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PREDICTING THE PROPAGATION OF GROUND VIBRATIONS FROM UNDERGROUND RAILWAYS

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1. METHODS OF PREDICTION

Prediction of vibration propagation from underground railway tunnels to nearby receptors can be done by the following means:

Calculation of levels of vibration using algebraic expressions for propagation.
This approach involves the calculation of levels of vibration in a manner analogous to the calculation of sound levels at distances from sound sources. Source levels are assumed, either in terms of source power or amplitudes at a stated distance, or on the tunnel wall, and a propagation law is applied, for example a cylindrical line source equation for the propagation of dilatational (compression) waves.

Application of empirical methods, by adapting results of measurements in similar circumstances.
This may take two general forms. The first is the use of specific measurement results, for example obtained by tests carried out at a relevant site for the purpose of discovering soil propagation constants, and adjusting the results for site-specific properties such as distance. The second is the use of empirical prediction methods derived from statistical consideration of large numbers of data measured in a variety of field surveys, from which prediction algorithms have been derived.

Numerical modelling of vibration propagation, taking full account of all relevant physical properties of the soil, source and receiving structures.
Numerical models most frequently take the form of Finite Element Models or Finite Difference Models. They are capable of a high level of accuracy subject to the accuracy of the parameters assumed. FEM and FDM models are beneficial in cases where the ground is layered, particularly when the propagation direction is oblique and conversion between compression, shear and Rayleigh, Lamb or Stoneley waves occur.

They also can take account of variation of source levels of vibration around the perimeter of a tunnel.

2. PROPAGATION PARAMETERS

The propagation of vibration from underground railway tunnels is dependent on a set of parameters which describe the properties of the soil. The principal properties are the shear modulus, G, Poisson's ratio, ν, and the soil damping β. Many other properties can be derived from these three parameters, as follows:

$$E = 2(1+\nu)G$$

where E is Young's modulus

$$D = E\frac{(1-\nu)}{(1+\nu)(1-2\nu)} = 2G\frac{(1-\nu)}{(1-2\nu)}$$

where D is the longitudinal stiffness

$$\lambda = \frac{2\nu G}{(1-2\nu)}$$

where λ is the coefficient of dilatation, one of two Lamé constants, the other being the shear modulus G given the symbol μ.

$$D = \lambda + 2\mu$$

Damping can be expressed in a variety of ways:

$$\beta = \frac{\eta}{2} = \frac{1}{2Q} = 2\frac{W}{\Delta W}$$

where Q is amplification at resonance; W is the elastic energy with maximum strain during one cycle and ΔW is the energy loss during this cycle. Damping can also be expressed in terms of complex moduli, e.g.

$$\bar{G} = G(1 + i\beta_s)$$

$$\bar{D} = D(1 + i\beta_p)$$

where β_s and β_p are the shear and dilatational damping respectively.

The shear wave speed is given by

$$c_s = \sqrt{\frac{G}{\rho}}$$

where ρ is the mass per unit volume.

The dilatational wave speed is given by

$$c_p = \sqrt{\frac{D}{\rho}}$$

Waves exist at interfaces between solids and fluids, e.g. the ground surface in which case the waves are described as Rayleigh Waves, and at interfaces between two solids (Lamb waves and Stoneley waves). In the frequent case where $\lambda = \mu$, i.e. $\nu = 0.25$, the Rayleigh wave speed is 0.9194 c_S.

It is particularly important that the values of the elastic constants are appropriate to the small strains involved in vibration propagation, and geotechnical data for large strains give results for E, for example which can be many times smaller than the true small-strain value. In the prediction of ground-borne noise E is not directly used, and since the conversion to D and G is strongly dependent on the value of ν, great care is necessary in assembling appropriate data for a prediction exercise. A key consideration is the presence of water in, for example, gravels, which can significantly raise the value of ν. For the limiting case of a liquid, $\nu = 0.5$, with the result that E and G become zero, and for gravels below the water table derivation of D from measured values of E whether drained or undrained is unreliable.

3. CALCULATION OF VIBRATION USING ALGEBRAIC EXPRESSIONS FOR PROPAGATION.

In cases where the soil is homogeneous, the tunnel is deep enough in relation to the lateral distance of interest for surface waves to be a second order effect, and free-field results are required (neglecting the effect of the presence of a building on ground vibration), the propagation of vibration from an underground railway tunnel can be calculated algebraically.

The first point to consider is the fact that the impedance of the tunnel wall or invert will be finite, and this will both influence the behaviour of any vibration isolation system installed in the tunnel both by changing resonant frequencies through coupling, and by altering transmissibility by the introduction of mobility to the foundation of the system. It will also determine the distribution of vibration amplitude around the perimeter of the tunnel.

While some tunnels in rock may not have separate linings, most tunnels have linings of concrete, steel or iron which form single or multiples boxes, or tubes. The walls of box tunnels act like plates with either clamped or hinged edge conditions. The plates are damped primarily as a result of radiation of vibration away into the surrounding soil, although this effect is dependent on the degree to which the tunnel wall is in intimate contact with the surrounding soil, and the presence of intermediate layers such as grout.

The tunnel is a more complex structure when it is a bored tunnel. It is a special kind of plate, 'rolled-up' so that two edges are joined. The consequences are that in addition to ring-like standing waves occurring at frequencies at which the tunnel circumference is an integral multiple of the wavelength of bending waves in a plate the thickness of the tunnel wall, wave propagation takes place along the tunnel in which wave fronts (line joining points of equal amplitude at any instant) are spiral. At the frequencies represented by the ring modes, the tunnel wall responds freely subject only to the damping provided by the soil and the tunnel wall structure.

The tunnel alignment affects the parameters on which the propagation variables are dependent. Because the tunnel is not an infinite straight tube, and the source function varies along its length due to variations in train speed and in some cases trackform, it is necessary to integrate the contributions made at each receiving location by each section of the tunnel.

In the prediction of vibration propagation the rate of decay of wave amplitude with distance tends to accord with the general solution

$$A_r(\omega) = \frac{A_0(\omega) r_0}{r^n} \exp i(\omega t - kr)$$

where $A_0(\omega)$ and $A_r(\omega)$ are the source amplitude and transmitted amplitude at distances r_0 and r respectively, both being functions of angular frequency, ω, and of time, t. The wave number k is given by

$$k = \frac{\omega}{c'}$$

and is a complex number since c' is complex when damping is taken into account.

The exponent n is dependent on the source geometry (e.g. whether point or line source) and on wave type. It may vary between ½ (for Rayleigh waves) and 2 for some cases of shear wave propagation. For the most common case of body waves $n = 1$.

Damping is not necessarily always viscous, and in some soils may be hysteretic. This can be represented by substituting a value for η which is frequency dependent.

In many cases where the soil is homogeneous, dilatational waves predominate and the distance function can be simplified to

$$L'(\omega) = L(\omega) - 10\log_{10}\frac{r_0 + x}{r_0} - 4.34\frac{\omega\eta x}{c}$$

where $L(\omega)$ is the velocity level in decibels at distance r_0 from a cylindrical source and $L'(\omega)$ is the velocity level at distance x. This does not, however, allow for the fact that $L(\omega)$ is also a function of position around the tunnel wall, and will be greater in the invert than in the walls or the crown.

Typical values of η are 0.01 for rock; 0.1 for sand, silt, gravel and loess, and 0.1 to 0.5 for clay soils. They range from 1100-1700 m/s for clays, 300 to 600 for gravels and sands and 3000 to 4000 for rock. The value of ν depends on the water content of the soil and may be from 0.25 to 0.49. Where possible η should also be measured from soil samples.

These propagation equations apply only to homogeneous soil conditions. Since there is always a ground surface above an underground railway, they are in fact never strictly applicable since the arrival of *p*-

waves and *s*-waves at the surface will cause the appearance of Rayleigh waves which will propagate along the surface at a different speed and different rate of attenuation to the *p*-waves and *s*-waves. In fact, for shallow tunnels, excitation of the surface above the tunnel, followed by Rayleigh wave propagation to the sides, may be the dominant method of propagation.

Where there are horizons or layers in the soil, and at a watertable, not only do reflections between interfaces occur which, for the limited case of normal incidence (i.e. directly above the tunnel) increase attenuation, but also wave conversion occurs at all other angles of incidence. Incident *p*-waves are transmitted and reflected as both *p*-waves and *s*-waves, and incident *s*-waves are also transmitted and reflected as *p*-waves and *s*-waves, all at their respective angles. Bending (Stoneley) waves will propagate along the interface in a manner somewhat analogous to Rayleigh waves on the surface. This effect is particularly pronounced in layered soil, and bending waves will propagate laterally within layers, with in some cases minimal attenuation due to geometric spreading and losing energy only through radiation to adjacent layers.

In layered media, and in the presence of the ground surface, it is also necessary to consider shear waves as separate waves. Those involving strain in the same plane as the layer or ground surface are *SH* waves; those involving strain normal to the plane are *SV* waves which are generally coupled to *P* waves. The reflection, and transmission coefficients, and the extent of wave conversion, are different for *SV* and *SH* waves.

In the case of normal incidence, the effect of layered media can be calculated algebraically using techniques such as transmission line theory. This involves calculating reflection and transmission coefficients starting at the termination (the ground surface) as follows:

$$R = \frac{Z_t/Z_i - 1}{Z_t/Z_i + 1}$$

where R is the reflection coefficient, i.e. the complex ratio of reflected wave amplitude to incident amplitude and where Z_t is the impedance of the medium into which the wave is transmitted and Z_i is the impedance of the incident medium. Ignoring damping, $Z = \rho c$ where ρ is the soil density and c is the appropriate wave velocity. To include damping, c' should be used and Z becomes a complex number. Working back from the termination, the reflection coefficients are successively calculated for each layer as follows:

$$R = R_i + \frac{R_t(1-R_i^2)\exp(-ik2l)}{1+R_tR_i\exp(-ik2l)}$$

where R_t is the reflection coefficient previously calculated for the far side of the layer in question; $R_i = \frac{Z_t/Z_i - 1}{Z_t/Z_i + 1}$ is the reflection coefficient at the near side; l is the thickness of the layer and $k = \omega/c'$. All reflection coefficients are calculated for waves travelling towards the termination.

In cases of oblique incidence, calculation of the wave transmission is generally impracticable, and numerical techniques should be used.

4. APPLICATION OF EMPIRICAL METHODS.

This method involves the acquisition of measured data from one or more sites and either adapting the results to a specific site, or the generation of a set of multivariate equations for the purpose of fitting predicted results to the measured data.

Actual propagation conditions may be established by site measurements, for example by detonating an explosive charge in the base of a borehole and measuring ground velocity at locations successive distances from the source using buried transducers, if necessary at the base of other boreholes. It should be borne in mind, however, that while both the pressure amplitude and the particle velocity amplitude decays in inverse proportion to the distance at large distances, there exists a near field close to the source such that the ratio of pressure to velocity (the radiation reactance) is given by

$$\frac{X}{\rho c} = \frac{1+(1-K)(\omega a/c)^2}{K(\omega a/c)[1+(\omega a/c)^2]}$$

where

$$K = \frac{(1-\nu)}{2(1-2\nu)}$$

and a is the radius of a cavity within which the pressure is generated; X is the radiation reactance; ρ is the soil density and c is the relevant wavespeed.

In considering the applicability of measured data, the following matters should be taken into account:

i) Tunnel type: whether circular of box, and, if box, whether single or multiple;

ii) Tunnel lining;

iii) Track characteristics;

iv) Rolling stock characteristics;

v) Operating conditions, speed, train length, load;

vi) Soil surrounding tunnel (G, ρ, ν and η);

vii) Tunnel depth;

viii) Horizontal distance from tunnel to receiver;

ix) Presence of layers and watertables;

x) Presence of other inhomogeneities (faults, boulders, lenses, other tunnels, pipes and structures); and

xi) The effect of the presence of the receiving building

Particular care is required in the measurement and signal processing technique. The type of transducer employed, whether accelerometer or geophone, will affect the bandwidth of the results, and the method of attachment of the transducer to the ground may also affect the results.

The choice of signal processing method should be related to the criterion to be used for the assessment of vibration and ground-borne noise. If maximum weighted acceleration or velocity using r.m.s. or VDV based indices are to be used, care is required to ensure that the time-weighting and frequency weighting methods used in the analysis of the signal are the same as, or can be corrected to, those required for assessment.

Care is required to ensure that the axis of vibration is properly taken into account. In limited circumstances, vertical ground vibration may predominate, but in most cases triaxial measurements are required, at least to demonstrate the predominance of vertical vibration if this is the case.

Where a large quantity of data are used to generate regression equations, the correlation coefficients and confidence limits should be stated in the resulting predictions.

5. NUMERICAL MODELLING OF VIBRATION PROPAGATION

Numerical modelling is recommended where geological conditions are too complex for algebraic prediction, and comparable measured data are not available. Finite Element Models have the advantage of being widely available as standard packages of computer software, but have some limitations including the size of the computing resources required to include the effect of a moving train as the source of excitation. Finite Difference Models are not widely available as standard packages, but have the advantage of being able to model a moving train without difficulty. FEM models are not confined to orthogonal grids, whereas FDM models are best suited to orthogonal grids and are less easily adapted to other cases. Modelling in the time domain tends to require a lesser amount of computing resources for an FDM model, in which case it is normally the primary output, than an FEM model.

FEM and FDM models may be created in 1, 2 or 3 dimensions. One-dimensional models are of very limited used and only appropriate to restricted problems such as propagation from the crown of a tunnel vertically through a layered soil. Two-dimensional models are adequate for many cases, provided that the receiver distance is large compared to the length of the rail vehicles or the spacing between bogies, and short compared to the length of the train. When these two conditions are not satisfied, worst case approximations can be made taking the tunnel wall velocity during the passage of a bogie as applying to the whole length of the tunnel.

Three-dimensional models can take account not only of the finite length of train elements, but also can model the receiving building more accurately.

In either case, it is necessary to ensure that the mesh or grid sizes used are appropriate to the bandwidth required, and the degree of complexity of the soil structure. In particular it is necessary to achieve reflection-free boundary conditions, for instance by progressively increasing the loss factor of the soil.

Numerical models should be validated against cases which can be solved algebraically, including the case of a beam, a plate and a tube submerged in a fluid with point excitation. In the case of plates and beams validation for free, hinged and clamped end and edge conditions should be carried out.

Numerical models should also be validated against actual underground railways in circumstances where the principal parameters are known.

Both FEM and FDM models normally represent damping as viscous. There is evidence that in many cases soil damping is a viscous process, but in some cases hysteretic damping occurs, and some track components have characteristics such as stiffness and damping which are non-trivial functions of frequency. In these cases, modelling should be carried out at discrete frequencies with the appropriate parameters chosen unless the model is capable of replicating the phenomena concerned.

Many numerical models are designed to deal with the case of railways at grade, with the ground consequently modelled as a half-space. There are many differences between vibration generation and propagation from at-grade railways and railways in tunnels, and these models should only be used where the effect of transferring from at-grade to tunnel can be adequately taken into account.

In addition to FEM and FDM models, boundary element techniques, as an extension of the algebraic method descried above are possible, particularly where it can be shown that the principal mode of propagation over the distances of interest is through *P*-waves. In cases where tunnel vibration varies greatly around the perimeter, dilatational waves arising from the radial or normal displacement of the tunnel wall will also be accompanied by shear waves which are difficult to model using boundary element techniques. In layered ground boundary element techniques involving ray-tracing are an alternative to FEM and FDM.

6. CONCLUSIONS

The appropriate method for the prediction of ground vibration propagation from railway tunnels is dependent on the complexity of the site and the purpose for which the predictions are to be used. In cases of homogeneous soil, algebraic calculation can be adequate if tunnel vibration data are available for comparable conditions and free-field results are of interest. Empirical methods are appropriate in cases of complex geological conditions, for example where there is layered ground and/or complex building foundations, when it is possible to obtain experimental data specific to the site, or data from a site sufficiently similar and well understood for adaptation of the results to the case of interest to be reliably possible. Numerical modelling techniques such as finite element or finite difference models are necessary if it is required to predict propagation in complex site conditions.