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TOWARDS AN ANALYTICAL MODEL OF WAVE PROPAGATION IN SANDWICH BEAMS

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1. INTRODUCTION

Sandwich materials find numerous applications where high strength and low-weight are important criteria. The main obstacle which holds back the otherwise anticipated growth in the utilization of sandwich materials is the lack of a general theory concerning its acoustical and dynamical behavior. The main aim of this work is to develop an analytical model for prescribing the wave propagation in sandwich constructions, whether subjected to external excitation or not, and thus increase the optimizing possibility of constructions in very early stage during design process. A model describing wave propagation in sandwich beams is developed. Measurements have been carried out on a sandwich beam. The agreement between predicted and experimental results have been found to be satisfactory.

2. ANALYTICAL FORMULATION

The geometry of the sandwich beam treated and the notations for the displacements are shown in Fig. 1. Stress components acting on laminates and core can be found in the same figure. Both laminates and core are assumed to be isotropic. The displacement of laminates is according to propagation of bending and longitudinal waves only, in other words—Euler beam theory is used. The following differential equations can then describe the motion of the laminates:

$$\frac{d^4}{dx^4}\eta_n(x) - \kappa_n^4\eta_n(x) = \frac{\sigma_{y_{n2}}}{D_{xn}}, \qquad \frac{d^2}{dx^2}\xi_n(x) + K_{ln}^2\xi_n(x) = -\frac{\tau_{xy_{n2}}}{E_{xn}t_n}, \tag{1}$$

On the other hand the wave propagation in the core will be described through the general field equations (see [1]). Thus bending, rotation and shear as well as longitudinal deflection are included through a combination of longitudinal and transverse waves:

$$\frac{\partial^2}{\partial x^2}\phi(x,y) + \frac{\partial^2}{\partial y^2}\phi(x,y) + K_{l2}^2\phi(x,y) = 0, \tag{2}$$

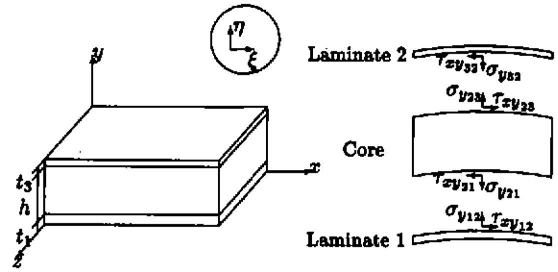

Fig. 1: The notations for the displacements and geometry of sandwich beam.

$$\frac{\partial^2}{\partial x^2}\psi(x,y)+\frac{\partial^2}{\partial y^2}\psi(x,y)+K_{t2}^2\psi(x,y)=0. \tag{3}$$

The fact that, the coupling/boundary conditions between laminates and core (*i.e.* continuity in displacements and stresses between layers) and the whole sandwich beam with two free surfaces must be satisfied at every instant of time, requires that all propagating waves in structure have the same time-dependency, here assumed as $e^{i\omega t}$. But these coupling/boundary conditions must also be satisfied at all locations between layers and free surfaces. Therefore it is assumed that all propagating wave types in the sandwich plate have the same propagation constant along x-axis. The following expressions are then solutions of equations of motion in laminates and core (Eqs. 1- 3), with time dependence omitted:

$$\begin{aligned}
&\eta_n(x)=A_n(\alpha)e^{\alpha x}; \qquad \xi_n(x)=B_n(\alpha)e^{\alpha x}; \qquad n=1,3;\\
&\phi(x,y)=e^{\alpha x}\left(C(\alpha)e^{\beta_L y}+D(\alpha)e^{-\beta_L y}\right);\\
&\psi(x,y)=e^{\alpha x}\left(E(\alpha)e^{\beta_T y}+F(\alpha)e^{-\beta_T y}\right),
\end{aligned}$$

where $\beta_L=i\sqrt{\alpha^2+K_{l2}^2}$; $\qquad \beta_T=i\sqrt{\alpha^2+K_{t2}^2}$.
The continuity of displacements between laminates and core requires:

$$\begin{aligned}
&\text{at } y=t_1 && \text{at } y=t_1+h\\
&\xi_1(x)-\frac{t_1}{2}\frac{d}{dx}\eta_1(x)=\xi_2(x,t_1); && \xi_2(x,t_1+h)=\xi_3(x)+\frac{t_3}{2}\frac{d}{dx}\eta_3(x);\\
&\eta_1(x)=\eta_2(x,t_1); && \eta_2(x,t_1+h)=\eta_3(x).
\end{aligned}$$

Moreover, using the continuity of stresses in the adjacent plates and inserting the assumed solutions in Eqs.1–3 leads to the following system of linear equations:

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14}\\ V_{21} & V_{22} & V_{23} & V_{24}\\ U_{31} & U_{32} & U_{33} & U_{34}\\ V_{41} & V_{42} & V_{43} & V_{44}\end{bmatrix}\begin{bmatrix} C\\ D\\ E\\ F\end{bmatrix}=0, \tag{4}$$

where

$$\begin{aligned} U_{n1} &= \left((-1)^{\frac{n+1}{2}} u^{+}_{11_n} + u^{+}_{12_n}\right) \\ U_{n2} &= \left((-1)^{\frac{n+1}{2}} u^{-}_{11_n} - u^{-}_{12_n}\right) \\ U_{n3} &= -\left((-1)^{\frac{n+1}{2}} u^{+}_{21_n} + u^{+}_{22_n}\right) \\ U_{n4} &= \left((-1)^{\frac{n+1}{2}} u^{-}_{21_n} - u^{-}_{22_n}\right) \\ &\text{for} \quad n = 1,3 \end{aligned} \qquad \begin{aligned} V_{n1} &= \left(v^{+}_{11_n} + v^{+}_{12_n}\right) \\ V_{n2} &= -\left(v^{-}_{11_n} + v^{-}_{12_n}\right) \\ V_{n3} &= -\left(v^{+}_{21_n} - v^{+}_{22_n}\right) \\ V_{n4} &= -\left(v^{-}_{21_n} - v^{-}_{22_n}\right) \\ &\text{for} \quad n = 2,4 \end{aligned}$$

The functions u and v can be found in the Appendix. There is only one non-trivial group of solutions for the above presented system of equations. This is when the determinant of the system matrix is equal to zero. This requirement leads to the so-called dispersion equation. Solving this equation gives wave propagation constants for the structure. The numerical method used for solving this complex-valued equation was based on so-called argument principle (see [2]). But despite the efficiency of this method one should not underestimate the numerical problems caused by, for instance, ill-conditioned matrix and complexity of the system of equations.

3. MEASUREMENTS

For this research, proper modal analysis of the sandwich-beams is vital. In order to determine the natural frequencies and the modal parameters accurately, modal analysis was performed on a sandwich-beam, its core and its laminates. The sandwich-beam was made of glass-reinforced plastic laminates, abbreviated to GRP laminates and PVC core material. The transfer functions com-

	ρ (kg/m^3)	E (N/m^2)	t (mm)	L (m)	b (mm)	δ $(\%)$	ν
Laminates	1509	8.3E9	2.5	2.0	115	1.1	0.4
Core	109	96E6	50	2.0	115	1	0.12

Table 1: Dimensions and material parameters.

puted by the spectrum analyser were given as input data to the modal analysis program, STAR, and the natural frequencies, corresponding mode shapes, and half power bandwidth damping ratio were determined. The material properties and dimensions of sandwich-beam are listed in Table 1.

4. RESULTS

Solving the dispersion equation, *i.e.* the determinant of the system matrix (4), the wave propagation constants for the sandwich beam will be obtained. Figure 2 depicts these solutions for the sandwich beam in question and its dimensions and material parameters can be found in Table 1. As expected, the character of dispersion curves are similar to the ones found by Nilsson [3].But the newness of our findings are in the heavily damped propagating evanecscent solutions. Although the comprehension of these kind of waves' properties is still limited, but one can ensure that the importance of these waves increases at bounderies. The measured wave propagating constants follow closely the predicted values for the first propagating mode.

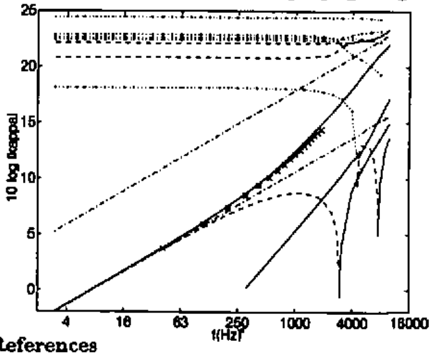

Fig. 2: Predicted and measured propagation constants for the beam. — propagating waves, - - - evanecscent waves, ... heavily damped waves, -.-.- asymptotes for the first propagating mode, ××× measured propagating wave constants.

References

[1] L. Cremer and M. Heckl. *Structure-borne Sound.* Springer Verlag, Berlin, 1988.

[2] I. Karasalo. Computation of modal wavenumbers using an adaptive winding-number integral method with error control. *Journal of Sound and Vibration*, 161:173–180, 1993.

[3] A. C. Nilsson. Wave propagation in and sound transmission through sandwich plates. *Journal of Sound and Vibration*, 138(1):73–94, 1990.

APPENDIX

We use the same notations as in Figure 1 concerning the geometry of sandwich construction. Furthermore if E_n, ρ_n and ν_n denote E-modulus, density and Poisson's ratio respectively, then we can define shear modiolus, bending stiffness, longitudinal-, transversal- and bending wavenumber as:

$$G_n = \frac{E_n}{2(1+\nu_n)}; \qquad D_{xn} = \frac{E_{xn}t_n^3}{12}; \qquad K_{ln} = \left(\frac{\rho_n\omega^2}{E_{xn}}\right)^{.5};$$

$$K_{tn} = \left(\frac{\rho_n\omega^2}{G_n}\right)^{.5}; \quad \kappa_n = \left(\frac{m_n\omega^2}{D_{xn}}\right)^{.25}$$

$$E_{xn} = \begin{cases} E_n(1+i\delta_n) & \text{if } n = 1,3 \\ E_n\dfrac{(1+\nu_n)}{(1+\nu_n)(1-2\nu_n)}(1+i\delta_n) & \text{if } n = 2 \end{cases}$$

By introducing the following parameters:

$$L_{2n} = \frac{E_{x2}}{D_{xn}}; \qquad S_{2n} = \frac{G_2}{E_{xn}t_n}; \qquad \nu = \frac{\nu_2}{1-\nu_2};$$

the functions u and v are defined as:

$$u_{11_n}^{\pm} = L_{2n}\left(\beta_L^2 + \nu\alpha^2\right)e^{\pm\beta_L y}; \qquad u_{12_n}^{\pm} = \beta_L\left(\alpha^4 - \kappa_n^4\right)e^{\pm\beta_L y};$$
$$u_{21_n}^{\pm} = L_{2n}(1-\nu)\alpha\beta_T e^{\pm\beta_T y}; \qquad u_{22_n}^{\pm} = \alpha\left(\alpha^4 - \kappa_n^4\right)e^{\pm\beta_T y};$$

for $n = 1, 3,$

$$v_{11_n}^{\pm} = \alpha\left(\alpha^2 + K_{l(n-1)}^2\right)\left((-1)^{\frac{n-2}{2}}\frac{t_{(n-1)}}{2}\beta_L \pm 1\right)e^{\pm\beta_L y}$$
$$v_{12_n}^{\pm} = 2\alpha\beta_L S_{2(n-1)}(-1)^{\frac{n-2}{2}}e^{\pm\beta_L y}$$
$$v_{21_n}^{\pm} = \left(\alpha^2 + K_{l(n-1)}^2\right)\left((-1)^{\frac{n-2}{2}}\frac{t_{(n-1)}}{2}\alpha^2 \mp \beta_T\right)e^{\pm\beta_T y}$$
$$v_{22_n}^{\pm} = S_{2(n-1)}(-1)^{\frac{n-2}{2}}\left(\beta_T^2 - \alpha^2\right)e^{\pm\beta_L y}$$

for $n = 2, 4,$

where

$$y = \begin{cases} t_1 & \text{if } n = 1,2 \\ t_1 + h & \text{if } n = 3,4 \end{cases}$$