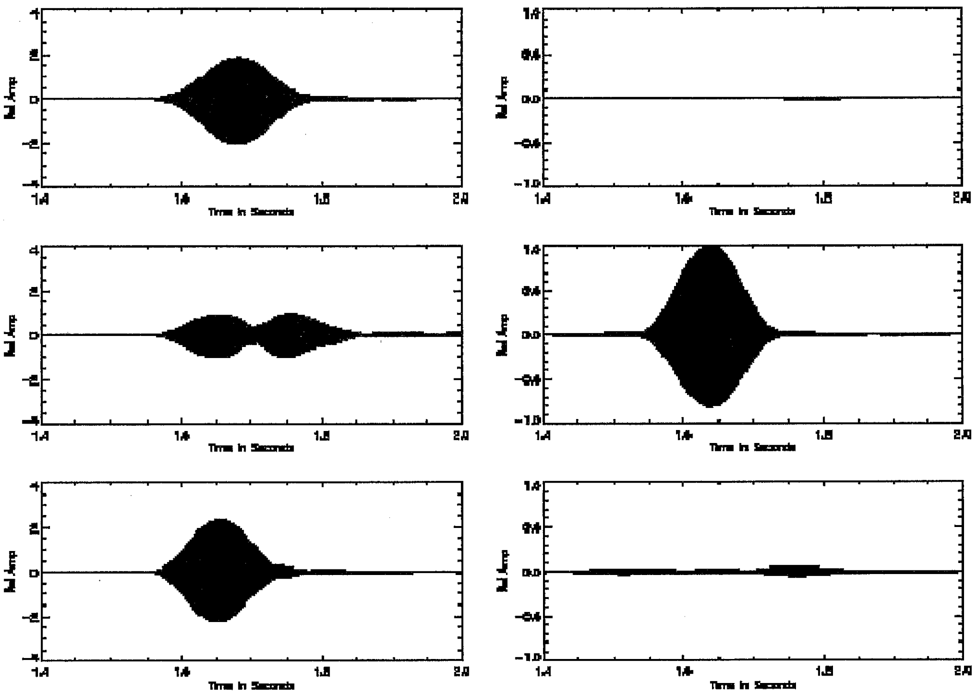

Fig. 2: Pulses in a two-way transmission path case: at the top, middle and bottom of the array (left) and at the source/receiver range after time-reversal and return propagation (right).

top, middle and bottom of the array. On the right are the time series of the time-reversed transmission at the range of the focus, at the surface, mid water and seabed. The plots are normalised by the peak amplitude at the mid water depth. The probe transmission is severely distorted by multipath, particularly at the middle of the array, and there is not much variation between energy at different depths. The temporal dispersion is reversed in the time-reversed transmission, and the energy concentrated at the focus and away from the boundaries. Dramatic increases in the energy at the receiver are predicted (greater than 20dB). Time compression is modest. Higher time compressions would be observed if shorter pulses were modelled (but then run-times would increase).

Examples of results for the four-way transmission path case are shown in Fig. 3, for different numbers of elements (for a fixed array length) and different environments. Gains are less than in the two-way path case. Performance generally increases with number of elements, as the array is better able to resolve the multipath.

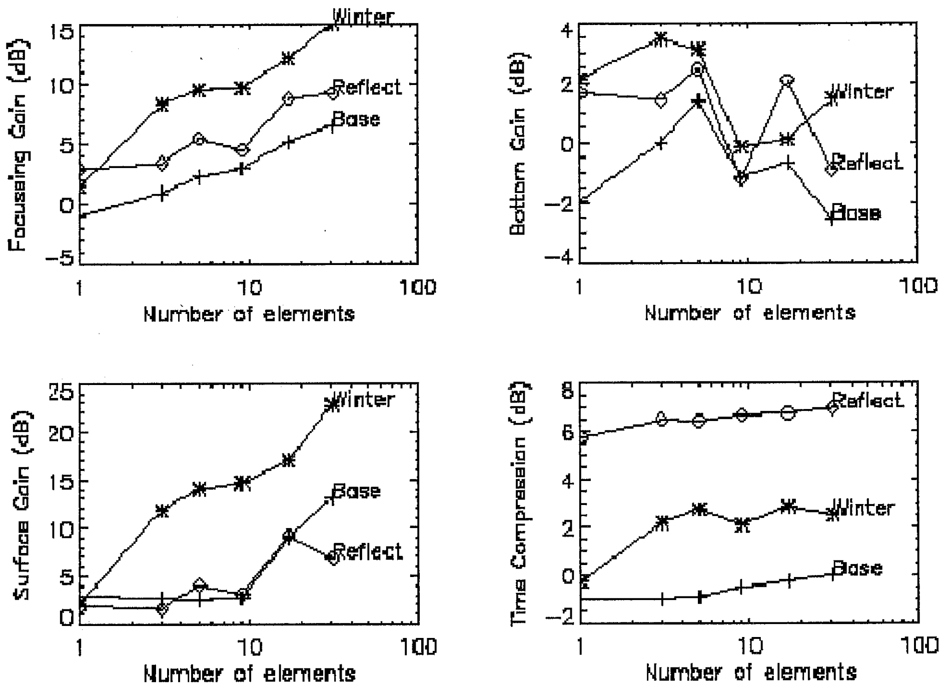

Fig. 3: Measures of performance in a four-way transmission path system for a vertical line array

Since the signal which is time-reversed includes energy that has been scattered by the surface and bottom scatterers, a portion of the energy in the time-reversed transmission would be expected to focus on these. The relative amounts of energy directed onto the target and the surface and bottom scatterers therefore depends as much on where the initial transmission goes (little energy reaches the surface scatterer in a downward refracting environment), as on the focusing ability of the array. The surface gain increases as the number of elements increases, but the bottom gain does not show any clear trend. The gains are strongly dependent on the environment. The time compression is greatest with the reflective sea-bed, because this supports more multipath.

A method of representing the effect of a non-stationary rough surface in the model was developed. This was achieved by modelling the fields in both the water and the air, and the reflection at the boundary between them, for snapshots of an evolving surface profile (fixed for each propagation leg). Calculations were performed for two-way and four-way systems in a 100m depth environment similar to the baseline environment. A pure 2kHz tone at 4.7km range and 94.5m depth was modelled (reducing bottom reverberation when looking for a near-bottom target is an important and demanding objective).

When the time-reversed transmission was made immediately after reception, the effect of a slightly rough surface was to improve focusing relative to the flat surface case. Levels away from the focus relative to levels at the focus were predicted to be lower than when the surface was flat (although absolute levels at the focus itself were reduced). This is somewhat surprising but was thought to be because the roughness scattered energy into higher angle modes, leading to a wider range of angles of arrival at the array. When the roughness increased, however, focusing started to degrade, because a significant amount of energy was now scattered into higher order modes which interacted with the seabed and were attenuated rapidly with range.

Fig. 4 shows the focusing achieved in a two-way system in the presence of a 0.25m RMS roughness for different time delays between reception and the time-reversed return. Here, the roughness is small and focusing does not degrade much as the time delay increases. When the roughness is larger, focusing was found to be degraded when the time delay was zero, and to degrade further as the time delay increased.

The focusing performance of a phase conjugate array at moderate range was found to be reasonably unaffected by surface motion except at high sea states. This is because the sound that propagates to these ranges is that which travels at low angles and only interacts weakly with the surface (the rest is stripped out by attenuation in the seabed). The low angle sound is affected by long wavelength components of the sea surface spectrum, which are stable for relatively long periods.

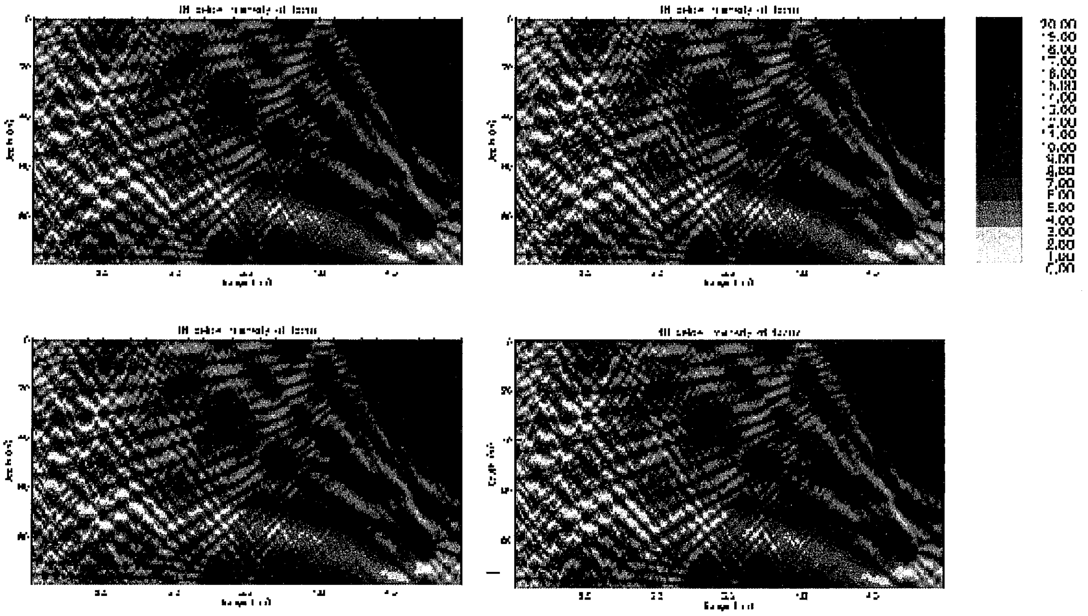

Fig. 4: Focusing with 0.25m RMS roughness for various time delays, top left 0s, top right 2s, bottom left 10s, bottom right 50s

3. INVERSE FILTER METHODS

Time-reversal techniques have been shown to be effective in enabling a desired signal to be transmitted to the focus point. To do this it is necessary to characterise the environment using a transmission from the focal point. There can be problems with time-reversal with broadband signals, however, when there is significant variation in the magnitude of the transfer function of the waveguide across the band.

Inverse filtering (or equalisation) is a common technique used in room acoustics [5]. Pre-filters are applied to the input(s) to compensate for the transfer function of the system, with the intention that the signal at the receiver is equal to the desired signal. In a non-square system, the pre-filter is calculated using a least squares approximation. In general, the simple least squares solution does not lead to a useable time domain form of the filter as it is non-causal and has long duration in forward time. These properties lead to temporal aliasing if a practical filter length is adopted.

Here, inverse filter techniques have been applied to focusing in the ocean environment and have been shown to overcome some of the limitations of time-reversal when applied to broadband signals. Tikhonov regularisation and cyclic shifting has been used to achieve a stable and causal inverse filter. The presence of low levels of noise has been shown to have little impact, indeed noise acts to regularise the matrix.

$$H(\omega) = \left[C^H(\omega) C(\omega) + E\left[N^H N\right] + \beta(\omega)\, I \right]^{-1} C^H(\omega)$$

The effectiveness of the inverse filter method with a vertical array has been assessed using transfer functions calculated using the OASES propagation model for the site off Italy where experiments on phase conjugate focusing have been conducted by MPL and SACLANTCEN [3].

An example of the temporal shape of the pulse, from an impulse signal from a conventional transmission (a broadside-steered array), from inverse filters, and from phase conjugation, for a two element array, is shown in Fig. 5. Also shown is the spectrum of the received signal. The

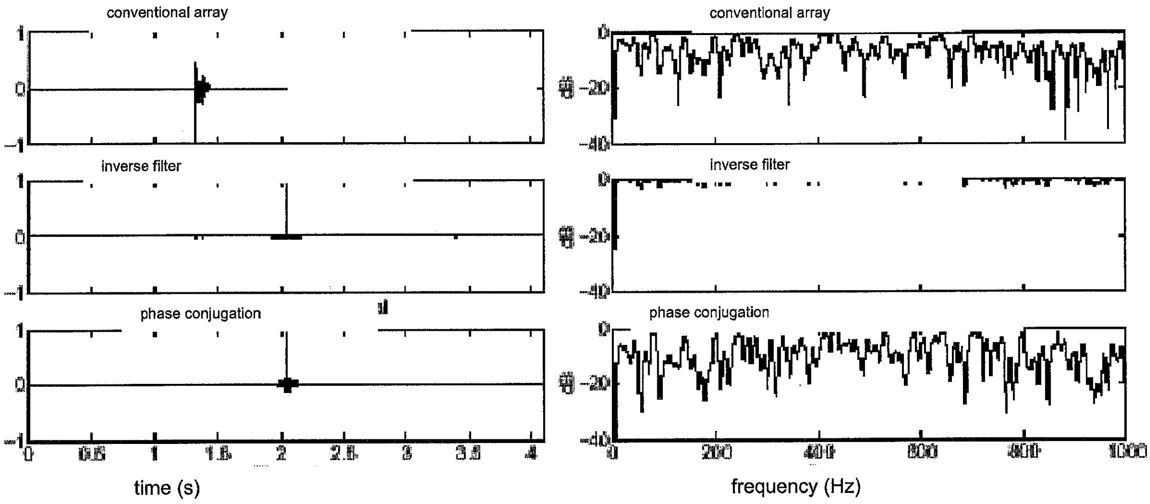

Fig: 5: Time series and spectrum of received pulse from conventional, inverse filter and phase conjugate transmissions

conventional transmission is significantly distorted as evidenced by the reduced amplitude and extended duration. In the model here, the inverse filter gives a near perfect impulse and a uniform spectrum. The phase conjugate transmission has significant low level sidelobes before and after the main impulse, and an uneven response in the frequency domain. The benefit of inverse filters is most marked when the number of elements in the array is small (as in the example here).

4. EXPERIMENTS

The relative performance of inverse filter and time-reversal focusing in the ocean has been investigated in numerical simulations and the inverse filter method has been shown to have important potential advantages. Focusing depends on a number of subtle environmental effects and several important simplifications have been made in the modelling. The results therefore need validation with data taken in a real environment.

Both methods will be degraded by changes to the environment and to the array during the period between the transmission of the probe signal and the final reception of the focused transmission. Noise at the array will also degrade focusing, although low levels of noise can have benefits in the calculation of the inverse filter. Flow noise, which is incoherent on the elements of the array, and coherent noise from ships in the vicinity are components of the noise field. Slight changes to the environment can be thought of as noise, provided they do not change the transfer function of the channel significantly. The susceptibility of a practical system to the changes to the environment and to noise needs to be assessed. In room acoustics, improved estimates of the plant matrix (the transfer function of the channel) are routinely obtained by averaging different transmissions. The extent to which this will be possible in the ocean in the presence of the underlying variability of underwater environment needs to be explored.

At the same time as demonstrating the focusing obtainable in a real environment, any experiment must be sufficiently controlled so that the impact of the different effects can be separated and identified. For example, steps must be taken to ensure the array is completely stable in order that the focusing achievable with a stationary array can be determined. The effect of translation and deformation of the array can then be determined. Modelling has shown that the type of environment, particularly the nature of the sea-bed, will have an important effect. This should be borne in mind when interpreting results and ideally measurements in several different types of environment should be undertaken

Any operational system using the focusing concept will want to minimise the number of sources and receivers employed in order to minimise costs and the effort required to deploy the system. An experimental system must be flexible to allow different number of elements to be tested. The focusing methods are unusual in that the performance can only be determined by playing back the transmissions through the environment itself, at the time in question. Post-trial analysis of data is limited to displaying the field in the region of interest measured at the time.

The MPL/SACLANTCEN experiments have been extremely successful in demonstrating the phenomenon of phase conjugate focusing in an ocean environment. These have used vertical strings of sources and receivers spanning a large fraction of the water column. Experiments at both 445Hz and 3.5kHz have been performed. Extensive environmental measurements were taken to assist in the interpretation of the results. An experimental system is being assembled in the work reported here which will enable a side-by-side comparison of the time-reversal and the inverse filter focusing methods to be made. The frequency selected is intermediate between the frequencies used by MPL/SACLANTCEN so will produce information in a different frequency regime. The MPL/ SACLANTCEN experiments have provided a benchmark, which has guided the design of our system [7].

A schematic of the system is shown in Fig. 6. Two 16-element 80m long source-receiver arrays are being produced. The arrays are intended to be deployed off the seabed in 100-120m of water. Each array is made up of four interchangeable modules. Arrays with less than 4 modules can be deployed. The arrays are enclosed in a 120mm hose filled with silicon oil. The elements are spaced much further apart than in a conventional array. Modelling has shown that at the ranges of interest most of the energy in the propagating field arrives at angles close to the horizontal and that aliasing will therefore not be a problem.

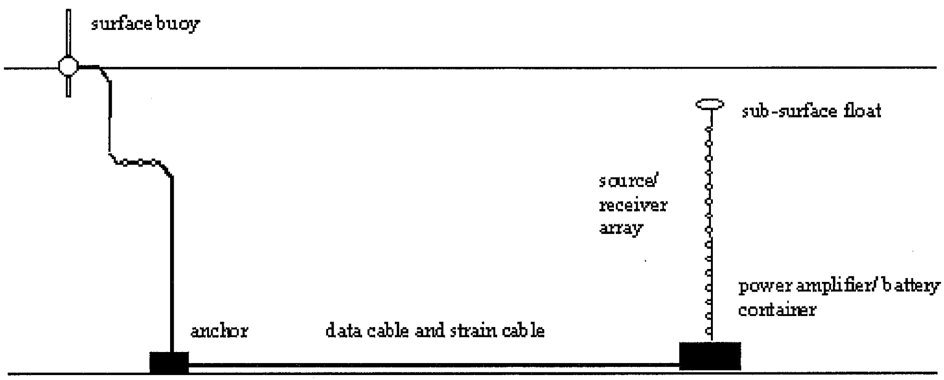

Fig. 6: Experimental system for phase conjugate array experiments

The sources are Marine Acoustics Sonoflex 1900 benders. The output of these is within 3dB over the 1.8 - 2.0kHz band and is approximately omnidirectional in azimuth and elevation. The response of the benders depends on depth, and varies a little from source to source. This has to be compensated for in the matching transformers and the pre-filters applied to each channel, as the method relies on transmitting a specified signal at each source (a time-reversed replica or the inverse filter). The sources were individually calibrated at the depths they are will be deployed in the arrays. The response was found not vary appreciably with depth except in the upper 20m of the water column (which will not apply here).

The receivers are pairs of Benthos AQ2 hydrophones mounted 100mm above and below the sources. The use of pairs of hydrophones does not incur an appreciable variation in response to the propagating field, but provides some rejection of noise arriving from above and below.

The arrays are autonomous and will be moored up from seabed units which act as anchors and contain the signal controllers, power amplifiers and batteries for the transmit system. This will ensure the arrays are decoupled from motions of the surface. The tops of the arrays will be supported by sub-surface floats. The advantage of having the power amplifiers and batteries in the seabed unit is that the power loss to the sources will be much reduced. The disadvantage is that it will be more difficult to access the batteries for recharging.

The arrays communicate with the trials ship through a separate cable from the seabed unit to a surface buoy and a RF data link. The arrays will be controlled from the trials ship. Data received on the receiver channels will be transmitted to the trials ship, where time-reversed replicas or inverse filters will be prepared. These will be passed to the arrays for transmission.

A single array can be used with a separate source and receiver unit lowered from the trials ship. The source will generate the probe signal and the receiver will measure the field in the focal region. In this arrangement the field will only be monitored at one point hence only information on amplitude at the focus and pulse shape will be obtained. Alternatively, the two source-receiver arrays can be used together, with an element of one of the arrays acting as the probe source and the field at the focal range being monitored at the receivers of the same array. This arrangement will enable the field at different depths at the focal range to be measured.

5. APPLICATIONS TO SONAR

Focusing techniques have obvious applications to underwater communication. As multipath dispersion is reversed, inter-symbol interference will be reduced and achievable data rates increased. Since energy is focused at the receiver, power consumption will be reduced. The transmitter and receiver need to be fixed in the period between the probe signal and the transmission of data, and this may be a constraint in some applications. The application of focusing techniques to sonar is less straightforward. Several possible concepts for a submarine detection system that uses focusing to obtain improved performance have been examined.

First is the use of previously acquired data, or data obtained in situ, to determine the transmissions required to focus at a particular range and depth. A source deployed from a helicopter or an AUV could be used as a probe source to acquire this data. The system would then scan a volume of water by making transmissions directed to a sequence of search cells. The echo from a submarine in the focal region would be stronger as more energy would be directed there, and reverberation would be lower because less energy would be directed at the surface and the seabed. The processing gain of a FM system would be higher as the dispersion of the transmission would be unravelled as it propagated to the search cell.

Modelling has indicated that gains from focusing from a long array in this sort of system could be dramatic. However, tight focus regions would have disadvantages as data at a greater number of focus points would need to be gathered and a greater number of transmissions would need to be made. All that is really required is that the field at the boundaries is substantially reduced relative to points in the middle of the water column. A smaller array will require fewer search cells and is therefore likely to be more effective in practice, as well as being cheaper and easier to deploy.

In an azimuth-independent environment the focus obtained from a vertical array would be an annulus. If the array was not vertical, or the environment was not the same in all azimuths, a focus would only be obtained in direction of the source. In these cases, data would have to be obtained for a number of azimuths as well as range and depth, the resolution being determined by the spatial extent of the focus. Another factor is the time validity of the focusing information. This would depend on the stability of the environment but in most environments it is likely to be limited.

The echo from a submarine could be received on the array generating the focused transmission, or on another array. Each would have advantages. Reception on the array generating the transmission would mean that the returns in the same time gate as the echo from a submarine in the search cell would be from the other points on the annular focus, hence reverberation would be low. If reception is on another horizontal array, reverberation in the same time gate from other directions could be rejected and the position of the contact could be determined.

Because of the difficulty collecting and processing the focusing information for a large number of search cells, this type of system is likely to be of most use in a scenario where the submarine is likely to be in a fairly restricted area, for example in a choke point. The requirement for a large number of transmissions on a continuing basis and the fact that the array making the focused transmissions needs to be stable, mean that this concept is most likely to be suitable for a fixed installation with a link to an external power supply.

A second concept is the use of an echo of an initial transmission from a submarine to determine the transmission required to focus on it. This is an acoustics version of the technique used successfully in laser-based directed energy weapons to overcome imperfections in the optical components and distortions and divergence of transmitted pulses due to atmospheric turbulence. The initial transmission in an acoustics system could be from the array that will make the focused transmission or from another array. If the initial transmission is from the array that will make the focused transmission and this array also makes the final reception, the situation is the 4-way transmission path case that was considered in the modelling. This predicted more modest (but still valuable) gains than for a system transmitting over a 2-way path.

An advantage of this system is that the transmission does not need to be scanned over a set of search cells covering a volume of water: the system will focus on any echo that is present. However, the system will only focus on the strongest scatterer(s) that contributed to the echo. The signal to noise of the echo from the submarine therefore needs to be positive initially. In current active sonars, the signal to noise threshold required to declare a detection with a 50% probability of detection and an acceptable probability of false alarm is quite substantial and therefore this is not a major limitation. The focused transmission will raise an echo which is on the limits of detectability, above the threshold. The process can be repeated and further gains obtained [8]. The system will focus on the strongest scatterer in the echo. The use of a focused transmission should be seen as a means to examine/investigate a possible contact, or a volume of water in which no detectable contacts are apparent with a conventional transmission.

The third concept is an acoustic barrier system in which a focus is generated using a source-receiver array and a probe source, and the field at different depths at the focal range monitored. The focus would probably be placed near the bottom in this case. A submarine crossing the propagation path will degrade the focus. Energy will be scattered to depths other than the focus. The MPL/SACLANTCEN experiments have shown that focus ranges of 30km are possible in some environments. The method provides a means of detecting the scattered field from a submarine in the presence of the direct pulse, which would normally mask it. The detection and false alarm performance of such a system will depend on the strength of the field from the submarine and natural variation of the focus, which is difficult to predict in advance.

6. CONCLUSIONS

A numerical model for predicting the focusing obtainable from a phase conjugate array (time-reversal mirror) has been developed. Measures of performance have been defined to quantify the advantages of different array configurations. Calculations have been made for systems that work over a 2-way and a 4-way propagation path. Gains in systems working over a 2-way path are substantial while gains in systems working over a 4-way path are more modest (but still valuable). A method of modelling the effect of a time varying rough surface has been developed. Focusing performance at moderate range was found to be reasonably unaffected by surface motion.

The application of inverse filters to focusing in the underwater environment has been investigated. It has been shown that inverse filters overcome some of the limitations of time-reversal when applied to broadband signals.

Focusing depends on a number of subtle environmental effects. An experimental system is being produced which will allow data to be obtained to validate the models.

A number of possible concept for using focusing in a submarine detection system have been identified. Systems could use focusing information obtained with a deployed source, or use the echo from an initial insonification of an area. An acoustic barrier detection system which monitors for a degradation to a focus caused by the presence of a submarine is also possible.

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