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A 3-DOF SEA ELEMENT FOR MODELLING ONE-DIMENSIONAL STRUCTURES WITH VERY HIGH MODAL OVERLAPS

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1. INTRODUCTION

SEA [1] provides estimates of space and frequency averages of vibration energy in built up structures. The applications of SEA to beam and rod structures are much studied because of the engineering significance. Also, such systems are computationally simple, so they are suitable for investigating concepts in SEA, as e.g. in 11 of the quoted references in [2]. Some of the results are discussed in conjunction with Fig. 1, the details are found in Sec. 3.

The application of SEA to this rod structure is governed by four criterias. *i*, Whether there are any resonances. In the coupled rod structure the first resonance is at 30 Hz, applying SEA below this frequency is not appropriate. *ii*, Whether there are enough resonances to give significance to a frequency average. Below 300 Hz, the expected number of resonances in each element is less than one per third octave band, hence, large errors are expected. *iii*, Whether coupling is weak. Using the criteria in [2, 3], coupling is strong for frequencies below 650 Hz. If coupling is strong, SEA over-estimates coupling power on average. For structures with many elements, the implications of strong coupling are not known, but standard SEA sub-structuring technique does not apply [2]. *iv*, Whether energy density is constant in the elements. From approximately 2 kHz, because of damping, energy decays exponentially away from the excitation. In such a situation, SEA estimating the coupling power from average energy results in severe errors, as seen in Fig. 1.

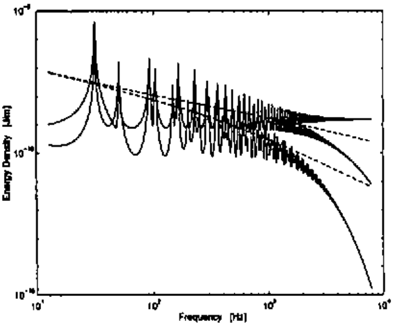

Fig. 1 Energy density in two-rod structure. —, Dynamic stiffness method: top, at excitation in rod 1; middle, far end of rod 1; bottom, far end rod 2; – –, standard SEA: top, rod 1; bottom, rod 2.

To sum up, for the investigated structure standard SEA should apply with good accuracy for frequencies in between, perhaps only, 500 Hz - 2 kHz. Hence, it is difficult to find applications for the method, particularly for larger structures where all the elements rarely meet all the criteria in the same frequency interval. The aim of the present work is to circumvent criterion *iv.* A 3 d.o.f. SEA-element is developed which could handle large damping within rod and beam elements. With this element there is no upper frequency limit for SEA, so more applications could be found for this convenient method.

2. FORMULATION OF 3-D.O.F. SEA ELEMENT

Consider a rod, excited by a harmonic force at one end and obeying an impedance boundary condition at the other end. Assuming a stationary time dependence, $e^{-i\omega t}$, the governing equations are

$$\begin{gathered} E(1-i\eta_e)\varepsilon = \sigma;\ -\rho\omega^2(1+i\eta_v)u = \partial\sigma/\partial x;\ \varepsilon = \partial u/\partial x \\ f_0 = -\sigma A,\ x=-L;\ \sigma = i\omega\rho c\zeta u,\ x=0 \\ e = e_p + e_k;\ e_p = \frac{EA}{2}\varepsilon^*\varepsilon;\ e_k = \frac{\rho A\omega^2}{2}u^* u \end{gathered} \tag{1}$$

where u is the displacement, ε strain, σ stress and ζ specific impedance. E is Young's modulus, A cross sectional area, ρ density, c sound velocity, e energy density, e_p and e_k are potential and kinetic energy densities. u^* is the complex conjugate of u. Losses are described by the visco-elastic loss factor η_e and the viscous loss factor η_v.

Energy distribution in rods. The equation for conservation of energy is $\mathrm{div}(I) = -\omega\left(\eta_e e_p + \eta_v e_k\right)$; where I is the intensity. Upon this, using equations (1), neglecting terms $O(\eta^2)$, and assuming $\eta_e = \eta_v = \eta/2$, it follows that

$$\frac{\partial e}{\partial x} = -\frac{\omega\eta}{c^2}I;\quad \frac{\partial^2 e}{\partial x^2} = \left(\frac{\omega\eta}{c}\right)^2 e \tag{2}$$

Now, the aim is to develop a 3 d.o.f. element that could be used in a larger structure, also when the modal overlap in the rod is large and hence the energy density is not constant. To that end, the rod is divided into 3 parts and relations between the total energy in these parts and the input power are sought. With reference to [2] and [4], the energy conservation between the three parts of the rod is tentatively written

$$\frac{1}{\pi}\begin{bmatrix} D_1 + C_{12} + C_{13} & -C_{12} & -C_{13} \\ -C_{12} & D_2 + C_{12} + C_{23} & -C_{23} \\ -C_{13} & -C_{23} & D_3 + C_{23} + C_{13} + Q \end{bmatrix}\begin{bmatrix} Em_1 \\ Em_2 \\ Em_3 \end{bmatrix} = \begin{bmatrix} P_{in} \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

where P_{in} is the input power, C_{ij} are coupling coefficients, Q models coupling losses at the end and where the damping parameters D are

$$D_j = \omega\eta L_j / c \tag{4}$$

The d.o.f. for the proposed SEA element are chosen to be the modal energies at the ends of the element Em_1 and Em_3 and the averaged modal energy in the element, Em_2. Accordingly: $L_2 = L$, $D_2 = D$ and $L_1, L_3, D_1, D_3 \to 0$. Then, the modal energies are found from the solutions to equations (2):

$$\begin{gathered} Em_j = \frac{\pi c}{L_j}\int e(x)\,\mathrm{d}x\,; \quad Em_1 = \frac{\pi P_{in}}{\tanh(D)}\left(1 - \frac{\kappa}{\sinh(D)\cosh(D)}\right) \\ Em_2 = \frac{\pi P_{in}}{D}\left(1 - \frac{\kappa}{\sinh(D)}\right); \quad Em_3 = \frac{\pi P_{in}}{\sinh(D)}\left(1 - \frac{\kappa}{\tanh(D)}\right) \\ \kappa = \frac{\tanh(a)}{1 + \tanh(a)/\tanh(D)}\,; \quad a = \operatorname{atanh}\left(\frac{2\,\mathrm{Re}(\zeta)}{1 + |\zeta|^2}\right) \end{gathered} \tag{5}$$

Inserting these expressions in equation (3), it is seen that this equation eventually is satisfied and that the coefficients are

$$\begin{gathered} C_{12} = C_{23} = \frac{\sinh(D)}{1 + \cosh(D) - 2\sinh(D)/D} \\ C_{13} = -C_{12}\,\frac{\sinh(D)/D - 1}{\cosh(D) - 1}\,; \quad Q = \tanh(a) \end{gathered} \tag{6}$$

The coupling coefficients C_{12}, C_{13} do not depend on the boundary conditions. This means that the element formulation has (using FE terminology) compact support, that is, it is made without any need for considering connected elements. This is, in fact, essential for computer implementation; without compact support, complexity would increase dramatically.

Element formulation. The 3 d.o.f. element is governed by the matrix

$$\mathbf{A} = \frac{1}{\pi}\begin{bmatrix} C_{12} + C_{13} & -C_{12} & -C_{13} \\ -C_{12} & D + 2C_{12} & -C_{12} \\ -C_{13} & -C_{12} & C_{12} + C_{13} \end{bmatrix} \tag{7}$$

where C_{12}, C_{13} are defined in (6) and D in (4). The element d.o.f. are $[Em_1 \quad Em_2 \quad Em_3]^T$ where Em_1 and Em_3 are the modal energies at the ends and Em_2 is the average modal energy in the element. They are, in equation (5), related to the energy density e. When they are independent variables in an SEA model, the matrix **A** can be directly assembled.

Bending Beam Formulation

For a bending beam the analysis in the previous section does not apply directly, since, in a standing wavefield the total energy varies over each half wavelength. Consequently the change of energy density could not be governed by equation (2) where the length scale is a factor $1/\eta$ larger. For a wave component, however, the relations (2) apply. That is, assuming

$$w(x) = w_0\, e^{ikx}\,; \quad k^4 = \left[\frac{\rho A(1-i\eta_v)\omega^2}{EI_y(1+i\eta_e)}\right] \tag{8}$$

then, neglecting terms $O(\eta^2)$ the equations (2) are recovered if the sound velocity is substituted for the beam group velocity, c_g.

One of the standard assumption in SEA is that different wave components are uncorrelated and may be energetically superimposed. Upon this assumption, the subdivision of a bending beam element as 3 strongly coupled SEA elements is equally described by equation (7) when the damping parameter D, modal energy E_m, modal density n and group velocity c_g are

$$D = \frac{\omega\eta L}{c_g};\quad Em_i = \frac{1}{n_i}\int e\,\mathrm{d}x\,;\quad n_i = \frac{L_i}{\pi c_g};\quad c_g = \frac{\partial\omega}{\partial k} \tag{9}$$

In this case, the modal energy describes only the energy in propagating waves and these are assumed uncorrelated. It is believed that, perhaps, this formulation applies for any wave type in one-axial uniform waveguides.

3 CALCULATION EXAMPLES

Rod structure. Consider a structure with 2 rods connected with a spring. The structure is excited at one end by a point force and the SEA estimate of the input power is $\Pi = \rho c A|f_o|^2$. The travelling wave estimate of the coupling loss factor is, as in [2], given by

$$\omega n \eta_c = C/\pi\,;\; C = \frac{\tau}{2-\tau} \tag{10}$$

where τ is the transmission factor. Repeating the calculations in [2], this time solving for the ensemble averaged modal energy at the two sides of the coupling, the ensemble average coupling loss factor is given by

$$\omega n \eta_c = \frac{C/\pi}{N - C/\tanh(D_1) - C/\tanh(D_2)} \tag{11}$$

where N is defined in [2, equation (3.11)] and where D_1, D_2 are the damping parameters in the two rods, defined by equation (4).

The energy densities at the excitation and at the right hand ends of the two rods are calculated using a dynamic stiffness method. The data are found in Table 1. The coupling spring is chosen to be very stiff so that the transmission factor is almost frequency independent. The results are shown in Fig. 1, as are the results from a standard SEA calculation using the coupling loss factor (10). Also, calculations are made using the 3 d.o.f. SEA element (7) and coupling loss factor (11). Considering the discussion in the introduction, the improved SEA formulation should provide a correction for strong coupling at lower frequencies. However, because of low modal density, results are unreliable, so, for a particular configuration this correction is not necessarily an improvement. For intermediate frequencies, 500 Hz - 2 kHz, the two SEA models should agree. For even higher frequencies only the improved model should provide accurate results. These findings are confirmed in Fig. 2, where the coupling power is shown in third octave bands and in octave bands.

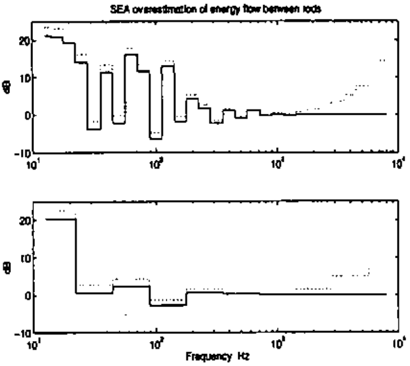

Figure 2. SEA overestimation of energy flow between rods. Upper, 1/3 octave bands. Lower octave bands. • • , standard SEA; ——, 3 d.o.f. SEA elements using coupling loss factor (11)

Beam structure. Consider two beams, simply supported at their intersection and at one end of the structure and excited by a point force at the other end. At the intersection of the beams is a rotational spring connected to ground. The transmission factor is given in [5, eq. (35)]. The data, Table 1, are chosen to resemble steel pipes with radius of 150 *mm* and thickness 3.5 *mm*. For this structure the lowest resonance occurs already at approximately 0.1 Hz. There is in each beam at least 1 resonance per third octave band from 40 Hz. With the soft spring the beams are weakly coupled from approximately 150 Hz whereas with the strong spring they are weakly coupled at all the considered frequencies. The damping parameters D_1, D_2 become 1 at approximately 400 Hz. The results are shown in Fig. 3 and 4, confirming the findings for the rod, while in this case, because of higher modal density, SEA produces much better results at lower frequencies. At higher frequencies, using the 3-d.o.f. SEA element results are on average accurate.

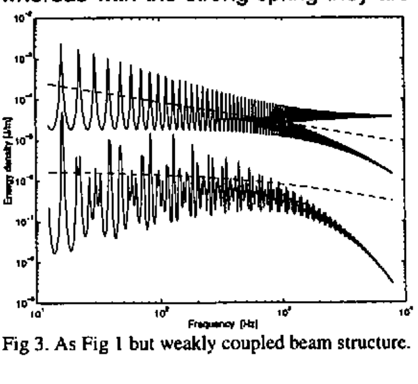

Fig 3. As Fig 1 but weakly coupled beam structure.

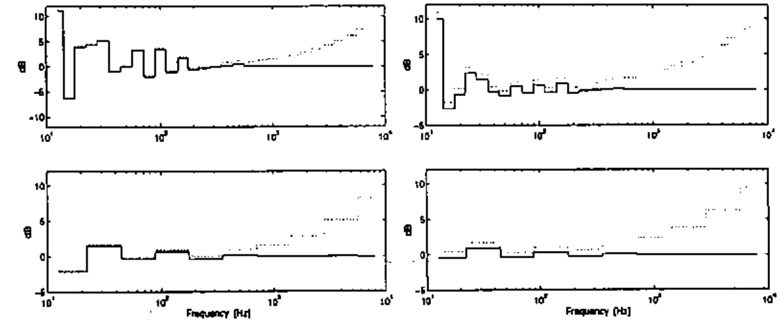

Fig. 4. SEA overestimation of energy flow between beams in 1/3- and 1- octave bands;
Left, weakly coupled, Right, strongly coupled; · · ·, standard SEA; ——, 3 d.o.f. SEA elements using coupling loss factor (11)

CONCLUSIONS

In long waveguides, because of damping energy decays away from the excitation. Thus, the fundamental assumption in SEA of uniform energy density is violated and large errors may result. To overcome this a 3 d.o.f. SEA element is proposed. The d.o.f. are the modal energies at the ends of the waveguide and the averaged modal energy in the element. Previous research suggest that when the modal overlap is larger than 1, SEA applies regardless of coupling strength. Hence, using the proposed element for long wave guides at high frequencies, very accurate results are expected. This is demonstrated with two calculation examples.

ACKNOWLEDGEMENT

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Table 1

both calculations		*rod calculation*	*beam calculation*
$E = 210\,10^9\ N/m^2$	$\eta = 0.15$	$A_1 = 0.0031\,m^2$	$A_1 = 0.0031\,m^2$
$\rho = 7850\ kg/m^3$	$L_1 = 40\ m$	$A_2 = 0.031\,m^2$	$A_2 = 0.0031\,m^2$
$\nu = 0.3$	$L_2 = 52.3\,m$		$I_y = 35\,10^{-6}\ m^4$

	rod coupling	*weak beam coupling*	*strong beam coupling*
Coupling spring	$7.55\,10^{10}\ N/m$	$10^8\ N/m^2$	$10^4\ N/m^2$