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STRUCTURAL COUPLING THROUGH THE FRAME OF A DOUBLE WALL

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INTRODUCTION

A common form of construction for an internal partition is to have a frame of timber or profile steel channel with one or more layers of plasterboard on each side as can be seen in Fig.1. The frame provides structural coupling between the plasterboard and so provides a transmission mechanism between the two rooms separated by the partition. This structural coupling is often best modelled as a line connection between the two sheets of plasterboard in which case the standard method of calculating the transmission is to use a wave model in which a (bending) wave is assumed incident on the joint and the amplitude of the waves leaving the joint are computed.[1]

There are many variations that can be made on the basic model and which can be used for cases where there is a line connection between plates some of which are shown in Fig.1.

A number of studies of structure borne sound transmission for this type of wall have been carried out [1,2,3,4,5,6]. In addition to the basic model of parallel plates connected at a line there have been studies of the effect of the mass of the coupling element (which may or may not be located symmetrically at the joint)[1]. One of the limitations of this type of model is that the two plates are assumed to be rigidly connected. This is reasonable if the coupling element is thin such as in a cavity wall where the cavity depth (typically 50 mm) is small compared to the wall thickness (typically over 100 mm). However, in a lightweight partition the cavity depth may be 10 times the plate thickness.

One method of including the additional effects of widely separated plates is to use the model of Battacharya *et al* [4] however this is unnecessarily complicated and includes many terms that are not necessary for accurate prediction of the performance.

The purpose of this paper is to examine transmission between the plates specifically looking at factors affected by the offset caused by the separating beam.

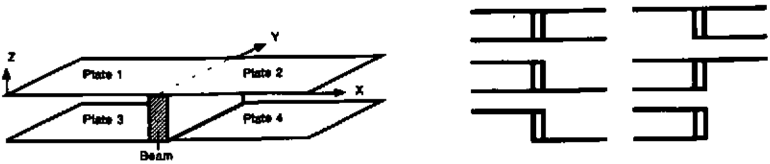

Fig.1 Standard 'H' structure and alternative constructions.

THEORY

The basic theory assumes that there are four semi-infinite plates connected by a line composed of an infinite beam with mass, inertia, stiffness etc as shown in Fig.1.

Using the co-ordinate system shown in Fig.2 the conditions that exist at the boundary can be written in terms of the displacement (ξ,ζ,η), slope (ϕ), the moments M and the forces (Fx,Fy,Fz).

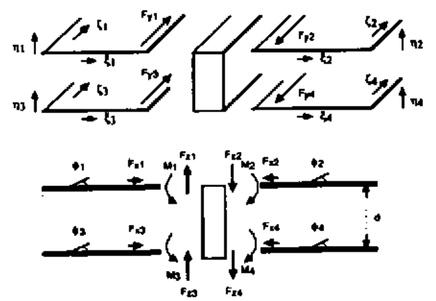

Fig 2 Coordinate system used for the theoretical model.

The continuity of displacement requirement at the joint requires that at the joint,

$$\eta_1=\eta_2,\ \eta_1=\eta_3,\ \eta_1=\eta_4,\ \xi_1=\xi_2,\ \xi_3=\xi_1+\left[\frac{\phi_1+\phi_3}{2}\right]d,\ \xi_3=\xi_4,\ \zeta_1=\zeta_2,\ \zeta_1=\zeta_3,\ \zeta_1=\zeta_4$$

where d is the depth of the beam. The term involving the slopes of plates 1 and 3 adjusts the in-plane displacement of plate 3 to account for the rotation of the beam.

The slope of plate 1 is equal to the slope of plate 2 but if the beam is deep enough then it will not be equal to the slope of plates 3 and 4. The difference between the slope of plate 1 and 3 is equal to the moments applied at the top (or bottom) of the plate divided by the bending stiffness D_y. Thus the slopes are given by,

$$\phi_1 = \phi_2,\ \phi_3 = \phi_1 + \frac{M_1 - M_2}{D_y},\ \phi_3 = \phi_4,$$

where D_y is given by ($E_r b^3/12d$) where b is the width of the beam. Due to the orthotropic nature of timber the modulus of elasticity can vary by a factor of over 20 between the axial (E_L) and radial (E_r) direction measured relative to the grain. In this equation it is the softer E_r that is used.

The sum of the bending moments about the y-axis acting on the beam must equal the beams resistance to rotation (its inertia) and twisting along its length (its torsional stiffness) to give [1]

$$M_1 - M_2 + M_3 - M_4 - F_{x1}\frac{d}{2} + F_{x2}\frac{d}{2} + F_{x3}\frac{d}{2} - F_{x4}\frac{d}{2} + T\frac{\partial^3}{\partial y^3}\left[\frac{\phi_1+\phi_3}{2}\right] = -\omega^2\Theta\left[\frac{\phi_1+\phi_3}{2}\right]$$

where $(\phi_1 + \phi_3)/2$ is the average rotation of the beam. T is the torsional stiffness of the beam and Θ is the mass moment of inertia per unit length of the beam.

The sum of the forces in x,y and z-direction acting on the beam must equal the force resisting linear displacement (its inertia) and the bending or compression along its length (its bending stiffness or Youngs modulus) to give

$$F_{z1} - F_{z2} + F_{z3} - F_{z4} - B_z\frac{\partial^4\eta_1}{\partial y^4} = -\omega^2\rho_l\,\eta_1$$

$$F_{x1} - F_{x2} + F_{x3} - F_{x4} - B_x\frac{\partial^4}{\partial y^4}\left[\xi_1 + \left(\frac{(\phi_1+\phi_3)}{2}\right)\frac{d}{2}\right] = -\omega^2\rho_l\left[\xi_1 + \left(\frac{(\phi_1+\phi_3)}{2}\right)\frac{d}{2}\right]$$

$$F_{y1} - F_{y2} + F_{y3} - F_{y4} - E\frac{\partial^4\zeta_1}{\partial y^4} = -\omega^2\rho_l\,\zeta_1$$

where properties of the beam include, ρ_l the density per unit length, B_z bending stiffness ($E_L bd^3/12$), B_x bending stiffness ($E_L db^3/12$) and E_L is the axial modulus of elasticity.

The equations giving angular displacement ϕ, moments M and respective forces $F_{x,y,z}$ of the plates in terms of the displacements are given in reference [1]. Once the 16 equations have been determined they can be solved numerically to give the amplitude of the waves leaving the joint and hence the transmission coefficients. These can then be used in a standard SEA model [6] to give the plate response for different forms of excitation.

RESULTS

In order to test the theories given above experiments were carried out on a variety of structures like those shown in Fig.1. The results given in this paper are for an 'H' joint but other joints were also tested.

The structures were composed of two sheets of 13 mm plasterboard (1.2 x 2.4 m) connected along a line of screws at 60 mm centres, to a timber softwood beam (1.2 x 0.05 x depth m) of varying depths 200, 100 and 50 mm or to a 70 mm steel channel (1.2 x 0.032 x 0.070 m). The screws formed a line connection dividing the two plates on either side of the beam into four plates (1.2 x 1.2 m). The frequency where the connection could be modelled as a point was around 3150 Hz, where the first half wavelength fitted between the screws [6].

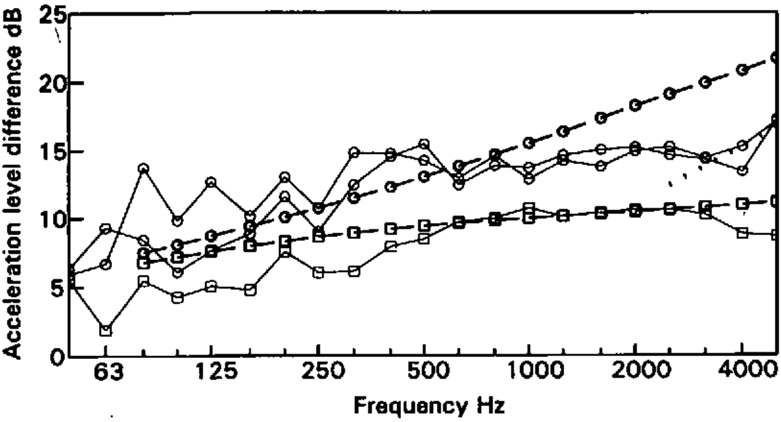

Fig 3 Measured and predicted results for a joint with a 100mm timber beam. ———, measured; - - - -, predicted; □, plate 1 to 2; ○, plate 1 to 3 and 4.

The results for a 100 mm depth timber beam are given in Fig.3. The measured and predicted transmission from plate 1 to 4 is much less than for transmission from plate 1 to 2 (the level difference is higher) and this is a consequence of the rotation of the top plates (1 and 2) being different from the lower plates (3 and 4). This difference can not be achieved if the joint is rigid.

At high frequencies (above about 3150 Hz) the spacing between the screws is such that the joint is better modelled as a series of point connections and so the theory for point connections is also shown. It can be see that there is a transition region from about 1000 Hz where the measured data crosses from one theory to the other.

A feature of the timber that is commonly used to form the frame is that the elastic modulus in the axial direction (usually the one quoted in textbooks) is much higher than the elastic modulus across the grain. This difference can be a factor of over 20 and means that the timber is much softer when being bent across the grain. This factor must be taken into account and the model is sensitive to changes in this value (which affects D_y in the equations).

The transmission to plates 3 and 4 is very similar in both measured and predicted results.

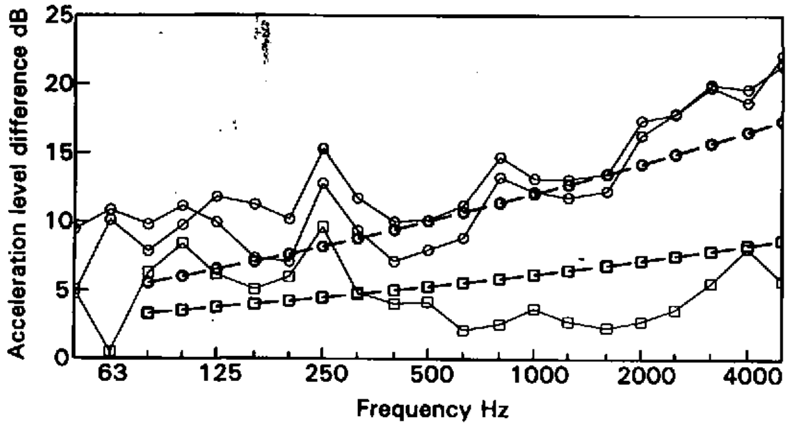

Fig.4 Measured and predicted results for a joint with a 70mm steel channel. ———, measured; - - - -, predicted; □, plate 1 to 2; ○, plate 1 to 3 and 4.

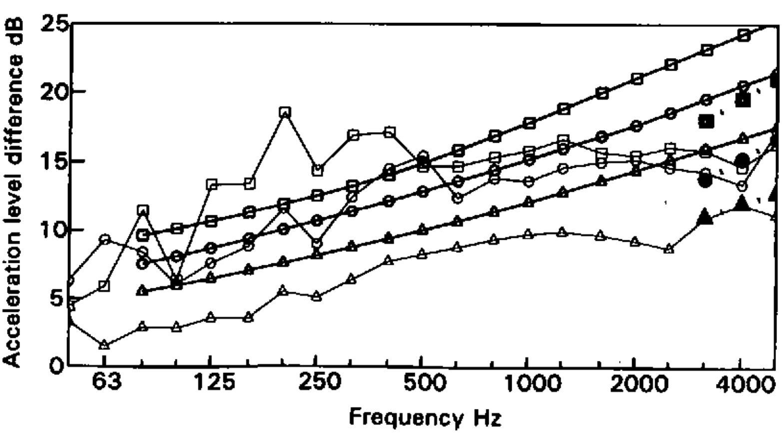

Fig 5 Measured and predicted acceleration level difference from plate 1 to plate 3 in a standard 'H'joint with varying beam depth. ———, measured; ———, predicted; □, 200mm;○, 100mm;△, 50mm.

Similar results can be seen in Fig.4 for a test structure made with a 70 mm steel channel. Again the agreement between the measurements and the theory is good and in this case the model where the joint behaves as a series of point connections is higher than for a line connection.

The effect of varying the depth of the timber beam can be seen in Fig. 5 which shows both the measured and predicted results for a variety of different timber beams. As the depth (offset) doubles so the acceleration level difference increases by 3-4 dB, particularly at the low frequencies. Thus the increase in offset results in weaker coupling across the beam to plates 3 and 4.

CONCLUSIONS

The theory presented in this paper is a development of the standard theories for rigid joints that are commonly used to predict structural transmission across line connections. The models work well across the full frequency range where it is appropriate to model the joint as a line connection.

This theory has application to a variety of different types of parallel plates ranging from stud partitions and basic timber floors to any form of double skin connected by beams.

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