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STATISTICAL ENERGY ANALYSIS: THEORY INTO PRACTICE

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1. INTRODUCTION

Statistical energy analysis, SEA, is a framework of analysis for modelling sound transmission through large complex structures [1, 2]. It is somewhat unusual as its proponents do not claim that it will give the correct answer for a particular application. Indeed, they would claim that SEA will always give the wrong answer for any specific application though it will give a good estimate of the true answer.

Any theory that attempts to model a large and complex structure must make approximations and so not getting the answer correct is not unique to SEA. What is unusual is that uncertainty is central to the way SEA is structured. This uncertainty in the accuracy of the answers is one of the many difficulties faced by a new user deciding on whether or not to select a theory for a particular application. An estimate of the time it will take to learn the theory has to be made and this has to be balanced with the benefits of having that understanding. In structural dynamics there are many different approaches used. If a new user decides that SEA may provide the answer to a problem or problems then where should that person go for information on how long it will take to understand enough to get started, how long it will take to be proficient and how long to master any software. Then there is the question of whether or not the approach will solve the problem and will it be flexible enough for the application. Learning a new technique requires an enormous investment of time and effort and the rewards are often poorly documented. Guidance is therefore required about where a theory fits in with other activities and criteria are required against which the approach can be judged.

This paper looks at some of the underlying concepts of statistical energy analysis, describes some typical applications and discusses some of the properties of large systems that can be useful for judging whether or not SEA is useful for a specific problem.

2. APPROACHES TO DESIGN

Engineering design is largely evolutionary. Each design tends to be based on a previous design so that, for example, each new car design is based on a previous design. For effective evolutionary design a theory has to be able to predict changes in design very accurately and it must be capable of refinement as more features are added. It is desirable but not always essential that the theory can model the entire structure as often only parts of the design are being changed.

However, there are other types of problem for which design is not evolutionary where there are no previous designs to develop and these require a different kind of theoretical model. One example is the design of a spacecraft. When the first spacecraft were designed there were no previous models to evolve from and a new design had to be undertaken with only limited experimental data to support the model. Statistical energy analysis was developed for this type of complex problem where information is limited and often exact details of construction unknown.

Buildings design tends not to be evolutionary as all buildings are different and even in domestic houses which look the same there are sufficient differences in construction, occupation and furnishings for apparently identical houses to be acoustically different. Ships are also built in this way and even nominally identical sister ships will be sufficiently different due to variations in construction and in fittings for them not to be exactly identical.

For manufactured objects that come off a production line there can be step changes in design that make all the old evolutionary models obsolete. One example is the change in design of railway carriages from a steel shell to extruded aluminium. The new construction is sufficiently different that design has to be begun from scratch.

A common feature of these types of project is that they are large and complex and so a particular type of theoretical model is required. It needs to be robust and insensitive to small design changes, and it must be able to model the entire structure. These were desirable features of the evolutionary model but not essential. On the other hand, the ability to predict the noise level accurately is not critical and an ability to endlessly refine the model is useful but not essential.

This difference in design approach leads to two types of engineer who have radically different approaches to design. There is often little that they have in common in terms of working practice and often little understanding of the benefits that the other can bring to the design process.

3. MODELLING WITH STATISTICAL ENERGY ANALYSIS

Statistical Energy Analysis was developed in the 1960's to help understand the problems of vibration on spacecraft and was developed in parallel with the Apollo programme to land a man on the moon. It builds on several branches of acoustics including room acoustics and building acoustics.

One of the observations we make in every day life is that the sound level in a room does not vary as we move from one seat to another when watching the television. We also notice no change in the acoustics either in the quality of sound or in the level as other people enter the room or as furniture is moved about. A rise in temperature of 5^0 will significantly alter the properties of a room yet we do not normally notice any difference in noise level. Measurements also show that for many structural elements such as walls and floors similar observations are valid for vibration. Whilst the vibration level on a wall will vary with position, this variation is relatively small being typically only 2 or 3 dB and is sufficiently small that a wall can be described by a single number. Observations also show that the vibration level of a floor is not particularly sensitive to where the furniture is put in a room nor where the people are standing.

Spacial variations in sound or vibration level are not ignored but treated statistically For example, Schroder [3] and others in the 1960's showed that the sound pressure level variation with position in a room is random with the pressure squared values having a χ^2 distribution and a standard deviation in dB, s, given by

$$s = 5.57/(1 + 0.238\,\Delta f T)^{1/2} \qquad (1)$$

where T is the reverberation time. It is therefore not only unnecessary to attempt to model exactly the sound pressure level in a room and to calculate the SPL at every position but it is a process that adds no extra useful information.

This approach to modelling in which large physical objects are considered as a single element is not a question of approximating the system and omitting important detail. Rather it is a systematic approach to averaging out information which cannot be known. It is in this sense that statistical energy analysis is statistical. It takes important information such as fittings, furnishings, usage and occupation which cannot be known in advance of construction and treats them in a statistically meaningful way to give an ensemble average performance for a system without being able to give details for any specific implementation.

The unwritten rule of SEA is that nothing should be more complicated than it needs to be. This starts at the beginning with the subdivision of a system into sub-elements or subsystems. Each room, wall or floor (or the equivalent components in a car, ship or aircraft) will be a subsystem. Each subsystem will have a different response for a given source or sources, however, within each of these subsystems the vibration level or the sound pressure level will be sufficiently uniform that it can realistically be described by a single value at each frequency band.

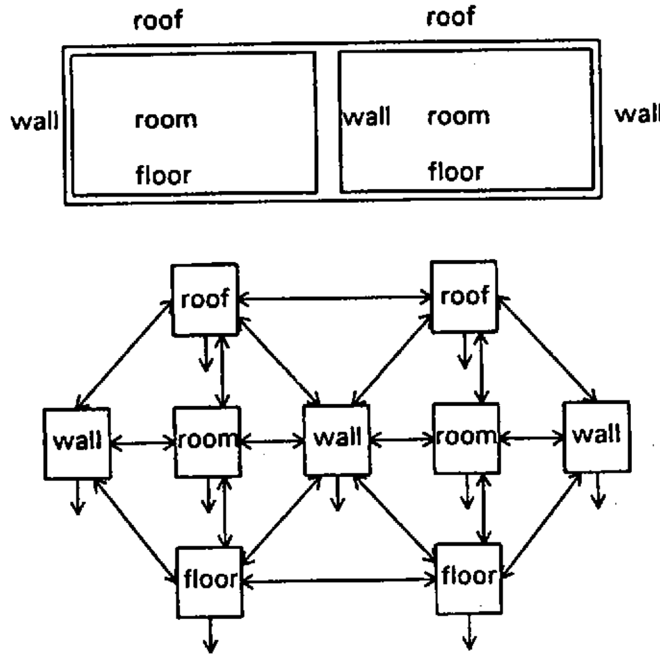

Figure 1. Section through a sound transmission laboratory with common walls and floors together with the equivalent SEA model. Subsystems for walls shown in elevation are not included for clarity.

For a two room sound transmission suite which has common flanking walls, floor and roof as shown in Figure 1, the SEA model has only 13 subsystems and is independent of the physical size of the construction (and is also shown in Figure 1).

The power flow between each of the subsystems is described by a parameter called the coupling loss factor and again the underlying principle in SEA is that this should be a simple as possible. Therefore, for most types of coupling simple algebraic expressions are sufficient such as the equation for coupling from a wall to a room as [1]

$$\eta_{wall-room} = \frac{\rho_0 c_0 \sigma}{\omega \rho_s} \qquad (2)$$

where ρ_0 is the density of air, c_0 is the wavespeed in air, ρ_s is the surface density and σ is the radiation efficiency. Similarly, the structural coupling between two walls along a line is given by [1]

$$\eta_{wall-wall} = 0.1365\left(\frac{hc_L}{f}\right)^{1/2}\frac{L}{S}\tau \quad (3)$$

where h is the thickness of the source plate, c_L is the longitudinal wavespeed of the source plate, L is the common boundary length, S is the surface area of the source plate and τ is the structural transmission coefficient. These equations are sufficiently simple that they can either be computed by hand or evaluated using a simple spreadsheet.

The final stage in the calculation process is the combination of all these loss factors together with information about damping and the acoustic sources to calculate the sound pressure levels in each of the rooms or the vibration level of each structural element. For a large model a matrix solution can be used to solve the equations (as many equations as there are subsystems).

Alternatively, if particular transmission paths are to be investigated then the transmission along a particular transmission path can be evaluated relatively simply. The airborne level difference due to transmission along a path 1-2-3-4 can be given by the equation [1]

$$D_{1-2-3-4} = 10\log\frac{\eta_2\eta_3\eta_4}{\eta_{12}\eta_{23}\eta_{34}}\frac{V_4}{V_1} \quad (4)$$

where η_{ij} is a coupling loss factor describing power flow between two subsystems, η_i is a subsystem total loss factor and V is the room volume. Again this equation is sufficiently simple that it can either be computed by hand or evaluated using a simple spreadsheet.

4. SOME APPLICATIONS OF STATISTICAL ENERGY ANALYSIS

Statistical energy analysis is most impressive when used to predict sound transmission through very large structures and an example of this can be seen in Figure 2 which shows a comparison of measured and predicted results for transmission through a large building [4].

The figure shows a graph of the difference between the measured response and the predicted response as a function of distance. The distance given is the distance from the centre of the source wall to the centre of the receiving wall and on average there is a structural joint every 2.5 m. The building (a section of which is shown in the insert) consists of 80 rooms on 3 floors. The section shows part of the building which is 30 m long and 7 m high. The building was excited with a structural source (shown by the arrow) and the vibration of the walls and floors throughout the building were measured. The attenuation varied from 10 dB across 1 structural joint to over 60 dB. Measurements were not possible over very large distances due to background noise limitations. The SEA model included all wave types.

It can be seen that there is a scatter of results about the zero line. Zero means that there is no difference between the measured and predicted result. Most of the results lie within about ±5 dB close to the source increasing to ±10 dB at large distances from the source. There are some large values (including one at 25 dB) but on the whole the results generally show good agreement. There is no tendency for the average of the measured values to either increase or decrease so that the values of the material properties must be about right (see discussion for Figure 6 and 7).

This result is typical of what might be expected for building acoustics. Getting the answer accurate to within 5 dB is generally considered good for transmission over long distances.

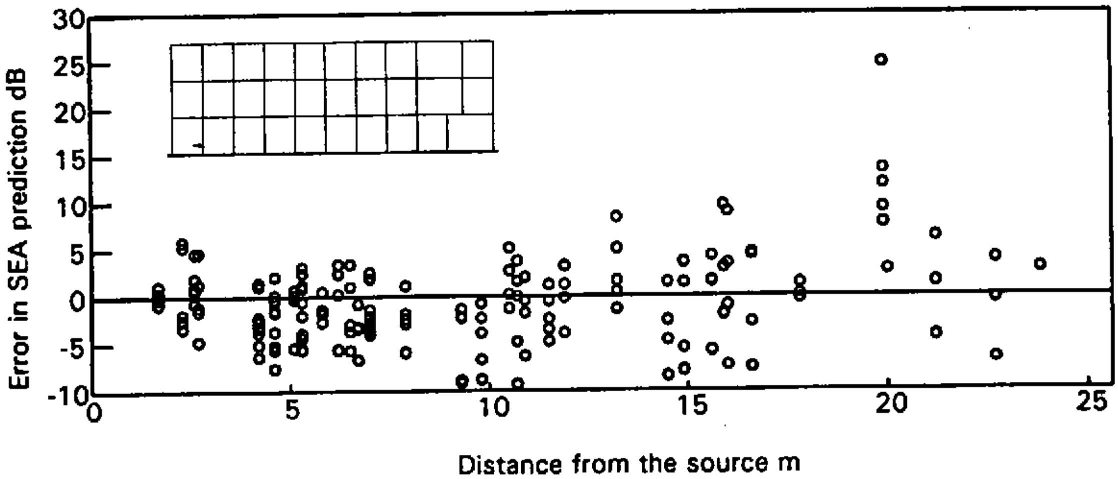

Figure 2. Measured minus predicted response as a function of distance. Data is 1/3 octave data at 125, 250, 500, 1000 and 2000 Hz [4].

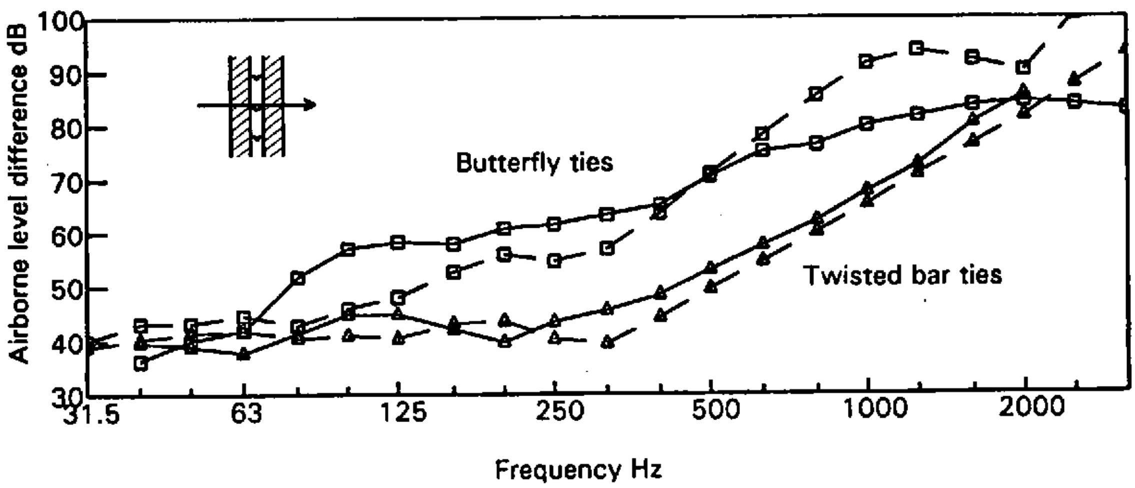

Figure 3. Measured and predicted sound transmission through a cavity wall where the leaves of the wall are coupled by wall ties [5]. ________, measured; - - - - -, predicted.

For simpler systems greater accuracy would be expected and is generally achieved as can be seen in the results for transmission between the two rooms in a sound transmission laboratory shown in Figure 3. This shows the measured and predicted results for transmission through a cavity wall where the two leaves of the wall were connected by cavity wall ties (both butterfly and twisted bar design) [5]. It can be seen that there is generally good agreement between the measured and predicted results for the case with the twisted bar ties but the results are not as good for the case with the butterfly ties due to the way in which coupling through the air was modelled. This poor agreement is not evidence that SEA is not an appropriate modelling approach but rather evidence that a specific mechanism of transmission is not understood.

A third example of the application of SEA is to sound transmission through a car and can be seen in Figure 4. This shows the measured and predicted sound pressure level in a car when the engine was running at a steady 5700 RPM [6]. Two results are shown. One results is for the case where there was a misalignment of the exhaust support causing additional noise transmission. In the other case the exhaust mount was disconnected resulting in lower noise levels. The results show good agreement between the measured and predicted results and show that the model can predict the effect of a faulty exhaust fitting which increases the power input into the floor of the car.

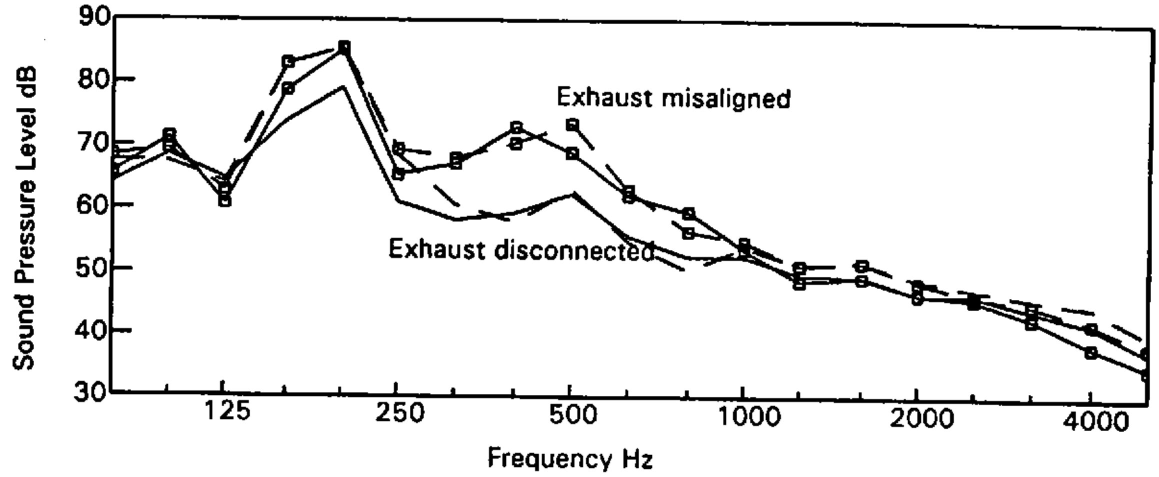

Figure 4. Measured and predicted sound pressure level in a saloon car with the exhaust mount misaligned and disconnected [6]. ________, measured, - - - - -, predicted.

A result with this level of accuracy would generally be considered as good in a building acoustics context but might be considered as insufficiently accurate for deciding whether or not a specific minor design change was going to be effective in a car. It would be sufficient for assessing a new design of car and the value of this analysis cannot be doubted when it is realised that making such a prediction represents only a few hours (or at most days) work.

5. IS SEA SUFFICIENTLY ACCURATE

One of the questions that needs to be asked when looking at the results given above is whether the answer is sufficiently accurate for the application. The answer to this question clearly divides those involved in evolutionary design and those involved in design from first principles. For those involved in the evolutionary design of a new car which you hope will be 0.5 dB quieter than the previous car, then an accuracy of ± 2 dB is not much use. On the other hand, if you are designing a new studio to house an orchestra in one room and a broadcast studio five floors up then being able to predict the noise level to within 2 dB would be considered excellent.

Design time can be very expensive and if the answer is only needed to within 5 dB there is little point spending months and months trying to achieve greater levels of precision. Since computer models can give answers to as many decimal places as can be imagined it is common for the number of decimal places printed out by the computer to be equated to the precision of the theoretical model. This is clearly a mistake and it is often only through experience that some of the limitations and accuracy of the model can be understood.

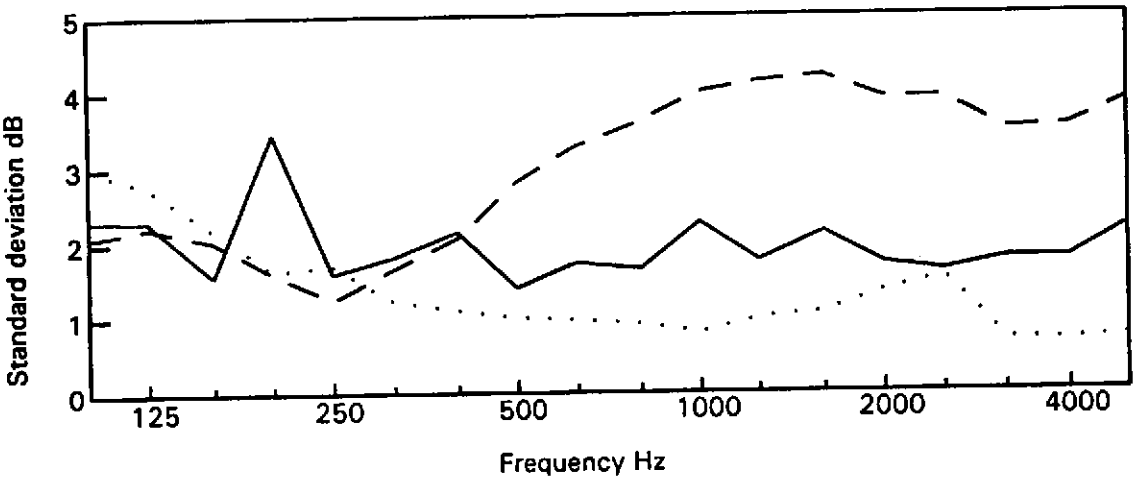

Figure 5. Standard deviation computed from different measurements made on nominally identical structures. ------ concrete floors; - - - - - - -, double leaf lightweight partitions; . . . . , single leaf lightweight partitions.

One method deciding how accurate the theory needs to be is to say that the theory need be no more accurate than the measured data against which it will be compared. Whilst the sound level at a specific point in a specific room can be measured to a fraction of a dB the measurement of parameters such as the airborne level difference is more complex requiring sampling of the SPL in two rooms for a variety of different source positions and measuring the reverberation time. ISO140 [7] estimates that the same person undertaking the same measurement over and over again (reproducibility) should expect differences from between 5 dB at low frequencies down to 1 dB at high frequencies.

There is also the expected variation between nominally identical examples of the same structure that has to be considered (termed repeatability in the ISO). Three sets tests have been carried out which demonstrate clearly this experimental uncertainty on the measurement of sound reduction index (SRI) of nominally identical structures. One series of measurements was made of the SRI of 10 concrete floors [8] and two series of measurements were made of the SRI of 24 lightweight partitions (both single leaf and double leaf) [9]. The standard deviation of these sets of measurements is shown in Figure 5. At low frequencies the three sets of data give similar answers with the standard deviation being about 2 dB while at higher frequencies there is a wide variation from 1 dB to 4 dB. Part of the high variation in the double wall can be attributed to small differences in construction but these differences can only be identified after the comparison identifies unusual results.

If the standard deviation is taken as being about 2 dB then there is a 95% probability that measurements on a single example would be within 2 standard deviations or ±4 dB from the true mean of an ensemble of many nominally identical systems. This level of measurement uncertainty is similar to the differences between the measured and predicted results shown earlier a places a limit on the achievable level of agreement between measured and predicted results.

Another approach to deciding the required accuracy of a theoretical model is to look at the sensitivity of the model to errors and uncertainty in the input data. One of the difficulties when modelling a large building is obtaining reliable values for the material properties. Manufacturers usually know the density of their products but properties such as Young's modulus or Poisson's ratio are not generally available.

Equally difficulty is estimating the internal damping which for many materials depends on the way the structure is built.

Errors in the material properties will lead to errors in the predictions and often tend to be cumulative as distance from the source increases. Examples of typical errors can be seen in Figures 6 and 7. Figure 6 shows a section through a building and in the centre of each room is the change in D_{nTw} (weighted standardised level difference [10]) due to a doubling of the value of Young's modulus for all elements [11]. A source was assumed in the room marked with an X. Young's modulus is rarely known to within a factor of 2 for building materials unless measurements are made on samples of material and there can be significant variations between individual samples of materials like brick. Therefore this magnitude of error would not be unusual. This is less of a problem for metals where the expected range of values for Young's modulus is smaller.

It can be seen that the change in D_{nTw} is relatively small with errors of less than 1.5 dB. Different patterns of change can occur if there is an error in only some of the elements (such as the floors) [11].

0.7	0.6	0.4	0.2	0.0	-0.2	-0.3	-0.5	-0.7	-0.9
1.0	0.9	0.7	0.5	0.3	0.1	-0.1	-0.3	-0.5	-0.8
1.4	1.0	0.8	0.7	0.5	0.3	0.2	-0.1	-0.3	-0.6
X	1.2	0.7	0.7	0.7	0.6	0.4	0.2	-0.1	-0.4

Figure 6. Change in D_{nTw} as the value of Young's modulus is doubled [11].

6.0	6.5	7.1	7.7	8.3	8.9	9.5	10.1	10.8	11.3
5.1	5.7	6.4	7.1	7.8	8.4	9.0	9.6	10.3	10.9
3.0	4.1	5.4	6.5	7.3	8.0	8.6	9.1	9.7	10.2
X	2.1	4.8	6.1	7.0	7.7	8.2	8.7	9.2	9.6

Figure 7. Change in D_{nTw} as the internal loss factor is doubled [11].

Similar results are shown in Figure 7 where in this case it is the internal loss factor (damping) of the materials that has been increased by a factor of 2 [11]. This increase in damping (from 0.015 to 0.03) results in a change of at least 2 dB in the room next to the source increasing to 11 dB at the opposite end of the building. As the energy is taken out of the building so less is transmitted to far parts of the building and so the calculated value of D_{nTw} increases.

Clearly if the input data is not known with any precision then the accuracy of the answer will also be uncertain and so there is little point in selecting a theory that is sensitive to changes in material properties.

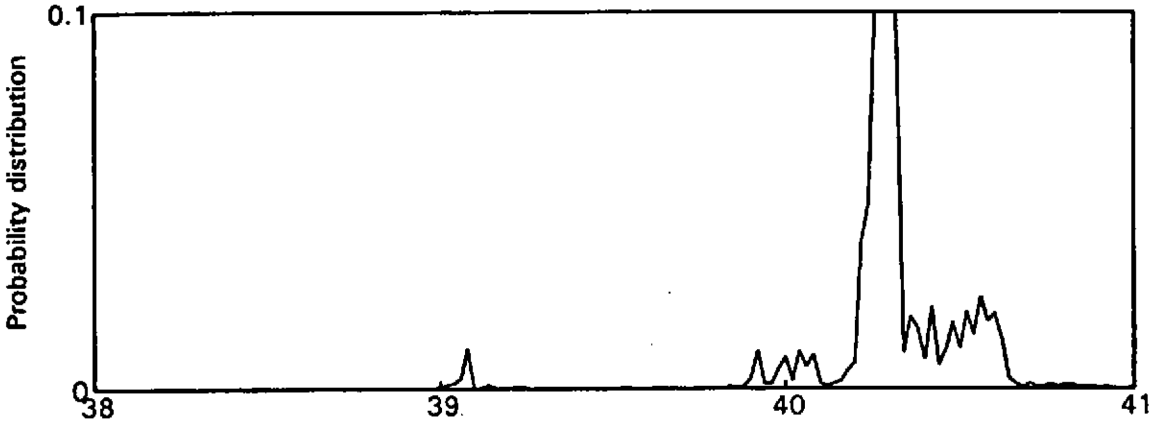

Figure 8. Probability distribution for sound transmission between two rooms.

Real structures are often not built exactly as originally designed. Building design constantly changes during construction and as a result the finished structure will not be the same as the one originally designed. In addition there may be differences between design and the actual structure because the contractor did not build what was designed and differences will also occur when there are faults such as when a crack occurs for example when a wall which is assumed to be connected to floor above is actually disconnected.

A theoretical model needs to be robust and insensitive to small changes in design if it is to be useful and assessing this robustness can be difficult as the effect of differences between the actual system and the assumed system can be complex since even if the original errors are random their effect on transmission will depend on their location.

An example of the influence of such random errors can be seen in Figure 8 which shows the probability distribution of the level difference D_{nTw} for transmission between two adjacent rooms when 1% of walls or floors are randomly disconnected at structural junctions simulating cracks. The figure shows the probability distribution normalised to a peak value of 1 (which occurs at 40.3 dB). It can be seen that such errors can either increase or decrease the level difference and that the results are not normally distributed. Over half the results lie within 0.1 dB of the "true" model. Every so often there are particular combinations of errors that give rise to errors of over 1 dB in the level difference.

On the whole it can be seen that the model is very robust and relatively insensitive to small design changes.

6. CONCLUSIONS

The results in this paper have shown that SEA is well suited to the kind of complex problem that involves large structures and limited data typified by building acoustics problems of sound transmission. There is uncertainty in the predictions of SEA due to the way in which parameters are defined as ensemble averages rather that situation specific. This leads to some uncertainty when the answers are applied to specific situations. However, this uncertainty is no larger than the uncertainty in the measurements of typical systems nor in the uncertainty arising from uncertain material properties.

SEA is a framework of analysis that is well suited to the types of problem that it aims to address. It is unusual in that it is a stated fundamental principle that there will be uncertainty in the answers obtained. The user is therefore always aware at least in a general sense that the answers have statistical uncertainty.

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