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VERIFICATION OF AN ANALYTIC SENSITIVITY CAPABILITY FOR THE FINITE ELEMENT STRUCTURAL-ACOUSTIC PROGRAM SARA-2D

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1. INTRODUCTION

SARA–2D [1] is a finite element program developed by BBN Acoustic Technologies which solves axisymmetric structural–acoustic problems. Radiated noise and scattering from submerged structures may be computed for symmetric or arbitrary excitations using SARA–2D.

Designers are often interested in the sensitivity of radiated noise and scattering levels with respect to various structural design parameters, such as shell thicknesses, loss factors, and material properties. Sensitivity analysis is now done by running an analysis on a baseline design, perturbing a single design parameter in the model by a small amount, reanalyzing the perturbed model, and computing the sensitivity using a first–order finite difference approximation:

$$Sensitivity = \frac{Baseline\ response\ -\ Perturbed\ response}{\Delta},$$

where Δ is the perturbation amount. This finite difference approach is approximate, since the error in sensitivity is theoretically proportional to Δ; and time consuming, since an additional finite element analysis is required for each design parameter of interest.

Methods exist for computing radiation and scattering sensitivities analytically. Analytic sensitivity calculations are more accurate than finite difference approximations, since no design perturbation is needed; and significantly faster, since additional finite element analyses are not needed.

2. ANALYTIC SENSITIVITY FORMULATION

Such an analytical sensitivity capability was implemented in SARA–2D. The complete formulation will appear in a future paper, but the general formulation is summarized here. The general matrix form of a finite element system of equations is:

$$[A]u = b,$$

where A represents the coupled fluid–structure system, u contains the structural velocities and acoustic pressures, and b is the load vector. Differentiating the matrix equation and assuming the gradient of the load vector b is zero, the sensitivity of u with respect to a design variable x_i is:

$$\frac{\partial u}{\partial x_i} = [A]^{-1}\left\{-\frac{\partial[A]}{\partial x_i}u\right\}.$$

Therefore, to compute the sensitivity of u, the sensitivity of $[A]$ with respect to x_i is needed. For physical and material design parameters like shell thickness, loss factor, Young's Modulus, and mass density, the $[A]$ sensitivities are simple to compute prior to solving the system of equations. The u sensitivities may then be computed easily following the solution for u itself.

In SARA–2D the sensitivities of normal velocities and fluid pressures at the wet surface are computed first. Then, the far–field radiated or scattered pressure sensitivities are computed from the wet surface sensitivities using the Helmholtz exterior integral equation.

3. VERIFICATION

A model of a submerged cylindrical shell was used to verify the analytic sensitivity capability. The model, used in a previous related study on structural–acoustic optimization [2], is shown in Figure 1. An axisymmetric ring load was applied at the center and radiated pressures were computed in the r (90 degrees) and z (0 degrees) directions. The radiated pressure spectra from 220 to 720 Hz are shown in Figure 2.

The sensitivities of the radiated pressures with respect to the center wall thickness t_1 and the center wall mechanical loss factor η_1 were computed both analytically using the new capability and approximately using the finite difference approach. The finite difference sensitivities were computed using four Δ levels. The baseline t_1 value is 10^{-2} m and the Δ values vary from 10^{-3} to 10^{-6} m. The baseline η_1 value is 10^{-2} and the Δ values vary from 10^{-2} to 10^{-5} m. For this study t_2 was set equal to t_1.

The convergence plots in Figure 3 (t_1) and Figure 4 (η_1) show that the finite difference sensitivities converge to the analytic sensitivities as Δ becomes small. Accuracy of the analytic sensitivities was verified for both pressure angles at the lower and upper frequencies of the radiated pressure spectra. Both real and imaginary components of pressure sensitivity are shown.

Similar convergence studies were done for the Young's Modulus and mass density of the shell material which is not shown here. The results of the convergence studies were similar to those of the thickness and loss factor studies. The analytic sensitivities were within $\pm 1\%$ of the converged finite difference sensitivities.

4. CONCLUSIONS

The new SARA–2D analytic sensitivity capability has been verified against converged, finite difference sensitivities of radiated pressures with respect to shell thickness, loss factor, Young's Modulus, and mass density design parameters. The analytic sensitivity calculation is much faster than the finite difference calculation – the additional analysis time was roughly 10–20% more than the basic analysis time. The additional analysis time required to compute sensitivities analytically is trivial compared to the additional full analysis for each design parameter required by the finite difference approach.

The new analytic sensitivity analysis capability not only provides information on what structural design parameters have strong effects on radiated and scattered pressures. The capability also enables the structural–acoustic optimization of larger–scale systems. General methods for structural–acoustic optimization of radiating structures have been developed, but demonstrated only on small–scale problems [2]. Optimization of larger problems was not attempted due to the computational effort required to compute radiated noise sensitivities using the finite difference approach. Hopefully, future optimization studies will be attempted on larger problems using the more accurate and computationally efficient analytic sensitivity capability.

5. ACKNOWLEDGEMENT

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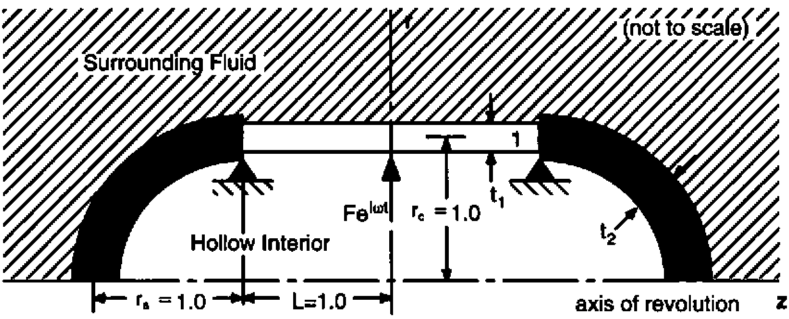

Structural Materials:	$E = 2.122\text{x}10^{11}\ N/m^2 \quad \nu = 0.3$ $\rho_s = 7995\ kg/m^3$ $\eta_1 = \eta_2 = 0.01$
Surrounding Fluid:	$\rho_f = 1025\ kg/m^2 \quad c = 1500\ m/s$

Figure 1. Simply Supported Cylindrical Shell with End Caps.

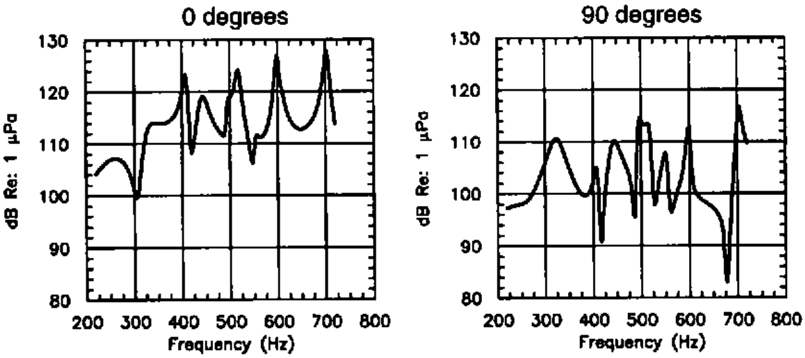

Figure 2. Far–field radiated noise (pressure at 1 yard).

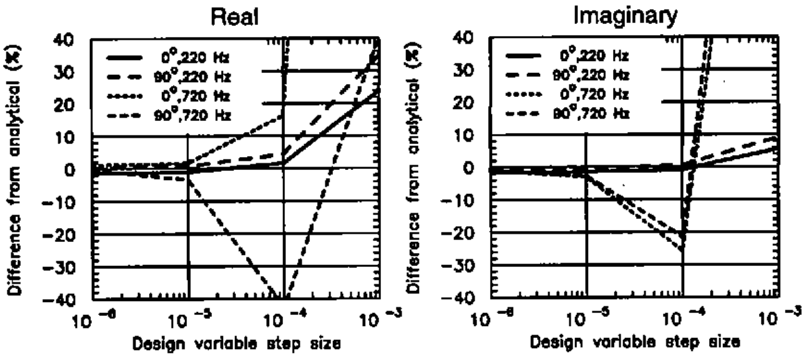

Figure 3. t_1 convergence (baseline value = 10^{-2} m).

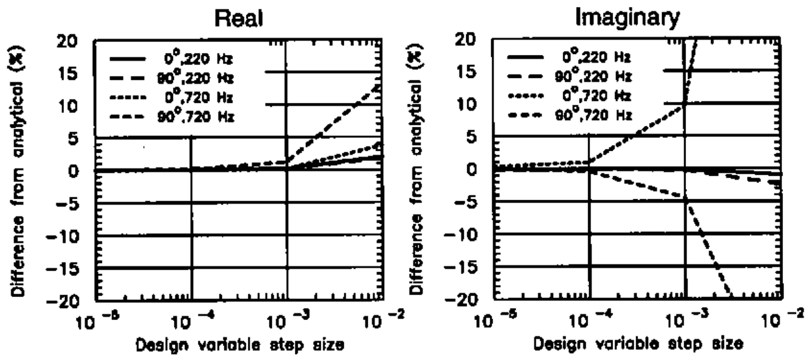

Figure 4. η_1 convergence (baseline value = 10^{-2}).