SIDE SCAN SONAR IMAGE RESTORATION USING SIMULATED ANNEALING AND ITERATIVE CONDITIONAL MODES

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1. INTRODUCTION

Side scan sonar belongs to the class of coherent image formation systems. That is the intensity of pixels in the image is proportional to the envelope of a reflected signal rather than the reflection strength of the scene. This envelope can vary radically even when the scene has a constant reflection strength. This variation in turn leads to the sonar image having a speckled appearance. For fully diffuse reflection this speckle[1,2] is unrelated to the underlying scene structure and therefore can be considered a noise process.

This is however not the only effect which tends to mask the reflection strength information. Most sonar images are also affected by blur. Blur is caused primarily by the relatively large beam widths of typical unfocused side scan sonar systems. The large beam widths result in not only signals from the desired broad side direction but also from the area encompassed by the beam pattern contributing to the echo for a given pixel.

In this paper the problem of recovering reflection strength information from a given degraded sonar image is considered. In their work Mason[3,4] et. al. describe an approach to removing these degradation's from the sonar image. The approach being based on first using a median type filter to suppress speckle and other noise then a constrained iterative filter to sharpen the image. In the algorithm described here a more formal approach is used in that the problem is approached as one of statistical optimisation. Specifically of Maximum A Posteriori (MAP) estimation.

In MAP estimation the desired restored image is defined as the maximum a posteriori estimate of the scene reflection strength given the observed image. This estimate is however not usually obtainable in closed form. In this paper therefore two indirect approaches to the problem are considered. These being the computationally efficient Iterative Conditional Modes (ICM) approach described by Bessag[5] and the globally optimal Stochastic Relaxation (SR) approach by Geman and Geman[6].

2. IMAGE FORMATION MODEL

If it is assumed that the sea bed is diffusely reflecting then the probability of observing a pixel value g_j where $j = (m,n)$ given the sea bed reflection strength f_{v_j} can in most cases be approximated using either a Rayleigh[7] or Normal[8] PDF:

$$P(g_j / f_{v_j}) = \frac{g_j}{\sigma_j^2}\exp\left\{-\frac{g_j^2}{2\sigma_j^2}\right\}, \quad \sigma_j^2 \propto \sum_{k \in v_j} b_{j-k} f_k^2 \quad (1)$$

$$P(g_j / f_{v_j}) = \frac{1}{\sqrt{2\pi\sigma_j^2}}\exp\left\{-\frac{(g_j - \mu_j)^2}{2\sigma_j^2}\right\}, \quad \sigma_j^2 \propto \sum_{k \in v_j} b_{j-k} f_k^2, \quad \mu_j = 2\sqrt{(1+B)}\sigma_j \quad (2)$$

Here b describes the beam pattern of the sonar and B where $B > 0$ the ratio of pre - to post - detection filter bandwidths. The choice of model depending on the degree of bandwidth restriction. Generally for $B > 0.5$ the Normal model provides the better approximation. These models are however only valid for a perfectly diffuse sea bed. For some beds other distributions may be more appropriate. Of particular note in his respect are the K - Distributions of Jakeman[9,10] et .al.

3. MAP FORMULATION

It is now desired to estimate the reflection strength $f = \{f_j : j \in S\}$ given the observed pixel values $g = \{g_j : j \in S\}$ where $S = \{(m,n) : m \in 1 \ldots M, n \in 1 \ldots N\}$ is the set of pixel locations. Like many other restoration tasks this problem is generally ill conditioned and under constrained. It is therefore necessary to augment the available observation data with some prior assumptions about the nature of the desired restoration. The approach to the problem described here incorporates this information using Bayes theorem:

$$P(f \backslash g) \propto P(g \backslash f) P(f) \quad (3)$$

The MAP estimate being defined as the estimate which maximises the a posteriori density function $P(f \backslash g)$. In sonar image restoration however this function is generally rather involved. The difficulty is caused principally by the correlated nature of the speckle process. The correlation is however generally quite small and it is useful to define an alternative pseudo MAP formulation of the form:

$$P(f \backslash g) \propto Q(g \backslash f) P(f) \quad (4)$$

$$Q(g \backslash f) = \prod_{j \in S} P(g_j \backslash f_{v_j}) \quad (5)$$

This function effectively ignores the correlation in the speckle process. The choice of prior model $P(f)$ is not restrictive, however for the problem here a number of desirable properties can be defined. First to minimise the effects of noise it should encourage smoothness in the solution. Secondly to minimise the computational cost it should have a limited support. That is the conditional distribution should be Markovian $P(f_j \backslash f_{S \backslash j}) = P(f_j \backslash f_{\partial_j})$. Here $S \backslash j$ is the set of all locations excluding pixel $j = (m,n)$ and ∂_j is the set of neighbours of pixel j where $j \notin \partial_j$, $j \in \partial_k \Leftrightarrow k \in \partial_j$.

Here we will assume that the model is a first order model for which ∂_j contains the pixels immediately above and below and to the right and left of pixel j. The simplest such model is the Gauss Markov Random Field (GMRF) which is defined by conditional distributions of the form:

$$P(f_j \setminus f_{S\setminus j}) = K\exp\left\{\frac{-1}{2\sigma_f^2}(f_j - \mu_f - \sum_{k\in\partial_j}\beta_{j,k}(f_k - \mu_f))^2\right\} \quad (6)$$

Here K is a normalising constant and for the field to be a valid Markov Random Field (MRF) $\beta_{j,k} = \beta_{k,j}$. In addition as with all such process the covariance matrix must be positive and definite. This type of model however has the disadvantage that it takes no account of the presence of sharp edges in the scene. This omission tends to produce blurred edges in the restored image. To avoid this problem the model can be augmented with a bond process which allows the connection between two pixels to be broken when an edge is present. One such model is the Compound Gauss Markov Random Field[11,12] (CGMRF) which may be expressed in the form[13]:

$$P(f_j \setminus f_{S\setminus j}, l) = K\exp\left\{\frac{-1}{2\sigma_f^2}\sum_{k\in\partial_j}\beta_{j,k}(1-l_{j,k})(f_j - f_k)^2 + (1 - \sum_{k\in\partial_j}\beta_{j,k})(f_j - \mu_f)^2\right\} \quad (7)$$

Here Z is the set of bond locations and $l_{j,k} = l_{k,j} = 1,0$ describes the bond between pixels j and k. It is observed that setting a bond to 1 indicating the presence of an edge removes any correlation between the pixels on either side of the edge. The actual bond process can itself be described by MRFs of varying degrees of complexity. Here the simplest possible process will be used for which:

$$P(l_{j,k} \setminus l_{Z\setminus j,k}) = K\exp\left\{-\frac{\alpha\beta_{j,k}}{2\sigma_f^2}l_{j,k}\right\} \quad (8)$$

4. IMAGE RESTORATION

It is now sought to maximise the probability $P(f,l \setminus g)$ with respect to f and l. This is generally a complex problem which can not be solved in closed form. The need to obtain the closed form solution can however be avoided through the use of either the ICM or SR algorithms. The ICM algorithm is based on the observation that the joint PDF can be factored into terms of the form $P(f,l \setminus g) = P(f_j \setminus g, f_{S\setminus j}, l)P(f_{S\setminus j}, l \setminus g)$. Any increase in $P(f_j \setminus g, f_{S\setminus j}, l)$ therefore results in an increase in the corresponding MAP function. By iteratively maximising $P(f_j \setminus g, f_{S\setminus j}, l)$ the algorithm must approach a maximum of this function. The ICM algorithm therefore consists of iteratively maximising:

$$P(f_j \setminus g, f_{S\setminus j}, l) \propto P(f_j \setminus f_{\partial_j}, l) \prod_{k \in \rho_j} P(g_k \setminus f_{v_k}) \quad (9)$$

$$P(l_{j,k} \setminus g, f, l_{Z\setminus j,k}) \propto P(l_{j,k} \setminus f_j, f_k) \quad (10)$$

Here $\rho_j = \{k : j \in v_k\}$ where v_k is the set of locations which contribute to the value of pixel g_k. This algorithm has however the disadvantage of all such algorithms in that it has a tendency to be susceptible to local minima. To avoid this problem the SR algorithm of Geman and Geman can be used instead. The SR algorithm is based on two theorems. The first theorem describes a method, termed the Gibbs Sampler (GS), for sampling (generating) MRFs. Its use in image processing is based on the observation that the a posteriori probability $P(f, l \setminus g)$ itself describes an MRF and therefore can be sampled using the GS. The second theorem describes a method, termed Simulated Annealing (SA), for investigating the base states of the system. To use SA to obtain the MAP solution a modified density function is defined:

$$P(f, l \setminus g)^{\frac{1}{T(t)}} \quad (11)$$

Here $T(t)$ is the temperature of the system. By reducing $T(t)$ slowly the samples generated using the GS will be drawn increasingly from the regions close to the modes of the a posteriori density. Geman and Geman show that, given certain conditions on the initial temperature and cooling schedule, the samples must eventually be drawn entirely from the uniform distribution of MAP solutions irrespective of the initial image estimate. The actual SR algorithm implementation is straight forward. First a random trial solution $f_j^{(t+1)}$ is generated and the energy change associated with it computed using:

$$E = \left\{ \frac{P(f_j^{(t+1)} \setminus g, f_{S\setminus j}, l)}{P(f_j^{(t)} \setminus g, f_{S\setminus j}, l)} \right\}^{\frac{1}{T(t)}} \quad (12)$$

Then if the new solution decreases the energy in the system, that is $E > 1$ the new solution is accepted otherwise it is accepted with probability E. The same steps being used for the bond process. This procedure is repeated at each pixel in the image. Then the temperature is reduced slightly and the process repeated. The whole cycle being carried out until no more changes are apparent.

5. RESULTS

The image in Figure 1 was obtained with a Klein 100kHz system and shows a rocky outcrop on the sea bed. The highlighted region shows the area used in processing. This area is shown at higher magnification in Figure 4. Examination of the pixel histograms shows that the actual distribution of the speckle in this image can be closely approximated using the Normal PDF model with $B \approx 0.7$. Before implementing the algorithms it is necessary to determine initial estimates of the edge locations. This estimate was obtained using a simple threshold operator on a heavily smoothed version of the original image. The initial image estimate was obtained using the same method from a lightly smoothed version of the original image.

These estimates were then used to initialise both the ICM and SR algorithms. The algorithms being implemented with $\sigma_f^2 = 1$, $\beta_{j,k} = \beta = 0.25$ and $\alpha = 20$. The cooling schedule in the SR algorithm had a logarithmic form with $T(t) = C / \log(1+t)$ where $C = 1$. The SR algorithm having 300 iterations and the ICM algorithm 10 iterations. The beam pattern was modelled using a Gaussian kernel with $\sigma_b^2 = 1.5$. The results of the processing are shown in the following figures.

Figure 1 Original 512 x 512 100kHz sonar image.

Figure 4 128 x 128 Fragment used in processing.

Figure 2 Processed image obtained using SR.

Figure 3 Processed image obtained using ICM.

6. DISCUSSION

As can be seen both the ICM and SR algorithms reduce considerably the speckle noise level in the images. Indeed in this case there appears to be little advantage in using the SR algorithm which is considerably more costly to compute. The principle reason for the efficacy of the ICM algorithm here is the choice of initial image and edge estimates. In this case the method used to obtain the initial estimates was chosen to steer the ICM algorithm towards the correct solution. With less carefully selected estimates the SR algorithm will generally produce better results.

It is worth noting that although theoretically it is possible to obtain the true MAP estimate using SR it is seldom practical. That is the high initial temperature and slow cooling schedule required to guarantee convergence make the algorithm to slow to be practicable. In addition it may not be the case that the true MAP estimate is preferable to a more subjectively pleasing sub - optimal solution.

The beam pattern estimate used here was chosen rather arbitrarily. It is doubtful however that given the heavily degraded nature of the images whether this significantly affects the result. The degree of sharpening which can actually be achieved is to some extent limited by the implicitly high level of degradation. One area which may provide some improvement in the performance of the ICM algorithm in this respect is the choice of initial edge estimate. It is possible that through a better choice of initial estimate a higher performance in terms of the degree of sharpening is obtainable.

7. REFERENCES

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