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RADIATION INTO A POROUS MEDIUM

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1. INTRODUCTION

Radiation from plates (such as walls in a building, the sides of a car or the fuselage of an aircraft) into air is well understood. However, in structures where the plate forms part of a double wall there may be absorption material in the air space or cavity and this will affect radiation from the plate. This paper examines radiation into a porous layer from a lightweight plate with a high critical frequency. The porous layer is assumed to extend to infinity so that comparisons can be drawn with radiation into free space.

A theoretical model is developed for radiation in which the porous layer is modelled as an equivalent fluid which includes dissipative losses. The theoretical results are then compared with measurements made on an aluminium panel.

If plate vibrates with a known velocity, v, and radiates acoustic power, W, then the radiation efficiency is related to the power radiated by the equation

$$W = v^2 \rho_o c_o \sigma \qquad (1)$$

where ρ_0 is the density of air and c_0 is the wavespeed in air. Two distinct frequency regions are of interest depending on the relative speeds of the bending waves on the plates and the wavespeed in air. The transition occurs when the two wavespeeds are equal and this frequency is known as the coincidence frequency, f_c . At frequencies where the wavespeed in the plate is higher than the wavespeed in air, above the coincidence frequency, then a plate is an efficient radiator and the radiation efficiency is close to 1. At frequencies below the coincidence frequency a plate is a poor radiator.

The theoretical approach for radiation into a porous layer uses a wave argument in a manner similar to Leppington *et al* [1] for radiation into air. This shows that a plate can be an efficient radiator at all frequencies.

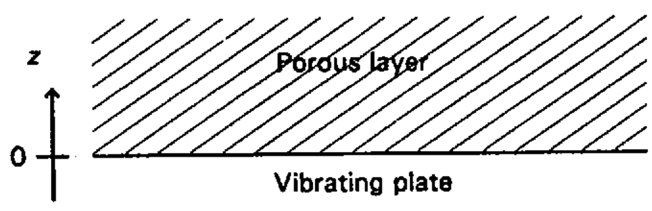

Figure 1. A plate with air on one side and a porous layer on the other. Radiation takes place into the porous layer.

2. RADIATION FROM A PLATE USING A WAVE APPROACH

The system being considered comprises a thin elastic plate which occupies the whole of the plane $z = 0$, as shown in Figure 1. The semi-infinite half space $z > 0$ contains a porous medium assumed to behave like a fluid with complex wave number $k_a = (1 - i\kappa)$ and complex density $\rho_a = \rho(1 - i\tau)$. The frequency dependent equivalent fluid properties can be found by semi-empirical methods such as those considered by Delaney and Bazeley [2, 3] and Allard and Champoux [4]. The plate is excited by a sound field from the region $z < 0$ in such a way as to vibrate with a transverse velocity

$$v(x,y,t) = \sin k_x x \sin k_y y \exp(-i\omega t) \tag{2}$$

where k_x and k_y are the wave numbers in the x and y directions respectively and ω is the angular frequency. The resulting sound field in the porous medium can be described by a velocity potential $\phi(x,y,z,t)$ which is governed by the wave equation

$$\nabla^2 \phi = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} \tag{3}$$

where c is the complex wave speed in the fluid.

The fluid motion described by equation (3) is coupled to the plate by the boundary condition

$$\frac{\partial \phi}{\partial t} = v(x,y,t) \quad \text{on } z = 0 \tag{4}$$

The solution is

$$\phi = \frac{1}{i\gamma_a}\sin k_x x \sin k_y y \exp i(\gamma_a z - \omega t) \tag{5}$$

where γ_a is the wave number in the z direction normal to the plates surface and satisfies the expression

$$\gamma_a^2 = k_a^2 - k_x^2 - k_y^2 \tag{6}$$

2.1 Radiation into air

The simplest radiation case to consider is radiation into air and this problem has already been examined by Leppington *et al* [1]. The problem is simplified as the density of air is real (1.18 kg/m^3) and the wavespeed is also real (343 m/s). There are therefore no dissipative losses in the fluid. The solution analogous to expression (5) is

$$\phi_o = \frac{1}{i\gamma_o}\sin k_x x \sin k_y y \exp i(\gamma_o z - \omega t) \tag{7}$$

where $\gamma_o^2 = k_o^2 - k_x^2 - k_y^2$. The form of γ_o will differ according to the position of k_o with respect to the plate wave number given by

$$k_p = \left(k_x^2 + k_y^2\right)^{\frac{1}{2}} \tag{8}$$

For $k_o > k_p$, the wave number $\gamma_o = \left(k_o^2 - k_p^2\right)^{\frac{1}{2}}$ is real and this is above coincidence. The actual waveforms in the air are given by the real part of ϕ_o which in this case is

$$\mathrm{Re}\{\phi_o\} = \frac{1}{\gamma_o}\sin k_x x \sin k_y y \sin(\gamma_o z - \omega t) \tag{9}$$

The term $\sin(\gamma_o z - \omega t)$ is oscillatory and is representative of a radiating sound field.

For $k_o < k_p$, (below coincidence), $\gamma_o = i\left(k_p^2 - k_o^2\right)^{\frac{1}{2}} = i\gamma$, say, so γ_o is purely imaginary. The waveforms are

$$\mathrm{Re}\{\phi_o\} = -\frac{1}{\gamma}\sin k_x x \sin k_y y \exp(-\gamma z)\cos \omega t \tag{10}$$

and the associated sound field consists of exponentially decaying surface waves and no sound is radiated.

The sound fields associated with equations (9) and (10) can be used to calculate the radiation efficiency as defined by equation (1). On substitution of the time-averaged power radiated, the following expression is obtained

$$\sigma = -4k_o \,\mathrm{Im}\left\{\iint \phi_o(x,y,0,t)\frac{\partial \phi_o^*}{\partial z}(x,y,0,t)dxdy\right\} \tag{11}$$

where the integral is performed to give power radiated per unit area of plate. The superscript * denotes complex conjugate.

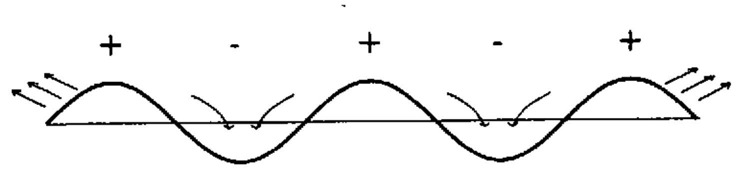

Figure 2. Air moves sideways as the plate vibrates at frequencies below the critical frequency where the waves in the air travel faster than the bending waves on the plate. On an infinite plate there is no radiation but on a finite plate there is radiation from the edges.

Above coincidence

The integrand is imaginary above coincidence (due to γ_0 being real) so the radiation efficiency is

$$\sigma = \frac{k_o}{\gamma_o} \tag{12}$$

This equation can be simplified to give the standard equation for radiation above the critical coincidence frequency as

$$\sigma = \left(1 - \frac{f_c}{f}\right)^{-\frac{1}{2}} \tag{13}$$

Below coincidence

Below coincidence γ_o is imaginary so the integrand is real and

$$\sigma = 0 \tag{14}$$

The physical explanation for this is described by the cancelling phenomenon associated with Figure 2. The sound in air travels faster than the bending waves on the plate and therefore disturbances in the air caused by plate vibrations result in the air moving sideways thus cancelling the pressure differences and resulting in no net radiation away from the plate.

However, in finite sized plates there is no cancellation of the acoustic pressure around the edges and so radiation can occur. The approximate equation given by Leppington *et al* [1] for radiation in this case is then

$$\sigma = \frac{Uc_o}{4\pi^2 f^{\frac{1}{2}} f_c^{\frac{1}{2}} S\left(\mu^2 - 1\right)^{\frac{1}{2}}}\left[\ln\left(\frac{\mu + 1}{\mu - 1}\right) + \frac{2\mu}{\mu^2 - 1}\right] \tag{15}$$

Where U is the perimeter length, S is the surface area and $\mu = (f_c / f)^{\frac{1}{2}}$.

2.2 Radiation into a porous layer

Sound radiation into a porous layer can be found in a similar manner to that into air by considering the form of γ_a as introduced in equation (6). Substitution of the complex form of k_a into this gives

$$\gamma_a = r^{\frac{1}{2}}\left(\cos\tfrac{1}{2}\varphi + i\sin\tfrac{1}{2}\varphi\right) \tag{16}$$

where $r = \left\{\left[k^2\left(1-\kappa^2\right) - k_p^2\right]^2 + 4k^4\kappa^2\right\}^{\frac{1}{2}}$ and $\varphi = \tan^{-1}\left\{\frac{-2k^2\kappa}{k^2\left(1-\kappa^2\right) - k_p^2}\right\}$; $-\pi < \varphi < 0$.

It was shown for air that $\sigma = 0$ when γ_o is imaginary. From expression (16) it can be seen that γ_a is only purely imaginary if $\cos\frac{1}{2}\varphi = 0$ which it cannot be in the range of definition. Therefore power will be radiated from the entire plate both above and below coincidence. The waveforms in the fluid are given by

$$\text{Re}\{\phi_a\} = \frac{1}{r^{\frac{1}{2}}}\sin k_x x \sin k_y y \exp\left(-r^{\frac{1}{2}}\sin\tfrac{1}{2}\varphi . z\right)\sin\left(r^{\frac{1}{2}}\cos\tfrac{1}{2}\varphi . z - \tfrac{1}{2}\varphi - \omega t\right) \tag{17}$$

This expression contains both the radiating sine term associated with expression (9) and a decaying exponential as in equation (10). There is therefore a radiating sound field, which decays with distance. The radiation efficiency can be found using a similar expression to equation (11) where the power radiated is normalised by that associated with a piston radiating into air. This gives

$$\sigma_a = \frac{\rho k_0}{\rho_0 r^{\frac{1}{2}}}\left(\cos\tfrac{1}{2}\varphi - \tau \sin\tfrac{1}{2}\varphi\right) \qquad (18)$$

for all frequencies.

Below coincidence the sideways motion of the fluid is impeded, due to the complex nature of the fluid, and so work is done to move the air and so the hydrodynamic short-circuiting effect is incomplete. Sound is radiated from the plate over the entire area giving a radiation efficiency that is a function of area and not boundary length. This also results in substantially higher values for the radiation.

3. EXPERIMENTAL RESULTS

In order to test the theory that was developed in the previous sections measurements were made on a thin aluminium plate radiating into a 200 mm thick rockwool slab with an air flow resistance of 40000 rayls. It is not possible to measure directly the sound waves generated in the porous layer partly due to the difficulties of placing a microphone in the porous layer and partly because the acoustic energy radiated is converted into heat within a short distance from the plate.

The approach adopted was therefore to measure the damping loss of the plate. The plate was placed on top of the porous layer and excited by striking with a hammer. The reverberation time was then measured from which the damping loss factor was found using the equation

$$\eta = \frac{2.2}{fT} \qquad (19)$$

The predicted damping loss factor was found from the equation

$$\eta = \frac{\rho_0 c_0 \sigma}{\omega \rho_s} \qquad (20)$$

Since σ was normalised by $\rho_0 c_0$ it is $\rho_0 c_0$ that is used in this equation and not $\rho_a c_a$.

To calculate the properties of the equivalent fluid, the Delaney and Bazeley expressions [2, 3] were used when f was greater than about 8000Hz and the Allard and Champoux [4] expressions were used when f was less than 8000Hz.

The results can be seen in Figure 3 and show excellent agreement between the measured and predicted results over the frequency range above 1000 Hz. Below about 1000 Hz the agreement is not as good. This is partly due to the difficulty of measuring high damping values by this method as damping of over 100 dB re 10^{-12} is close to the limit for the filter which is 112 dB due to filter ringing. There may also be a limitation because the application of the equivalent fluid concept is not strictly valid in this range. The lower line on the graph shows the loss factor of the plate when suspended by a thin string and is a measure of the internal loss factor of the plate. Laying the plate on the porous layer has increased the damping of the plate by about 10 dB showing that the radiation into the porous layer is the dominant damping mechanism.

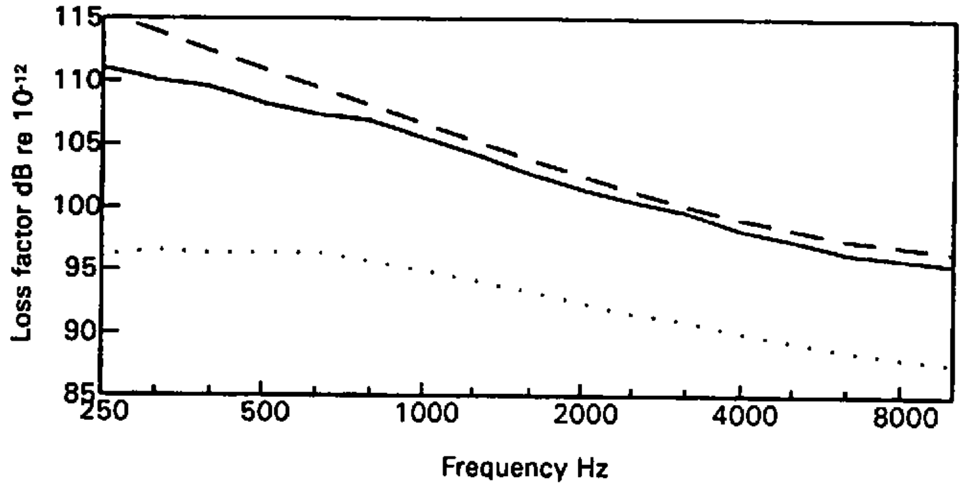

Figure 3. Measured and predicted damping loss factor of an aluminium plate when radiating into a porous layer. ________, measured damping when radiating into a porous layer; - - - - - -, predicted damping when radiating into a porous layer; . . . ., measured damping of a freely suspended plate.

4. CONCLUSIONS

The results of this work have shown that when a plate radiates into a porous layer then the radiation will be much higher than for radiation into air. Radiation takes place over the entire plate and so infinite plate theory can be used to calculate the radiation efficiency.

The porous layer was modelled as an equivalent fluid and this places a lower frequency limit on the model. However, now that the mechanism of radiation is understood a more detailed model of the porous layer could be introduced.

This theory can be used to predict the damping of plates in contact with porous materials.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

1. FG Leppington, EG Broadbent and KH Heron. The acoustic radiation efficiency of rectangular panels. *Proc. R. Soc. Lond.* **A382**, 245-271, 1982.
2. A Cummings. Sound attenuation in ducts lined on two opposite walls with a porous material, with some application to splitters. *J. Sound & Vibration.* **49(1)**, 9-35, 1976.
3. ME Delaney and EN Bazeley. Acoustical properties of fibrous absorbent materials. *Applied Acoustics.* **3**, 105-116, 1970.
4. J.-F Allard and Y Champoux. New empirical equations for sound propagation in rigid frame fibrous materials. *J. Acoust. Soc. Am.* **91(6)**, 3346-3353, 1992.