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A COMPARISON BETWEEN WAVE PROPAGATION TECHNIQUES AND TRANSIENT SEA FOR THE PREDICTION OF SHOCK RESPONSE IN ONE-DIMENSIONAL SYSTEMS

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1. INTRODUCTION

Many engineering structures are subjected to shocks which can be transmitted to remote sites causing damage to delicate equipment. The waveform at the remote site can have the original impulsive waveform and it can have a complex oscillatory form. The damage potential for either waveform can be estimated [1] if the peak velocity, peak energy and energy spectral content is known.

Two approximate means of finding these terms are presented here, namely Transient Statistical Energy Analysis (TSEA) [2] and a wave propagation analysis (WPA) [3]. The latter calculates the first wave that arrives at the remote site. The time-varying energy is described in terms of the peak energy and the time delay at the peak. The calculations are performed on a source beam driven at one end coupled to a receiver beam. The approximate response calculated at the far end of the receiver beam is compared with results from an exact modelling using a wave approach, which includes attenuation due to damping and across the connecting boundary. Both longitudinal and flexural waves are considered [4]. The results from a parameter study were compared in terms of modal overlap [5].

2. THE COUPLED RESPONSE OF TWO ONE-DIMENSIONAL SYSTEMS

Consider the coupled system in Figure 1. Two one-dimensional systems of respective finite lengths ℓ_s and ℓ_r are coupled via one-degree of freedom with a spring of stiffness $\overline{K}$. The source beam (subscript s) is subject to a force spectrum F_{s1}, with corresponding velocity responses in the frequency domain of V_{s1} and V_{s2}. The receiver beam moves with velocities V_{r1} and V_{r2}, at each end. End 2 of the receiver beam is free.

The kinematic quantities are related by the mobilities of the uncoupled systems: i.e., for the source system,

$$\begin{Bmatrix} V_{s1} \\ V_{s2} \end{Bmatrix} = \begin{bmatrix} M_{s11} & M_{s12} \\ M_{s21} & M_{s22} \end{bmatrix} \begin{Bmatrix} F_{s1} \\ F_{s2} \end{Bmatrix}, \qquad (1)$$

and likewise for the receiver, with s replaced by r. Inserting the boundary conditions, $F_{r2} = 0$, $F_{s2} = -F_{r1} = (V_{s2} - V_{r1})\,\overline{K}/i\omega$, the velocity response at the remote end of the receiver is defined by a transfer mobility, M_{sr}: i.e.,

$$V_{r2} = M_{sr} F_{s1}, \qquad (2)$$

where $M_{sr} = M_{s12} M_{r12}/(M_{s22} + M_{r11} + i\omega/\overline{K})$. This is the basis of the exact solution for longitudinal wave transmission, flexural wave transmission, and the WPA solution for these two wave types.

If the two beams are excited only axisymmetrically in the longitudinal direction, the longitudinal mobilities can be expressed in the forms

$$M_{s11} = M_{s22} = M_s(1 + \overline{\alpha}_s)(1 - \overline{\alpha}_s), \quad M_{s12} = M_{s21} = M_s(2\overline{\alpha}_s^{1/2}/(1 - \overline{\alpha}_s), \qquad (3)$$

where M_s is the characteristic mobility of an infinite beam:

$$M_s = (\rho_s S_s c_s)^{-1}, \quad \overline{\alpha}_s = e^{-2ik_s \ell_s}.$$

$\overline{\alpha}$ is the attenuation and phase change in a wave travelling $2\ell_s$ and ρ_s, S_s and c_s are the density, cross-sectional area and longitudinal phase wave speed, respectively. k_s is the complex wave number for longitudinal wave motion, given by $k_s \approx (\omega/c_s)(1 - i\eta_s/2)$, in which η_s is the hysteretic loss factor for the source beam. In this case the characteristic mobility also corresponds to the input mobility of a semi-infinite beam. Similar definitions apply to the receiver beam, with subscript r replacing s.

The velocity, $v_{r2}(t)$, in the time domain is found by using an inverse Fourier transform:

$$v_{r2}(t) = \int_{\alpha}^{-\alpha} V_{r2}\, e^{i2\pi ft}\, df\ . \qquad (4)$$

The procedure in wave propagation analysis is similar to that described for the exact solution. There is a difference in the calculation of the transfer mobility, M_{rs}, defining the ratio of the transmitted velocity to the remote end of the second beam to an input force on the source beam. In this approach only the first incident wave is considered, this wave being modified at each boundary encountered by a transmission coefficient, t. The ratio of final to initial wave amplitude is simply the product of transmission coefficients. The transfer mobility, M_{rs}, is therefore written as:

$$M_{rs} = V_{r2}/F_{s1} = (V_s/F_{s1})(V_r/v_s)(V_{r2}/V_r)\,. \qquad (5)$$

The first factor represents the travelling s wave at $x = \ell_s$ in a semi-infinite beam due to an input force, F_{s1}: i.e.,

$$V_s/F_{s1} = M_s e^{-ik_s \ell_s} . \tag{6}$$

The second factor is the ratio of the transmitted wave, V_r, to the incident travelling wave, V_s, at a junction between two semi-infinite beams, usually known as the transmission coefficient,

$$t = V_r/V_s . \tag{7}$$

For longitudinal wave motion, the transmission coefficient, t, is [4]

$$t = 2M_r/(M_r + M_s + i\omega/\overline{K}) . \tag{8}$$

The third factor, V_{r2}/V_r, is the ratio between the free velocity, V_{r2}, on the receiver beam at $x = \ell_r$ due to an incident travelling wave at x = 0 on the receiver beam:

$$V_{r2}/V_r = 2e^{-ik_r \ell_r} . \tag{9}$$

The exponential term describes the change in phase and amplitude of the wave travelling a distance ℓ_r from the junction. At ℓ_r, there is a free end causing a doubling of velocity amplitude due to the reflection. Very similar expressions are available for the flexural waves.

Figure 2 shows a comparison of the WPA solution and exact solution for the case of longitudinal wave excitation. The WPA solution obviously has no resonances. To make comparisons with TSEA these solutions were averaged later in bands of one decade. Ideally it would be best if a single band was used but it is necessary because the TSEA has frequency-varying coupling loss factors.

Figure 3 shows the Fourier transform version of the exact solution. The WPA solution is just the first peak.

3. TRANSIENT STATISTICAL ENERGY ANALYSIS

The time varying version of SEA for two coupled subsystems is shown in Figure 1. The time varying energies are E_s and E_r; η_s and η_r are the inherent loss factors of the two systems. The energy balance equations are

$$\begin{aligned} dE_s/dt + \eta_s\omega E_s + \eta_{sr}\omega E_s - \eta_{rs}\omega E_r &= 0, \\ dE_r/dt + \eta_r\omega E_r + \eta_{rs}\omega E_r - \eta_{sr}\omega E_s &= 0. \end{aligned} \tag{10}$$

η_{sr}, η_{rs} are coupling loss factors and ω a band centre frequency. The solution for an initial energy, E_{s0}, in the source and zero initial receiver energy is [1]

$$\begin{aligned} E_s &= (E_{s0}/2b)\, e^{-a\omega t} \left[(a + b - \eta_a)\, e^{b\omega t} + (-a + b + \eta_a)\, e^{-b\omega t}\right], \\ E_r &= (E_{s0}/2b)\, e^{-a\omega t}\, \eta_{sr} \left[e^{b\omega t} - e^{-b\omega t}\right], \end{aligned} \tag{11}$$

where $\eta_a = \eta_s + \eta_r$, $\eta_b = \eta_r + \eta_{rs}$, $a = (\eta_a + \eta_b)/2$ and $b = \left[(\eta_a - \eta_b)^2 + 4\eta_{rs}\eta_{sr}\right]^{1/2}/2$. The coupling loss factor for the source beam is shown in [4] to be η_{sr}, given by

$$\eta_{sr} = \tau_{sr}/\omega(1 - \tau_{sr}/2)n_s\,, \qquad (12)$$

4. NUMERICAL PARAMETERS STUDY – CONCLUSIONS

The preceding solutions – exact, WPA and TSEA – for the response of impulsively excited beams were applied in a numerical parameter study. The objective was to calculate and compare the relative responses of the receiver beam in terms of the peak velocity, the peak energy and the rise time for the peak energy, for a range of parameters: varying lengths, damping and cross-sectional areas. A suitable means of normalizing the data was sought and found to be the modal overlap factor [5], m, which is related to the energy attenuation of a wave travelling the length of both beams.

The beams were chosen to have slightly different lengths to ensure that the impulses did not arrive simultaneously at the junction from either beam. The two beams had different cross-sections and were coupled with a hinge. Longitudinal and flexural wave excitation occurred. Time averaging of the energy from the exact method and the WPA was necessary for comparison with the TSEA. This was done using both the end velocity alone, and also segments along the length. Such a comparison is seen in Figure 4 between the TSEA and a measurement. Figure 5 shows a comparison of the delay time to the peak energy. The TSEA is accurate at modal overlap less than unity, and the WPA for modal overlap greater than unity. Figure 6 gives the comparisons of peak energy. The peak energy is well modelled by both methods for modal overlap less than unity but at high modal overlap the TSEA overestimates on account of no allowance being made for the energy loss due to damping of the travelling wave between the input and output.

5. REFERENCES

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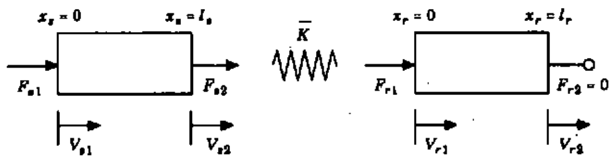

Figure 1. Schematic diagram of two coupled one-dimensional systems.

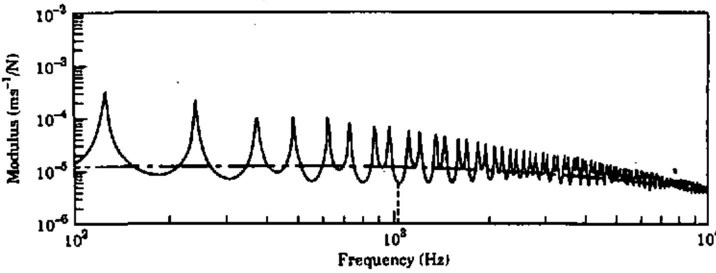

Figure 2. Frequency response function V_{r2}/F_{s1}; longitudinal excitation case A. —, exact solution; – — –, WPA solution; – – –, position of high pass filter.

Figure 3. Impulse response from the exact solution of the coupled longitudinal beams $V_{r2}(t)$, case A.

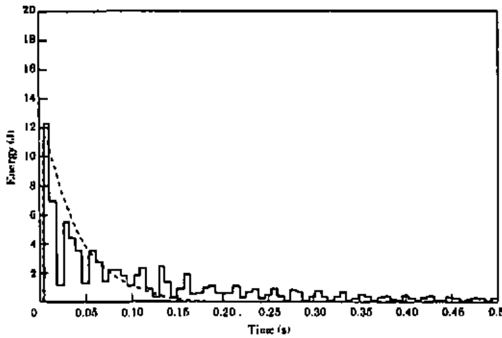

Figure 4. Comparison of measured energy level and TSEA estimate for the two coupled perspex beams.
—, measured energy; – – –, TSEA estimate.

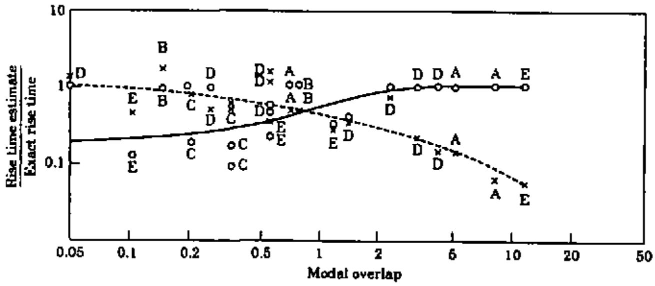

Figure 5. Normalised rise time for the peak energy in the receiver beam, tests A – F, against modal overlap.
o—o, WPA/exact; x– – –x, TSEA/exact.

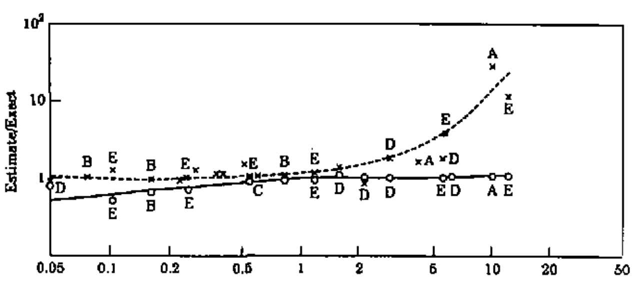

Figure 6. Normalised peak receiver beam energy from tests A – F, against modal overlap.
o—o, WPA/exact; x– – –x, TSEA/exact.