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A STUDY OF ONLINE PLANT MODELLING METHODS FOR ACTIVE CONTROL OF SOUND AND VIBRATION

S Laugesen

Department of Acoustic Technology, Technical University of Denmark, Building 352, DK-2800, Lyngby, Denmark

1. INTRODUCTION

Active control systems that use the popular filtered-*x* adaptive algorithm [1,2] require so-called plant models to describe the relations between the secondary source outputs and the error sensor inputs in order to ensure convergence [3,4]. For most laboratory experiments reported in the literature the plant models have been identified *a priori* and have remained fixed during the control experiment itself [5,6,7]. This approach is perfectly viable as long as the physics of the plant does not change drastically during the experiment [8-12]. However, for long-term practical applications an online plant identification scheme may be required, which continuously adapts the plant model to the possibly changing environment.

Several methods for online plant modelling have been suggested in the literature. The controller developed by Eriksson & Allie [13,14] uses a low level auxiliary "probe noise" and a standard LMS adaptive filter to model the plant. Successful applications have been reported for both single [15] and multichannel [16,17] systems. Sommerfeldt and his co-workers [18-22] have avoided probe noise by modelling both the primary and secondary plant responses by means of the projection algorithm [23], but this approach has fundamental problems with the convergence properties that—in this context—have received only little attention [24,25].

While the two above methods may in principle be used for control problems with both sinusoidal and broadband random disturbances other approaches focus on control of sinusoidal disturbances. Thus, Kuo et al. [26,27] have proposed two algorithms that feature online plant modelling without probe noise. However, their convergence properties need yet to be fully explored. Finally, the "curve fit algorithm" [28] or H-TAG [29] should be mentioned. This approach eliminates the need for a plant model, but at the cost of a higher complexity and slow convergence. Due to lack of space these latter methods will not be further discussed.

Apart from the survey of the existing methods for online plant modelling, the objective of this paper is to compare the performance of the two dominant approaches outlined above. This is with advantage studied by computer simulations because in that case optimal reference results can easily be established, to which the results obtained with the adaptive algorithms can be compared. The specific case considered is that of an active control system intended for reducing noise in a shallow enclosure.

2. DESCRIPTION OF THE MODEL CONTROL PROBLEM

The sound pressure p at a point $\underline{x}$ in the enclosure is described as

$$p(\underline{x},\omega) = \sum_{n=0}^{N} \psi_n(\underline{x}) a_n(\omega) \ , \qquad (1)$$

where ψ_n are the usual cosine mode shapes, $a_n(\omega)$ are the complex mode amplitudes [30,31] and ω the angular frequency of the sinusoidal excitation. Both the primary and secondary sources are modelled as square pistons mounted flush with an enclosure wall, as originally suggested by Bullmore [32]. Each mode has an associated damping ratio ζ_n, which is set to 0.04—a rather high value, chosen so as to keep the model impulse responses to within a reasonable length.

Control strategies

Although it is well known [32,5,31,33] that the total potential acoustic energy E_p is a more reliable global measure of the noise level in the enclosure, a practical controller will have to rely on minimising the quantity J_l, which is the sum-squared outputs of a number of sound pressure error sensors. When the enclosure is driven by broadband random noise a constraint comes into play, because the controllers must respond to the reference signal $x(n)$ in a causal manner. Thus, another formulation is required where the time domain cost function $J_l = \mathrm{E}\{\Sigma_{l=1}^{L} e_l(n)^2\}$ is minimised with respect to the coefficients a_{mi} of the causal I tap FIR control filters feeding the M secondary sources (m=1 to M) [34,2,31]. Here, E{·} denotes expected value and $e_l(n)$ is the lth discretised error signal (l=1 to L).

For the simulations, the adaptive control algorithm is the standard Multiple Error LMS [2], the main equations of which are

$$a_{mi}(n+1) = a_{mi}(n) - \alpha \sum_{l=1}^{L} e_l(n) r_{lm}(n-i) \ ; \ \ r_{lm}(n) = \sum_{j=0}^{J-1} \hat{c}_{lmj} x(n-j) \ . \qquad (2a,b)$$

Here, $a_{mi}(n)$ are the time-dependent control filter coefficients, α is the convergence gain, $r_{lm}(n)$ are the filtered reference signals, and $\hat{c}_{lmj}$ are the coefficients of J tap FIR filters used to estimate the impulse responses from the mth secondary source to the lth error sensor.

3. ONLINE PLANT MODELLING METHODS

The two most general methods for online plant modelling are the "probe noise" approach [13], presented in terms of the block diagram in Fig.1, and the "complete system identification (ID)" approach [18]. The former is readily extended to multichannel systems. This requires M probe noise generators that are uncorrelated with both each other and the primary disturbance, and an $M \times L$ matrix of LMS adaptive $\hat{c}$-filters.

The basic assumption of the complete system ID approach is that an estimate of the lth error signal may be expressed as

$$\hat{e}_l(n) = \sum_{k=0}^{K-1} \hat{g}_{lk} x(n-k) + \sum_{m=1}^{M} \sum_{j=0}^{J-1} \hat{c}_{lmj} u_m(n-j) \; ; \; u_m(n) = \sum_{i=0}^{I-1} a_{mi} x(n-i) \; , \qquad (3a,b)$$

where $\hat{g}_{lk}$ are the coefficients of K tap FIR filters used to estimate the impulse response from the primary disturbance mechanism to the lth error sensor, $\hat{c}_{lmj}$ are as above and $u_m(n)$ is the output from the control filter to the mth secondary source. With the definitions of the regression vector $\underline{\Phi}^T(n)=[u_1(n) \; u_1(n-1) \cdots u_M(n-J+1) \; x(n) \cdots x(n-K+1)]$, the vector of estimated error signals $\underline{\hat{e}}(n)$, and the parameter matrix $\underline{\underline{\hat{\Theta}}}(n)$:

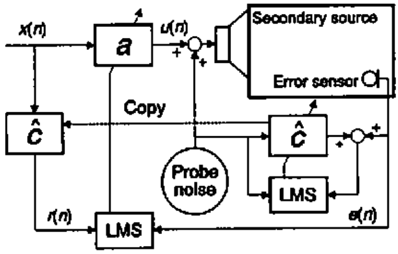

Fig.1. *Sketch of the "probe noise" approach to online plant modelling in a single channel active control system.*

$$\underline{\underline{\hat{\Theta}}}^T(n) = \begin{bmatrix} \hat{c}_{110} & \hat{c}_{111} \cdots & \hat{c}_{1MJ-1} & \hat{g}_{10} \cdots & \hat{g}_{1K-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{c}_{L10} & \hat{c}_{L11} \cdots & \hat{c}_{LMJ-1} & \hat{g}_{L0} \cdots & \hat{g}_{LK-1} \end{bmatrix} \; ; \quad \underline{\hat{e}}(n)=[\hat{e}_1(n) \cdots \hat{e}_L(n)]^T = \underline{\underline{\hat{\Theta}}}^T(n)\underline{\Phi}(n) \; , \qquad (4)$$

it follows immediately that $\underline{\underline{\hat{\Theta}}}(n)$ can be estimated recursively by means of e.g. the multichannel projection algorithm [23]:

$$\underline{\underline{\hat{\Theta}}}(n+1) = \underline{\underline{\hat{\Theta}}}(n) + \frac{a\underline{\Phi}(n)}{b+\underline{\Phi}^T(n)\underline{\Phi}(n)} [\underline{e}(n)-\underline{\hat{e}}(n)] \; ; \; 0<a<2 \; ; \; b>0 \; . \qquad (5)$$

It is important to note that convergence is only guaranteed in terms of

$$\lim_{n\to\infty} \| \underline{\underline{\hat{\Theta}}}(n)-\underline{\underline{\hat{\Theta}}}(n-\kappa) \| = 0 \text{ , for any finite } \kappa \; ; \quad \| \underline{\underline{M}} \| = (\sum_{i,j} m_{ij}^2)^{1/2} \; . \qquad (6)$$

One implication of this is that under certain circumstances the overall model error J_e=E{ $\| \underline{e}(n)-\underline{\hat{e}}(n) \|^2$} can be driven to zero although the individual models of the g_l and c_{lm} responses are erroneous [25]. This is a major problem, since the whole point of the system identification is to provide accurate estimates of the c_{lm} responses to the control algorithm (2a,b).

4. TEST CASES

A sketch of the enclosure and the positions of the primary and secondary sources and error sensors of the model control system is shown in Fig.2. Two configurations will be considered: A two input two output system with both secondary sources and error sensors 1 and 2 (S_{1-2};E_{1-2}), and a four input two output system with both sources and all four error sensors (S_{1-2}; E_{1-4}). When the control system comprises an equal number of sources and sensors it is theoretically possible to reduce the cost function J_f to zero for sinusoidal disturbances [5]. However, this does not necessarily lead to good reductions of the global measure E_p. When there are more sensors than sources (S_{1-2};E_{1-4}) only a limited reduction of J_f is obtainable, but the resulting reduction of E_p (not shown) is in this particular case very close to the ideal result obtained by minimising the energy E_p itself.

A time domain model has been obtained by Fourier transforming the transfer impedances from the frequency domain model outlined in section 2 above [31]. For a sampling frequency of f_s=1000 Hz the length of the impulse responses could be restricted to 250 taps. The results of causally constrained minimisations of the time domain cost function J_t for a white noise disturbance and control filters of length I=128 are included in Fig.3 below.

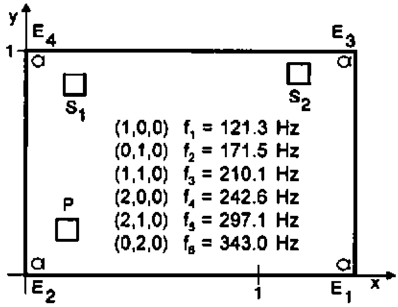

Fig.2. *Sketch of the shallow enclosure with a primary piston source (P), two secondary sources ($S_{1,2}$), and four error sensors ($E_{1,4}$). Modal integers and natural frequencies for the first six modes are also indicated.*

5. RESULTS FOR BROADBAND RANDOM DISTURBANCE

For these simulations a broadband pseudo-random disturbance with a flat spectrum was constructed by adding together 128 unit sine functions with random phases and frequencies evenly distributed between 0 Hz and $\frac{1}{2}f_s$, so that the reference signal had a period of 256 samples. The control filters and the secondary and primary plant model filters had I=J=K=128 coefficients each. Thus, the two online plant modelling approaches outlined above were tested for the two system configurations considered, and compared to an idealised case where the filtered reference signals were determined by means of the actual model impulse responses. The results are shown in Fig.3. It is easily seen that in both cases the results obtained with the probe noise approach are almost identical to those obtained with ideal plant models, which again are very close to the results of the causally constrained minimisation of J_t. The complete system ID approach provides some reduction but the results are not as good as the other. The explanation has already been mentioned. The secondary plant models ($\hat{c}_{lm}$) provided for the control algorithm are erroneous [25] and this leads to a steady state residual error which is greater than it ideally should be [9], although the plant models are still good enough to ensure convergence.

6. RESULTS FOR SINUSOIDAL DISTURBANCE

A set of simulations similar to those described above was carried out for tonal disturbances at three different frequencies. There are, however, several notable differences. When a tonal disturbance is considered, it is theoretically possible to reduce the cost function to zero exactly in the S_{1-2};E_{1-2} case. In the S_{1-2};E_{1-4} case only a limited amount of reduction is obtainable. Futhermore, when a tonal disturbance is considered two coefficients in each plant model and control filter (I=J=K=2) should, in principle, suffice to realise the correct amplitude and phase at the frequency in question. However, the probe noise approach requires that

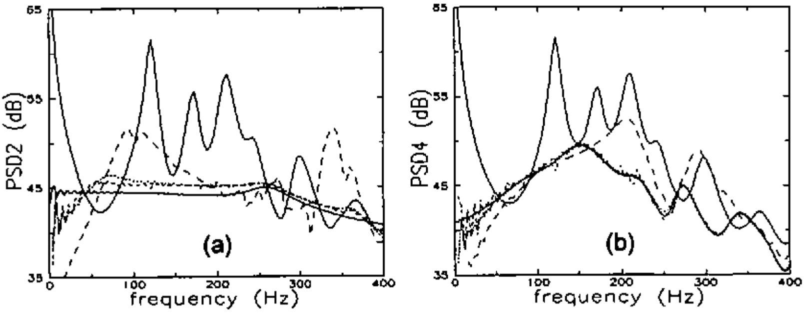

Fig.3. *Simulated results of adaptive control of a pseudo-random broadband disturbance. The mean sum of the power spectral densities (PSD) of the error signals ——before control and with fully converged controller obtained with ----ideal plant models, and online plant modelling by the ——probe noise and – – –complete system ID approaches. Finally, ——the reference result of the causally constrained minimisation of J_t. Control system: (a) $S_{1\text{-}2};E_{1\text{-}2}$ and (b) $S_{1\text{-}2};E_{1\text{-}4}$.*

the probe noise sources are uncorrelated with the primary disturbance, and hence it is still necessary to use broadband probe signals when the disturbance is sinusoidal. The spectra of the probe signals may be shaped around the frequency of the disturbance [35], which means that plant model filters with fewer coefficients can be used, than if the probe noises had flat spectra. The results for the $S_{1\text{-}2};E_{1\text{-}4}$ case will be dealt with first and are summarised in Table 1. The test runs with ideal plant models gave results that were identical to those obtained from direct minimisation in the frequency domain. After testing a wide range of spectral shapes and plant model filter lengths the probe noise approach gave results that were reasonably close to the ideal. The complete system ID approach again showed its tendency to converge to a sub-optimal steady state—most prominently at 125 Hz. In this case the problem can be described as lack of "persistent excitation" [23]. Even in the simplest single channel case $\hat{\underline{\theta}}(n)$ (eq.4a) contains four unknown parameters, and a single sinusoid is only persistently exciting of order two, which means that only two parameters can be uniquely determined.

Disturb. frequency	Ideal plant models	Probe noise	Complete system ID
125.0 Hz	15.2 dB	15.0 dB	13.4 dB
187.5 Hz	8.0 dB	7.6 dB	7.9 dB
375.0 Hz	3.2 dB	2.6 dB	2.5 dB

Table 1. *Reductions of J_t found with sinusoidal disturbances and different plant modelling methods in the $S_{1\text{-}2};E_{1\text{-}4}$ case.*

In the $S_{1\text{-}2};E_{1\text{-}2}$ case the probe noise approach gave reductions between 26 and 85 dB, again very much depending on the choice of probe signal shape and plant model filter length. For ideal plant modelling and complete system ID the obtainable reductions were only limited by the arithmetic precision. However, in the latter case it is notable that whereas the overall model error J_e at once would become very small, the controller cost function J_t would waver around the uncontrolled level for quite a long time before finally starting its descend towards the residual level. This is because there is an infinite subspace of plant model coefficients that yield a zero model error, and the plant models are drifting around in the J_e=0-subspace until they incidentally arrive at a point that allows the controller to converge fully. Apparently, once convergence of the controller was obtained, the control system was able to stay in the optimal state.

7. CONCLUDING REMARKS

The two dominant methods for online plant modelling in multichannel active control systems have been compared by means of computer simulations. The "complete system identification" approach [18] gave seemingly convincing results, as converge was obtained always. However, in some cases the steady state performance could be quite far from optimal, because the secondary plant models provided by the estimation algorithm were erroneous. For a broadband random disturbance the "probe noise" approach [13] gave results that were very close to the theoretically optimal, whereas the results obtained for sinusoidal disturbances were less impressive. In the latter case the choice of plant model filter length and spectral shape of the probe signals turned out to be critical.

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