MAP SIDE SCAN SONAR MOTION DISTORTION CORRECTION USING ITERATIVE CONDITIONAL MODES AND SIMULATED ANNEALING

R S Beattie (1), S C Elder (1)

(1) The Robert Gordon University, Aberdeen

1. INTRODUCTION

It is often difficult in practice to achieve the regular scanning pattern required to produce a distortion free side scan sonar image. Extraneous platform motion causes disruption of the scanning pattern, see Figure 1, and hence distortion in the image. In extreme cases this motion may lead to back scanning where the same area of sea bed appears three or sometimes five times in the image. In this paper the problem of removing the effects of this platform motion distortion is considered.

In his work Cobra[1,2] has shown how the local image covariance may be used to provide an estimate of the relative displacement of scans on the sea bed. He describes an algorithm which uses these displacement measurements to estimate the true scanning pattern using a Kalman filter based algorithm. He then shows how, once the scan pattern has been determined, it is possible to produce a corrected image by interpolating over the distorted pattern and then re-scanning the interpolated image on a regular grid.

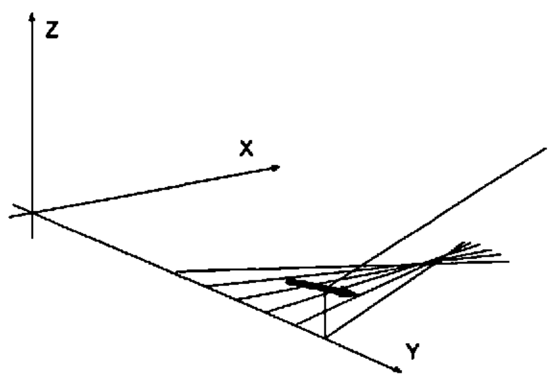

Figure 1 Platform motion causes disruption of the scanning pattern.

In this work an alternative approach to the scan pattern estimation task is examined. The scan pattern estimation algorithm developed here uses a non - linear Maximum A Posteriori (MAP) formulation of the problem. This formulation has a potential advantage over Cobra's approach in three respects. Firstly it is non - causal in nature and hence uses more of the available information. Secondly the non linear formulation uses more realistic statistical models and hence should produce improved results. Finally the formulation is insensitive to the displacement orientation and hence back scanning need not be estimated prior to application of the algorithm.

Two alternative approaches to computing the MAP estimate are suggested here. These being the globally optimal Stochastic Relaxation (SR) approach described by Geman and Geman[3] and the Iterative Conditional Modes (ICM) approach by Bessag[4]. It is also described how, through the combination of ML estimation techniques with the ICM algorithm, it is possible to simultaneously estimate both the scanning pattern and coefficients of the statistical models.

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2. MOTION MODEL

The sonar platform has six degrees of motional freedom. Three transitional and three rotational modes, see Figure 2. Theoretically five of these modes can cause distortion in the sonar image. The sixth, platform roll, does not alter the position of the sample point, corresponding to a given image pixel, on the sea bed and therefore can not contribute to motion distortion. If it is assumed that the sonar platform is stationary during acquisition of each scan and that the deviations of the platform from its nominal course and heading $\{\phi_{f(n)} = 0, \theta_{f(n)} = 0, z_{f(n)} = z, x_{f(n)} = 0\}$ are small the following expressions may be derived for the long and cross track sample displacement on the sea bed as a function of the sonar attitude and position:

$$\Delta x_{s(m,n)} = x_{s(m,n+1)} - x_{s(m,n)} = \Delta x_{f(n)} - z\Delta z_{f(n)} / l_{(m)} \quad (1)$$

$$\Delta y_{s(m,n)} = y_{s(m,n+1)} - y_{s(m,n)} = \Delta y_{f(n)} + l_{(m)}\Delta\theta_{f(n)} + z\Delta\phi_{f(n)} \quad (2)$$

Here $\Delta x_{s(m,n)}$ and $\Delta y_{s(m,n)}$, are the long and cross track sample point displacements on the sea bed between the sample points at a fixed range, or distance across the image, on adjacent scans n and n+1. The differential motion parameters being defined as $\Delta\phi_{f(n)} = \phi_{f(n+1)} - \phi_{f(n)}$, etc. The quantity $l_{(m)}$ is known as the slant range. These equations, which relate the sample displacements on the sea bed to the motion parameters of the sonar platform, will hence forward be referred to as the motion model.

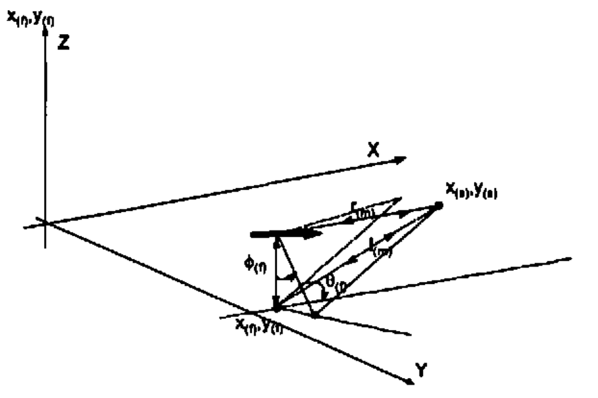

Figure 2 Sonar platform motion.

It should be noted that the motion model defined above is degenerate in the sense that changes in pitch $\Delta\phi_{f(n)}$ are indistinguishable from those in long track displacement $\Delta y_{f(n)}$. In order to remove this ambiguity it will be assumed in the following that the sonar platform proceeds a constant distance Δy_f between scans.

3. DISPLACEMENT ESTIMATION

In his work Cobra[1,2] describes how local covariance function measurements made in the sonar image may be used to estimate the magnitude of the sample displacements on the sea bed. The local covariance function estimate at pixel m,n having the form:

$$\rho_{(p,q)} = \sum_{j=-L}^{j=+L}\sum_{k=-W}^{k=+W}(g_{(m+j,n+k)} - \mu_{(m,n)})(g_{(m+p+j,n+q+k)} - \mu_{(m+p,n+q)}) / \sigma_{(m,n)}\sigma_{(m+p,n+q)} \quad (3)$$

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This function is a maximum when the cross track lag p is such that the effects of transitional motion have been cancelled. At that point the corresponding sample points on adjacent scans have their closest correspondence on the sea bed. The magnitude of the cross track distortion may therefore be estimated using:

$$\Delta\hat{x}_{s(m,n)} \approx -\frac{r_{(m)}\Delta r}{l_{(m)}} P_{(m,n)} \quad (4)$$

Here $P_{(m,n)} = \arg\{\max_p \rho_{(p,1)}\}$ and Δr is the cross track sample displacement which is a function of the sample rate of the sonar system. The expression above implies that the distortion can only be measured to within an integer multiple of Δr. In practice this limitation can be avoided by interpolating the covariance function about the maximum using a continuous polynomial, then finding the maximum of this polynomial. The point at which the maximum occurs now being a continuous rather than discrete variable.

Measurement of the long track displacement is considerably more complex. It is however reasonable to suppose that the magnitude of the covariance function is related in some respect to the scan displacement. That is the similarity, and hence covariance, will be greater when the scans are closer together and vice versa. The covariance function is however a complex function and there will be no simple relationship between scan spacing and covariance magnitude. There is however a simple relationship between the covariance length $y_{T(m)}$, that is the distance it takes for the covariance function to fall to a threshold value ρ_T, and the scan spacing $\Delta y_{s(m,n)}$. This relation is:

$$\hat{y}_{s(m,n)} \approx \left| \sum_{q=0}^{Q_{(m,n)}-1} \Delta y_{s(m,n+q)} \right| \quad (5)$$

Here $Q_{(m,n)} = \arg\{\rho_{(0,q)} = \rho_T\}$ is termed the covariance length. Again the covariance function may be interpolated to allow non integer values of $Q_{(m,n)}$. Note that this measure can not distinguish the orientation of the displacements. Displacement in both the forward and reverse directions giving the same result. When both cross and long track distortion is present it is generally simpler to treat the problem as two separate sub problems. That is to estimate first the cross track displacement, then produce a partially corrected image using this information. From this partially corrected image the long track distortion may easily be estimated and the final corrected image produced.

4. MAP ESTIMATION

Given the displacement χ measurements made at M ranges across the N scan image it is now desired to estimate the motion parameters ω. Here χ is the vector containing the observation data $\{\Delta\hat{x}_{s(m,n)}, \hat{y}_{s(m,n)} : m \in M, n \in N\}$ and ω contains the parameters to be estimated $\omega = \{\Delta\phi_{f(n)}, \Delta\theta_{f(n)}, \Delta x_{f(n)}, \Delta z_{f(n)} : n \in N\}$. The MAP estimate of these parameters is then defined as:

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$$\hat{\omega} = \arg\{\max_{\omega} \log P(\chi / \omega) + \log P(\omega)\} \quad (6)$$

Maximising this function is however generally quite involved and before attempting it a number of assumptions will be made. Firstly it is assumed that the motion model statistics are Markovian:

$$P(\omega / \omega_{S\setminus j}) = P(\omega_i / \omega_{\partial_j}) \quad (7)$$

Here $\omega_{S\setminus j}$ is the set of all parameters excluding the jth and ω_{∂_j} is the small set of neighbours on which ω_j depends. It is also assumed the joint conditional distribution can be approximated using a function of the form:

$$P(\chi / \omega) \approx Q(\chi / \omega) = \prod_{j \in Z} P(\chi_j / \omega_{v_j}) \quad (8)$$

Now noting that the MAP estimator can be factored into terms of the form:

$$P(\omega / \chi) = P(\omega_j / \omega_{S\setminus j}, \chi) P(\omega_{S\setminus j}, \chi) \quad (9)$$

Therefore an increase in $P(\omega_j / \omega_{S\setminus j}, \chi)$ results in an increase in $P(\omega / \chi)$ so by iteritively maximising the local distribution the algorithm must eventually converge to a maximum of the joint MAP estimate. What is more as the motion model is Markovian the conditional distributions are only locally connected and hence straight forward to maximise. The local distributions having the form:

$$P(\omega_j / \omega_{S\setminus j}, \chi) \propto P(\omega_j / \omega_{\partial_j}) \prod_{k \in \rho_j} P(\chi_k / \omega_{v_k}) \quad (10)$$

Here $\rho_j = \{k : j \in v_k\}$. This type of approach is known as an Iterative Conditional Modes (ICM) approach and was first suggested by Bessag[4]. In his original paper Bessag also suggests that the estimate $\hat{\omega}^{(k)}$ at each iteration k of the algorithm may used to form a complete data set from which to estimate the coefficients φ of the motion model and ϕ of the obscuring noise. The actual estimates being obtained at the end of each iteration using Maximum Likelihood (ML) estimates of the form:

$$\overline{\varphi}^{(k)} = \arg\{\max_{\varphi} \log\log P(\hat{\omega}^{(k)} / \varphi)\} \quad (11)$$

Although in most cases this approach has been found to converge satisfactorily, convergence to a global optimum can not be guaranteed. Indeed when the algorithm is used to estimate both the coefficients and motion parameters convergence to even a local optimum can not be assumed. This problem can however be avoided, at the cost of a considerable increase in computational burden, if the Stochastic Relaxation (SR) approach described by Geman and Geman[3] is employed.

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The stochastic approach is based on the Gibbs Sampler (GS) and Simulated Annealing (SA) theorems derived by Geman and Geman. The actual algorithm uses the Gibbs Sampler to generate random samples from the Markov Random Process (MRP) described by the modified a posteriori distribution:

$$P(\omega \setminus \chi)^{\frac{1}{T(t)}} \quad (12)$$

By slowly reducing $T(t)$, a process referred to as Simulated Annealing, the trial solutions are drawn increasingly from around the modes of the distribution and hence correspond to the MAP estimates. Geman and Geman show that if the initial temperature is high enough and the cooling rate slow enough, the samples must eventually be drawn from the uniform distribution over the MAP estimates. The actual sampling is straight forward and consists of generating a random trail $\hat{\omega}_j^{(t+1)}$, then if this trial decreases the energy of the system $E > 1$ then it is accepted as the new state, otherwise it is accepted with probability E where:

$$E = \left\{ \frac{P(\omega_j^{(t+1)} / \hat{\omega}_{S\setminus j}^{(t)}, \chi)}{P(\omega_j^{(t)} / \hat{\omega}_{S\setminus j}^{(t)}, \chi)} \right\}^{\frac{1}{T(t)}}$$

5. STATISTICAL MODELS

It is now necessary to specify the statistical models which describe the observation and motion models. In the simulation of his algorithm Cobra employs four fourth order Auto Regressive models to describe the platforms motion:

$$\Delta\theta_{f(n)} = \sum_{p=1}^{P} a_{\theta\theta(p)} \Delta\theta_{f(n-p)} + w_{\theta(n)} \quad (13)$$

Here $\{w_{\theta(n)} : n \in N\}$ is a sequence of N Independent Identically Distributed (IID) Gaussian random variables. The required conditional PDFs are therefore seen to have the form:

$$P(\Delta\theta_{f(j)} / \Delta\theta_{f(k)} : k \in \partial_j) = \frac{1}{\sqrt{2\pi\eta_\theta^2}} \exp\left\{ -\frac{1}{2\eta_\theta^2} \left(\Delta\theta_{f(j)} - \sum_{p=1}^{P} h_{\theta\theta(p)} (\Delta\theta_{f(n+p)} + \Delta\theta_{f(n-p)}) \right)^2 \right\}$$

(14)

$$\eta_\theta^2 = \sigma_\theta^2 / \sum_{p=0}^{P} a_{\theta\theta(p)}^2, \quad h_{\theta\theta(n)} = -\sum_{p=0}^{P} a_{\theta\theta(p)} a_{\theta\theta(p+n)} / \sum_{p=0}^{P} a_{\theta\theta(p)}^2, \quad a_{\theta\theta(0)} = -1 \quad (15)$$

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The ML parameter estimates for such a model can be determined straightforwardly and are found to be:

$$\hat{r}_{\theta\theta(n)} = \sum_{p=1}^{P} a_{\theta\theta(p)}\hat{r}_{\theta\theta(n-p)}, \sigma_{\theta}^{2} = \hat{r}_{\theta\theta(0)} - \sum_{p=1}^{P} a_{\theta\theta(p)}\hat{r}_{\theta\theta(n-p)}, \hat{r}_{\theta\theta(p)} = \frac{1}{N-P}\sum_{n=p+1}^{N}\Delta\theta_{(f)n}\Delta\theta_{(f)n-p} \quad (16)$$

These are the so called Yule - Walker equations. It will be assumed here that the measurement error has an approximately Gaussian form. This then gives for the long and cross track measurement error models:

$$\Delta\hat{x}_{s(m,n)} = \Delta x_{f(n)} + z\Delta z_{f(n)} / l_m = \Delta x_{s(m,n)} + v_{x(m,n)} \quad (17)$$

$$P(\hat{x}_{s(m,n)} / \Delta x_{f(n)}, \Delta z_{f(n)}) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{ -\frac{1}{2\sigma_x^2}(\Delta\hat{x}_{s(m,n)} - \Delta x_{s(m,n)})^2 \right\} \quad (18)$$

$$\hat{y}_{s(m,n)} = \left| \sum_{q=0}^{Q_{(m,n)}-1} (\Delta y + z\Delta\phi_{f(n+q)} + l_{(m)}\Delta\theta_{f(n+q)}) \right| = y_{T(m)} + v_{y(m,n)} \quad (19)$$

$$P(\hat{y}_{s(m,n)} / \Delta\phi_{f(n+q)}, \Delta\theta_{f(n+q)} : q \in Q_{(m,n)}) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left\{ -\frac{1}{2\sigma_y^2}(\hat{y}_{s(m,n)} - y_{T(m)})^2 \right\} \quad (20)$$

Here $\{v_{x(m,n)} : m \in M, n \in N\}$ and $\{v_{y(m,n)} : m \in M, n \in N\}$ are the cross and long track measurement errors respectively. The errors here being assumed to have a zero mean. Although it is feasible to implement the algorithms in this form it is computationally expensive. This expense is principally due to the relatively large support of the long track measurement error model. It is useful therefore to define a simplified model which uses the approximate displacement estimate $\Delta\hat{y}_{s(m,n)} \approx y_{T(m)} / Q_{(m,n)}$ which gives the modified observation model:

$$\Delta y_{s(m,n)} = \left| \Delta y + z\Delta\phi_{f(n)} + l_{(m)}\Delta\theta_{f(n)} \right| \quad (21)$$

$$P(\Delta\hat{y}_{s(m,n)} / \Delta\phi_{f(n)}, \Delta\theta_{f(n)}) = \frac{1}{\sqrt{2\pi\sigma^2\Delta y_{s(m,n)}^2}} \exp\left\{ -\frac{1}{2\sigma^2\Delta y_{s(m,n)}^2}(\Delta\hat{y}_{s(m,n)} - \Delta y_{s(n)})^2 \right\} \quad (22)$$

Here the covariance length can be measured either forwards, backwards or as an average of both. The average providing theoretically the best estimate. Although this estimate is an unbiased estimate only when the scan displacement is constant it was found in the simulations used here to produce an adequate measure. The ML estimate of the noise coefficient for this model is seen to be:

$$\overline{\sigma}^2 = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}(\Delta\hat{y}_{s(m,n)} - \Delta y_{s(m,n)})^2 / \Delta y^2_{s(m,n)} \qquad (23)$$

6. RESULTS

In order to evaluate the performance of these algorithms the simulation procedure described by Cobra was used. That is beginning with an undistorted sonar image an artificially distorted image was generated by re-scanning it in such a way as to produce an image equivalent to the one which would be generated by an unstable platform. The actual platform motion parameters were generated using the motion model adopted Cobra in his simulation. A fragment of the undistorted image is shown in Figure 3 together with the long track and cross track distorted fragments.

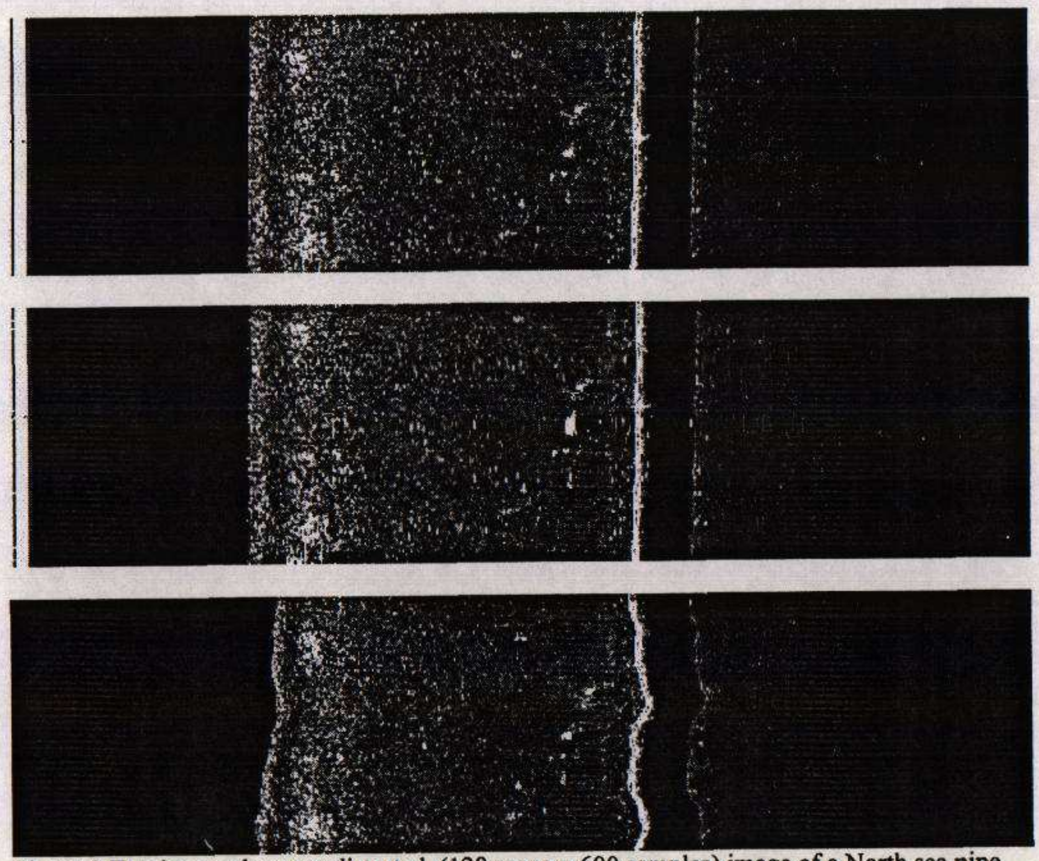

Figure 3 Top image shows undistorted (128 scans x 600 samples) image of a North sea pipe line and is followed by the rotationaly and translationaly distorted fragments.

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The displacement estimates were then made using the procedure described earlier with W = 10 (pixels), L = 0 and $\rho_T = 0.9$ for the long track displacement and W = 20, L = 2 for the cross track. A total of 60 measurements were made evenly distributed on the port and starboard sides with the first window centre at 150 and the rest at evenly spaced intervals of 11.

It was found however that the best results scan pattern estimates could be achieved if only the first 10 measurements on the port and starboard channels were actually used in the computation. The value of the correlation length was estimated by scaling the displacement estimates at each cross track range to ensure that the sum at each range is equal to the mean path travelled by the platform. This initial estimate however takes no account of back scanning and thus usually must be modified as the true orientation of the displacements becomes clear during computation. This was however not necessary in this case because of the restricted range of measurements used in the computation.

The actual maximisation was carried out by quantising the parameter ranges into 101 values and evaluating the likelihood function at each of these values for each parameter in turn. The initial parameter estimates being taken as the ML estimates which were also computed using the ICM algorithm. The ML estimate being computed as above using 10 iterations the results of which were then used as the initial state of the MAP algorithm which used a further 20 iterations. The final coefficient estimates are shown in Table 1 together with their true values. The cumulative parameter estimates are plotted in the following figures where the cumulative estimates are defined as:

$$\theta_{f(n)} = \sum_{j=1}^{n} \Delta\theta_{f(j)}, \ldots, z_{f(n)} = \sum_{j=1}^{n} \Delta z_{f(j)} \quad (24)$$

	$a_{(1)}$ (3.047)	$a_{(2)}$ (-3.600)	$a_{(3)}$ (1.950)	$a_{(4)}$ (-0.410)	σ^2
Yaw θ	3.013	-3.714	2.273	-0.593	(1e-8)2.244e-8
Pitch ϕ	2.937	-3.562	2.200	-0.579	(1e-8)2.040e-8
Horizontal X	2.110	-1.205	-0.174	0.243	(2e-6)4.508e-6
Vertical Z	2.147	-1.593	0.386	0.027	(5e-7)4.715e-6

Table 1 Motion model coefficient estimates the true values are shown in brackets.

7. DISCUSSION

As can be seen the ICM algorithm provides a reasonable estimate of both the rotational and transitional motion parameters. It is apparent however that the statistical parameter estimates are considerably better for the rotational model. The relatively poor performance in identifying the transitional coefficients is probably a result of the image formation geometry which tends to exaggerate the rotational motion thus making it easier to measure.

It is noted that there is not the clear distinction in the quality of the pitch and yaw angle estimates apparent in Cobra's work. This difference is due to the restricted range of long track displacement measurements which are used in the estimation. Using more measurements tends to improve the yaw estimates at the expense of the pitch estimates. The reason for this being that the effects of distortion due to pitching are swamped at long ranges because of the amplification in the effects of yawing as the

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range is increased. The relatively poor quality of the long track displacement measurements at long ranges is probably due to the way in which the correlation length was estimated. That is at long ranges where the effects of yawing are most severe the large measurement errors on scans with a wide spacing will tend to contaminate the rest of the data at that range.

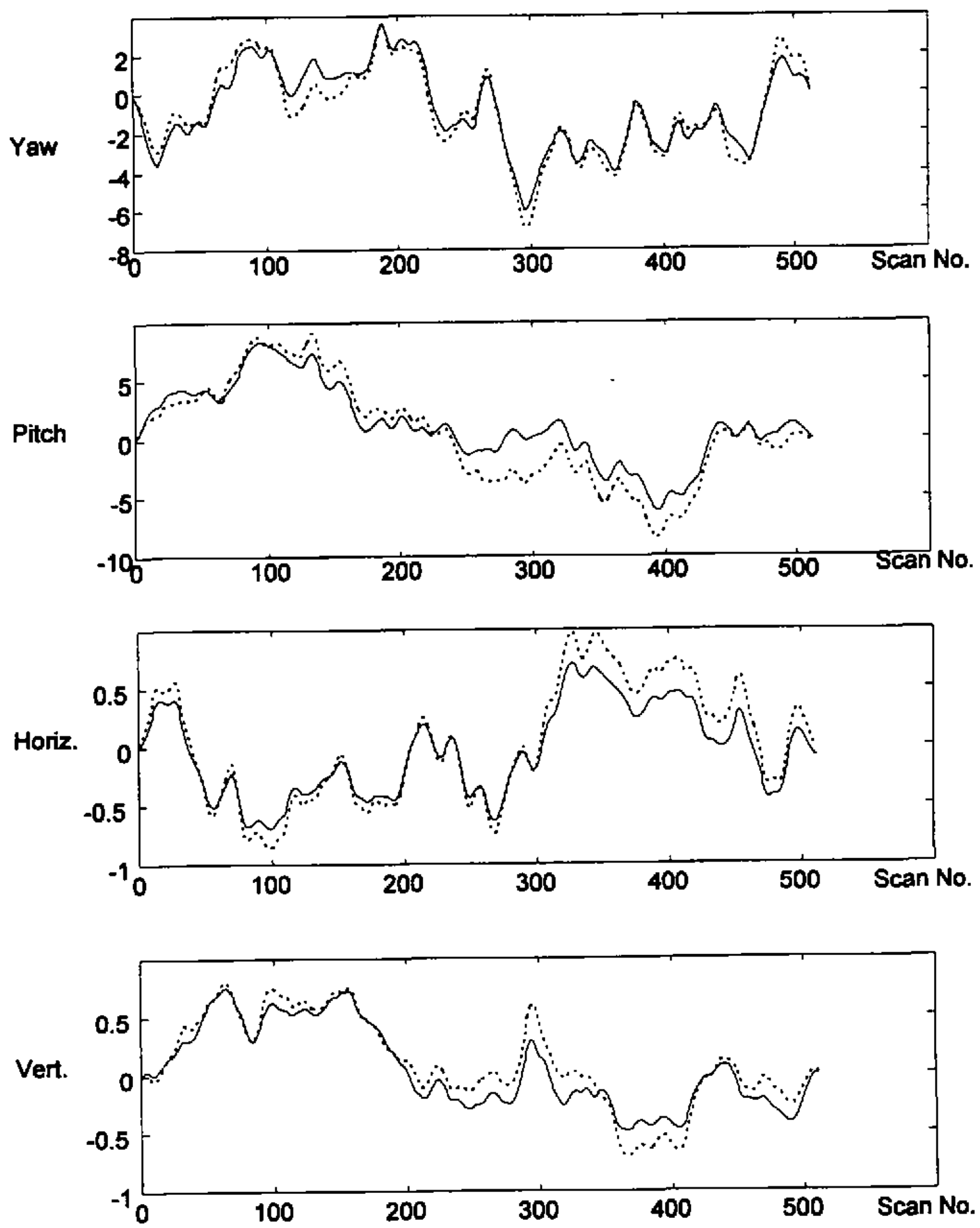

Figure 4 Cumulative ICM algorithm estimates from the top, yaw and pitch estimates in degrees followed by horizontal and vertical motion estimates in meters. Broken lines show actual values and solid lines show estimated values.

8. ACKNOWLEDGEMENTS

The authors with to express their thanks to Geoteam, UK for the data used in this work.

9. REFERENCES

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