

## POWER OPTIMAL CONTROL OF SOUND TRANSMISSION THROUGH PLATE BY USE OF PIEZOELECTRIC ELEMENTS

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### 1. INTRODUCTION

Recently an application of the active control systems has been considered promising in reducing sound transmission efficiency through plate because of the high cost-effectiveness of the active control in low frequency range<sup>(1,2)</sup>. The authors pursue an efficient active control system of sound transmission by use of piezoelectric ceramics in plate<sup>(3)</sup>. Piezoelectric elements are attractive, because their utility is not restricted on the actuators in control system but it can be developed on the sensors<sup>(4)</sup>. We discuss an active control system of thin homogeneous elastic plate, where eigen modes of bending vibration are found to prevail the sound transmission phenomena. Efficient countermeasures to resolve the spill-over resulting from truncation error of the higher order eigen modes are essential in the system. The objective of the active sound transmission control is expressed such as; to minimize the energy transmitted through plate excited by the incidental sound wave carrying unit energy. The objective is fulfilled by introducing the LQG control system. We suppose the objective function is constituted by the energies of the frequency weighted controlled variables. The purpose of the paper is to clarify the designing method of the control system and to prove the effectiveness of the system.

### 2. MODELLING OF PLATE/CONTROL SYSTEM

#### Configuration of a model

Figure 1 shows a configuration of an experimental model. A thin plate of stainless steel is clamped at the top of rectangular duct, size of which is about 310mm × 270mm × 1mm. Five pieces of piezoelectric ceramics are bonded on the both surfaces of the plate. In the frequency range under 500Hz incident sound is assumed as plane wave, since the wave length is very large compared with the duct cross-sectional dimensions. The sound wave is emitted from a

speaker installed in the lowermost part of the duct. The piezoelectric ceramics on the upper surface are sensors, while those on the lower surface are actuators.

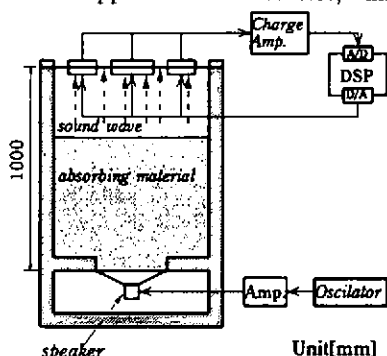


Fig.1 Experimental setup

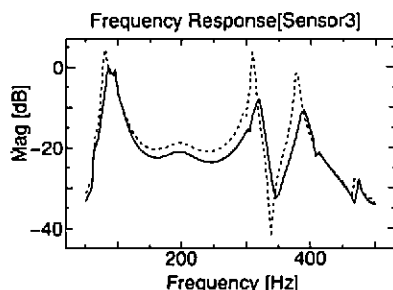


Fig.2 Vibration deflection in a piezo. sensor

### FEM dynamic model

The elastic vibration of thin rectangular plate is described by Finite Element Method (FEM) model as follows,

$$M\ddot{\eta} + D\dot{\eta} + Kq = f_s + f_c \quad (1)$$

where  $q$  is the bending deflection vector comprising  $3m$  components, which means three components  $[w, w_x, w_y]^T$  are assigned to each of  $m$  node points in the plate,  $f_s$  and  $f_c$  are the generalized force vector due to the incident sound pressure and that due to the control force, respectively.  $M$ ,  $D$  and  $K$  are inertia matrix, damping matrix and stiffness matrix, respectively. As the charge signal extracted from a piezoelectric element is composed of the slopes of bending deflection,  $w_x$  and  $w_y$ , at the edge<sup>(5)</sup>, then the output signal vector  $y$  is related with  $q$  as follows,

$$y = C_p q \quad (2)$$

On designing the control system the state variable is assumed as the truncated mode variable vector  $\eta_N$ , which is related with  $q$  by the equation,  $q = \Phi_N \eta_N$ . Since  $\Phi_N$  is the transfer matrix comprising  $N$  number of the eigen vectors, Eq.(1) is rewritten with diagonalized form in the left side such as,

$$\ddot{\eta}_N + \Lambda_N \dot{\eta}_N + \Omega_N^2 \eta_N = \Phi_N^T (f_s + f_c) \quad (3)$$

where  $\Phi_N$  is related with  $M$ ,  $K$ ,  $D$  such as,  $\Phi_N^T M \Phi_N = I_N$ ,  $\Phi_N^T K \Phi_N = \Omega_N^2$ ,  $\Phi_N^T D \Phi_N = \Lambda_N$ . When the control force vector  $f_c$  is derived by the control input signal  $u$  comprising  $k$  components,  $f_c$  is written by  $f_c = B_p u$ ,  $B_p$ ;  $3m \times k$ . A component of  $B_p$  matrix in the above equation is related with a discretized line moment on the edges of the actuator element<sup>(5)</sup>. The generalized load force vector  $f_s$  is expressed by the incident sound pressure  $p_i$  multiplied with

the load distributing matrix  $B_i$  such as,  $f_s = B_i p_i$ ,  $B_i : 3m \times 1$ . Equation (3) is transformed to the standard state equations by use of  $x = [\eta_N^T, \dot{\eta}_N^T]^T$  as the state variable, together with output equation such as,

$$\dot{x} = Ax + Bu + D_s p_i, \quad y = Cx \quad (4)$$

### Acoustic radiation power

Sound pressure field  $p(P)$  radiated from a baffled plate on a point  $P$  in semi-infinite space is expressed as follows on the assumption of far field meaning that distance  $R$  is far larger than the wave length<sup>(6)</sup>,

$$p(P; R, \theta, \phi) = j\rho_a \frac{e^{jkR}}{2\pi R} \int_{-L_x}^{L_x} dx' \int_{-L_y}^{L_y} \dot{w}(x', y') \exp[jk(\alpha x' + \beta y')] dy' \quad (5)$$

where the notation,  $\alpha = \sin \theta \cos \phi$ ,  $\beta = \sin \theta \sin \phi$ ,  $k$ ; wave number and  $\rho_a$ ; air density, are valid. As the acoustic power flow passing a unit area centered on the point  $P$  is expressed by  $I_R = \bar{p}p/2\rho_a c$ , where  $\bar{p}$  is the complex conjugate of  $p$ , then the total acoustic power  $P(\omega)$  is calculated by

$$P(\omega) = \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} I_R R^2 \sin \theta d\theta \quad (6)$$

Since  $\dot{w}$  is written by  $\dot{w} = \Phi_N \dot{\eta}_N$ , integration is executed by using FEM shape function of Hermite cubics for the expression of  $\Phi_N$  as follows.

$$P(s) = \dot{\eta}_N^H M_N(s) \dot{\eta}_N, \quad M_N(s) : N \times N \quad (7)$$

After  $M_N(s)$  is decomposed such as  $M_N(s) = G^H(s)G(s)$ ,  $P(s)$  is rewritten by

$$P(s) = z^H(s)z(s), \quad z(s) = G(s)\dot{\eta}_N \quad (8)$$

### 3. DESIGNING OF LQG CONTROLLER

When we introduce the new vector variable  $\bar{x} = [x^T, \zeta^T]^T$ , where  $\zeta$  denotes a state variable for realizing the relation  $z(s) = G(s)\dot{\eta}_N(s)$ , we get an expression of the extended plant for the new state variable  $\bar{x} = [x^T \zeta^T]^T$  as follows,

$$\begin{cases} \dot{\bar{x}} = A_{\bar{x}} \bar{x} + B_{\bar{x}} u + D_{\bar{x}} w_d \\ y = C_{\bar{x}} \bar{x} \\ z = C_z \bar{x} \end{cases} \quad w_d = p_i \quad (9)$$

The objective function  $J$  is defined by Eq.(10) representing the sound energy added by the control energy, which are frequency-weighted by  $C_z^H(\omega)C_z(\omega)$  and  $R(\omega)$ , respectively

$$J = \int_0^\infty [\bar{x}^T C_z^H C_z \bar{x} + u^T R u] dt = \int_{-\infty}^\infty [z^T(-\omega)z(\omega) + u^T(-\omega)R(\omega)u(\omega)] d\omega \quad (10)$$

The LQG controller that minimizes the above objective function is represented by the state feedback algorithm  $u = -K\hat{\bar{x}}$ , where the estimated state variable  $\hat{\bar{x}}$  is computed in real time by the Kalman filter.

#### 4. CONTROL EXPERIMENT

After LQG controllers are examined with computer simulation, the control experiment is conducted using the selected controller, where DSP is applied with a sample rate 10(kHz). The gain matrix  $R$  is determined, so that the stability of higher eigen modes deteriorated by truncation error may be recovered and the control input voltage to may be restricted under 10(V). Figure 3 shows the frequency response function curve from the sound signal for a loud speaker to the vibration deflection signal at the central piezoelectric sensor. A solid line in the figure denotes the frequency response with control, while a dashed line does the response without control. We can find significant reduction in magnitude at the eigen mode frequencies in the figure. Figure 4 shows the frequency response curve from the sound signal to the sound power that is calculated by Eq.(8) using estimated  $\hat{\eta}_N$ . Undistinguishable reduction in magnitude at the first eigen mode frequency comes from small magnitude in  $G(s)$  at the frequency. Figure 5 shows the frequency response curve from the sound signal to the sound pressure level detected by a microphone positioned at 50(cm) above the plate. The curve represents the same trend as that in Fig.3.

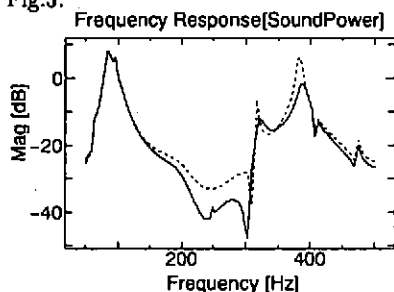


Fig.3 Sound power transmission

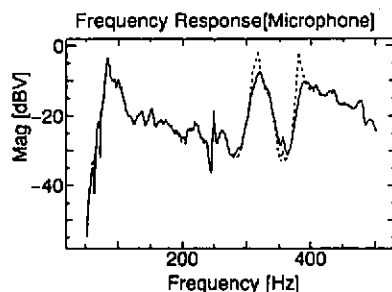


Fig.4 SPL measurement by a microphone

#### CONCLUDING REMARKS

The designing method for the power optimal control of sound transmission through plate is proposed assuming the application of piezoelectric elements as actuators and sensors. The effectiveness of the control system is proved by the experimental result presenting significant reduction in sound power mainly at the eigen mode frequencies.

#### Reference

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