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A FAST METHOD FOR DEDUCING GROUND IMPEDANCE PARAMETERS FROM MEASUREMENTS OF EXCESS ATTENUATION SPECTRA

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ABSTRACT

A method of obtaining ground-surface impedance parameters from measurements of short-range excess attenuation spectra is proposed. The algorithm uses a fast numerical method for indirect deduction of ground surface impedance [J. Acoust. Soc. Am. 105(3), 2039-2042, 1999]. Subsequently, a linear least squares fitting is employed to obtain estimates for ground impedance parameters from deduced impedance data using the Attenborough two-parameter impedance model. It is shown that a small modification to the model yields better agreement with measurements at grazing angles.

1 INTRODUCTION

Knowledge of acoustic impedance of ground surface is important for prediction of outdoor sound propagation. Intrusive methods such as sampling soils for subsequent laboratory tests[1] or use of impedance tube *in situ* are time consuming and laborious. Other, less intrusive methods include an indirect deduction of impedance from plane wave reflection coefficient[2] or by fitting impedance models to short-range measurements of sound spectra[3-8]. Most of these schemes employ some method of multi-dimensional minimization Recently, direct deduction methods (i.e. independent of a particular impedance model) have been proposed that compare measured and theoretical spherical reflection coefficient at each frequency point. One proposed scheme[9,10] is based on two-dimensional minimization of the difference between data and theory and the other technique[11] makes use of mathematical properties of the classical approximation for the spherical reflection coefficient to propose a fast, one-dimensional root-finding process.
After a brief review of the root-finding method in section 2, the impedance values deduced by this method will be used in section 3 to obtain estimates for the parameters in the Attenborough two-parameter model of acoustic surface impedance. Later in the same section a modification to the model is proposed that results in a better agreement of the model with the data at grazing angles. Two sets of data are used for the comparison; a set of short-range Excess Attenuation Spectra measured above a sand tray and the other measured over 5cm of snow laid on a hard ground.

2 THEORY

One starts by measuring Excess Attenuation of sound spectra defined by:

$$P = 1 + Q\frac{R_1}{R_2}e^{ik(R_2-R_1)} \qquad (1)$$

where Q is the spherical reflection coefficient of the ground surface, k is the wavenumber, and R_1 & R_2 are direct and reflected path lengths respectively. The reflection coefficient can be evaluated from measurement of the Excess Attenuation spectra using this expression. It should be noted that both the magnitude and the phase information (i.e. a complex quantity) are required for subsequent calculations.

One then attempts to minimize the difference between the measured value of the reflection coefficient and the theoretical value by optimizing the real and imaginary parts of the acoustic surface admittance, β. The expression for the reflection coefficient can be written as[11]:

$$Q = 1 + 2(w - \tau\cos\theta)\left[i\sqrt{\pi}W(w)\right] \tag{2}$$

with

$$W(w) = e^{-w^2}erfc(-iw) \tag{3}$$

and

$$\tau = \sqrt{\tfrac{1}{2}ikR_2} \tag{4}$$

The angle of incidence is θ and the numerical distance, w, is given by

$$w = \sqrt{\tfrac{1}{2}ikR_2}(\cos\theta + \beta) \tag{5}$$

The approach here is to consider the complex equation

$$\Gamma(\beta) = Q(\beta) - Q_{measured} = 0 \tag{6}$$

The reflection coefficient is an analytical function of the numerical distance which is itself a function of the admittance. This means that a simple and efficient solution technique such as Newton-Raphson method can be employed to obtain a solution in β. In the Newton-Raphson method a 'better' approximation, say x_1, to the solution of $\Gamma(x)$ can be obtained from

$$x_1 - x_0 = \delta x = -\frac{\Gamma(x_0)}{\Gamma'(x_0)} \tag{7}$$

where x_0 is an initial approximation and the prime denotes differentiation with respect to the argument. A final solution is obtained by iterating equation (7) until a desired accuracy is achieved. Hence in our case:

$$\delta\beta = -\frac{\Gamma(\beta)}{d\Gamma/d\beta} \tag{8}$$

Furthermore, the function $W(z)$ and its first derivative are well known. We thus have for the derivative of the function:

$$\frac{d\Gamma}{d\beta} = \frac{dQ}{dw}\cdot\frac{dw}{d\beta} = 2\tau\left[i\sqrt{\pi}W(w) - 2(w - \tau\cos\theta)\left(1 + i\sqrt{\pi}wW(w)\right)\right] \tag{9}$$

Both expressions for the reflection coefficient and the derivative term can be evaluated quickly by series expansion given in standard mathematical textbooks[12].

Less than ten iterations are needed to find the required answer in most cases of interest. Moreover, this method is easily implemented within a PC based acquisition system and can be performed in "real time". It is much more efficient than techniques based on minimization methods without any tradeoff in accuracy. It should be noted also that the predicted impedance values reproduce the measured Excess Attenuation Spectra almost exactly when used back in equation (1). This means that errors in the measurement will result in corresponding errors in the deduced impedance values.

3 PARAMETER FITTING

Having obtained the specific surface impedance as a function of frequency, the possibility of fitting an impedance model to the impedance data is investigated. Once again, the emphasis is on speed and efficiency. The impedance model chosen here for fitting purpose is a two-parameter model[13,14]. This model has been shown to give good agreement with data taken both indoors and outdoors. It is given by:

$$Z = \frac{1+i}{\sqrt{\pi\gamma\rho}}\sqrt{\frac{\sigma_e}{f}} + \frac{ic_0\alpha_e}{8\pi\gamma f} \qquad \textbf{(10)}$$

where $\sigma_e = s_p^2\sigma/\Omega$ and $\alpha_e = (n'+2)\alpha/\Omega$; σ, Ω, α, s_p, and n' are the flow resistivity and porosity at the surface, the rate of change of porosity with depth, the pore shape factor and the grain shape factor respectively. The two adjustable parameters are σ_e and α_e. This model has the added advantage that the real and imaginary parts of the impedance have a simple power dependence on frequency so that a least squares fitting can be used:

$$\mathrm{Re}(Z) = Af^{1/2} \qquad \textbf{(11a)}$$

$$\mathrm{Im}(Z) - \mathrm{Re}(Z) = Bf^{-1} \qquad \textbf{(11b)}$$

where *A* depends on σ_e and *B* on α_e (eqn. (10)). A simple Gaussian least squares fitting of the two parts of impedance should yield estimates for σ_e and α_e. The resultant values will depend strongly on the range of frequencies used in the fitting. Since the ground effect minimum is most sensitive to the changes in the parameters, the best choice would be the frequency range that encompasses the first ground effect minimum and excludes very low and high frequencies.

The first example is based on a set of two short-range excess attenuation spectra measured above a sand surface. The grazing angles for the three measurements are 11.3°, 21.8° and 31.0°. Figures 1a and 1b show measured excess attenuation spectra and the specific surface impance respectively, of the three settings.

Making use of equations (11a and b) and the impedance model given in equation (10), one arrives at the best-fit values given in Table I. It is seen that the deduced parameter values from the two nearest- grazing measurements are very close. The frequency range for the fitting process was chosen so that it would include the first minimum only.

Data	*grazing Angle* (deg)	*Frequency range* (Hz)	σ_e (Nsm^{-4})	α_e (m^{-1})
1	11.3	1k – 10 k	293,000	-159
2	21.8	800- 3 k	314,000	-57
Average			**303,500**	**-108**

Table I: List of best-fit parameters using the two-parameter impedance model. The frequency range indicates the range at which the least-squares fitting was implemented.

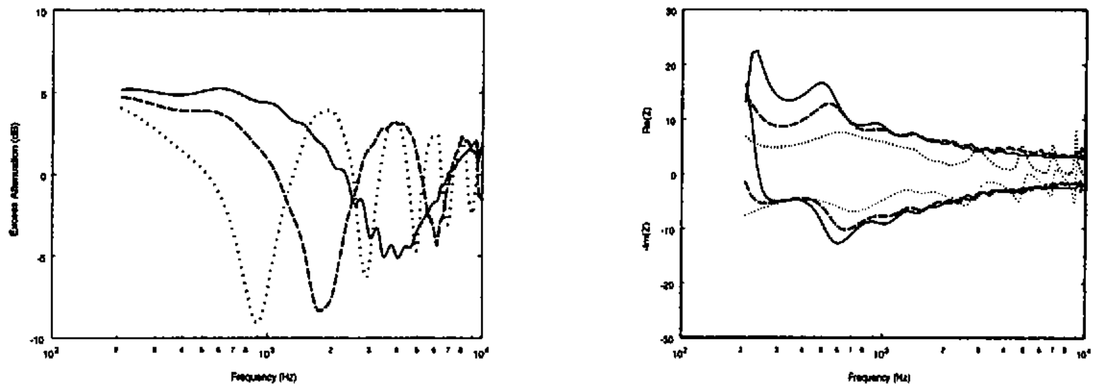

Figure 1 Measured excess attenuation spectra (a) and the corresponding deduced normalized impedance (b) of a sand surface. Solid line: source and receiver heights 0.1m (grazing angle 11.3°); dashed line: 0.2m (21.8°) and the dotted line: 0.3m (31.0°). The horizontal separation in all three cases was 1.0m. At frequencies below 300 Hz the source output was below the background noise, hence the deviation from the expected values.

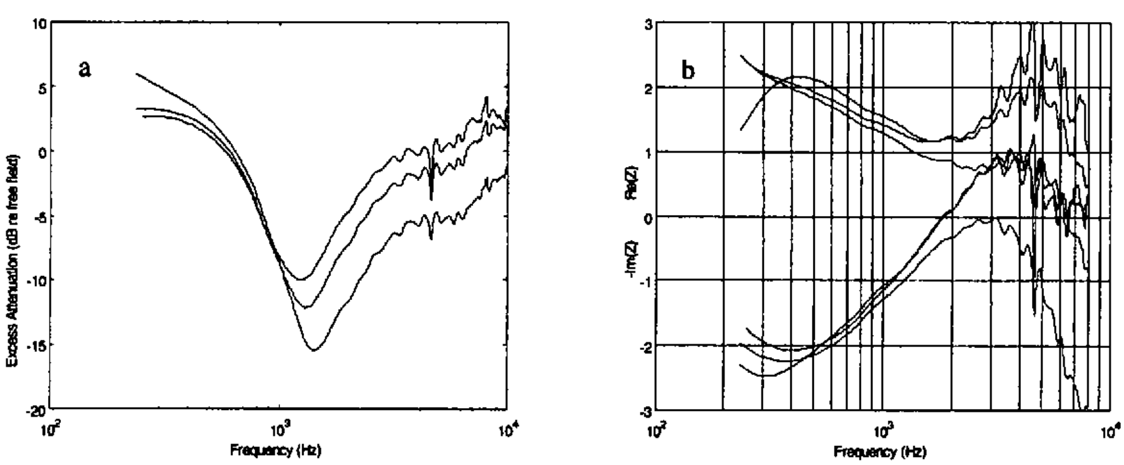

Figure 2 Measured excess attenuation spectra (a) and the corresponding deduced normalized impedance (b) of 4.5 cm snow covering a hard ground. Solid line: source and receiver heights 0.05m and 0.15m (grazing angle 11.3°); dashed line: 0.05m and 0.05m (5.7°) and the dash-dot line: 0.1m and 0.1m (11.3°). The horizontal separation in all three cases was 1.0m.

The second set of data corresponds to excess attenuation measurements taken above a snow-covered ground at grazing angles of 5.7° and 11.3°. The measured spectra and deduced specific surface impedance are given in figure 2. There are two measurements at grazing angle of 11.3° and the three data sets predict similar impedance values at a frequency range of 300-2000 Hz. However, above 2kHz the deduced values diverge significantly.

To improve the impedance fit at higher frequencies, a new approximation for impedance is suggested that introduce a third parameter. The additional parameter is an effective tortuosity and influences the high frequency values of the impedance.

The expressions for the complex propagation constant normalized with respect to that in air (k_b) and the specific characteristic impedance for a homogeneous rigid porous ground may be written[12]

$$k_b = \sqrt{\gamma\left[aT + \frac{4i\Omega^2\sigma_e}{\rho\omega}\right]} \tag{12}$$

$$Z_c = \left(\frac{4T}{3\Omega} + \frac{4i\Omega\sigma_e}{\rho\omega}\right)\times k_b^{-1} \tag{13}$$

where T is the tortuosity, Ω is the porosity, ρ is the density of air, γ is the ratio of the specific heats for air and a is a constant. If the first term under the square root in equation **(15)** for k_b is ignored, one arrives at

$$Z_c = \frac{\sqrt{2}}{\sqrt{\gamma}}\frac{\frac{4}{3}T_e + iS}{(1+i)\sqrt{S}} \tag{14}$$

with

$$T_e = \frac{T}{\Omega^2} \tag{15}$$

$$S = \frac{4}{2\pi\rho}\times\frac{\sigma_e}{f} \tag{16}$$

or with some re-arrangement

$$Z_c = \frac{1}{\sqrt{\pi\gamma\rho}}(1+i)\sqrt{\frac{\sigma_e}{f}} + \tfrac{2}{3}\sqrt{\frac{\pi\rho}{\gamma}}(1-i)T_e\sqrt{\frac{f}{\sigma_e}} \tag{17}$$

The second term in the equation (17) can be regarded as a high frequency correction term to the high flow resistivity/ low frequency approximation the surface impedance of a semi-infinite homogeneous porous medium. With the extra term arising from non-uniformity of the ground[7] (i.e. the second term in equation (13)), the specific acoustic impedance of a ground in which either the porosity decreases exponentially with depth or acts as a non-hard backed layer is given by

$$Z = Z_c + \frac{ic_0\alpha_e}{8\pi\gamma f} = \frac{1}{\sqrt{\pi\gamma\rho}}(1+i)\sqrt{\frac{\sigma_e}{f}} + \tfrac{2}{3}\sqrt{\frac{\pi\rho}{\gamma}}(1-i)T_e\sqrt{\frac{f}{\sigma_e}} + \frac{ic_0}{8\pi\gamma}\times\frac{\alpha_e}{f} \tag{18}$$

Although there are other three-parameter impedance models in the literature[6], the correction suggested here has the advantage that the least squares fitting technique can be used to obtain estimates of the three adjustable parameters, namely σ_e, α_e and T_e by means of

$$\mathrm{Re}(Z)\sqrt{f} = A + Cf \tag{19}$$

$$[\mathrm{Im}(Z) - \mathrm{Re}(Z)]f = B - Df^{3/2} \tag{20}$$

where A and B depend on σ_e and α_e respectively and C and D on both T_e and σ_e.

Data	*grazing Angle* (deg)	*Frequency range* (Hz)	σ_e (Nsm^{-4})	α_e (m^{-1})	T_e
1	11.3	1k – 10 k	357,000	-159	3.2
2	21.8	800- 3 k	320,000	-57	3.1
Average			**338,500**	**-108**	**3.15**

Table II: List of best-fit parameters using the three-parameter impedance model. The frequency range indicates the range at which the least-squares fitting was implemented.

To test the usefulness of the new three-parameter impedance model, equations (19) and (20) were used to produce a best fit estimate of the data discussed previously. Table II gives the list of best fit parameters for the two measurements, while figure 3 shows the measured and predicted spectra. The model produces better agreement with the data. However, it should be noted that although the best-fit effective flow resistivities from the two models are similar, there is a noticeable difference in the best-fit values of α_e. The values in Table I are somewhat different to those reported in reference 11 resulting in a better fit for the two parameter model. This is primarily due to a better fitting routine for α_e, which resulted in very different predicted values for it. The influence of the extra term introduced in equation (18) is confined to higher frequencies and grazing angles.

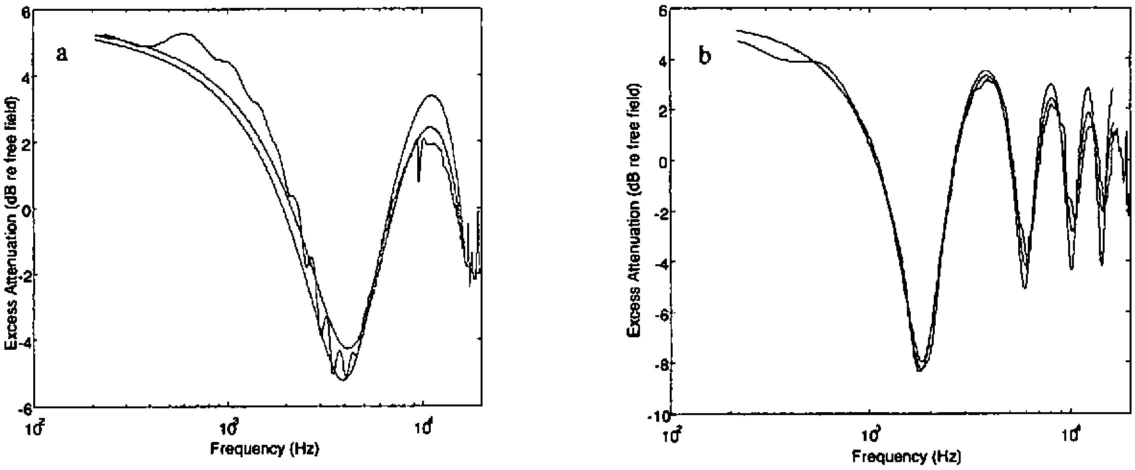

Figure 3 Measured and best fit Excess attenuation at grazing angle of 11.3° (a) and 21.8° (b) above a sand surface. The solid lines are measured data; dashed lines are the two-parameter model and the dotted lines are three parameter model predictions.

As a further example the surface impedance deduced for a snow-covered ground (figure 2) is used in the fitting routine to obtain estimates for the effective flow resistivity, rate of change of porosity and tortuosity of a layer of snow. The data set chosen has source and receiver heights both of 0.1m and separation of 1.0m (grazing angle of 11.3°). As expected from impedance values, there is a large discrepancy between data and predictions above 2 kHz.

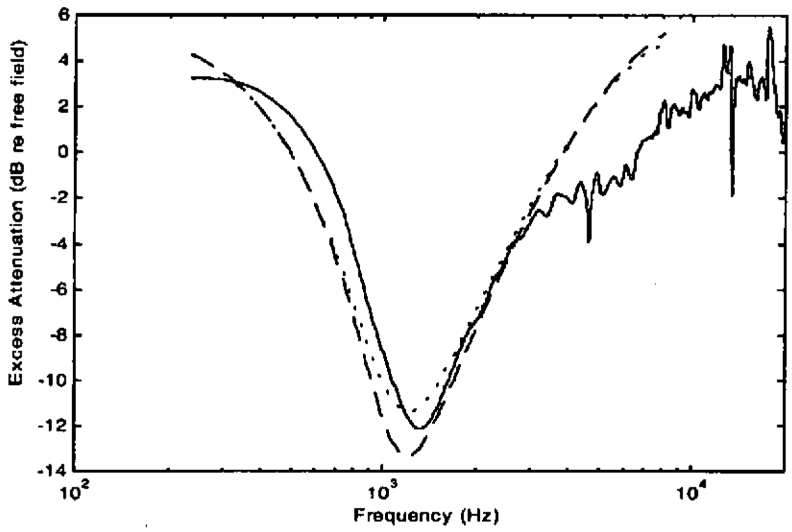

Figure 4 Measured (solid line) and predicted Excess Attenuation over 4.5 cm of snow. Solid line is the measured values, dashed line is two-parameter prediction and the dotted line is the three parameter prediction. The source and receiver heights are both 0.1m and separation is 1.0m.

Model	*Grazing Angle* (deg)	*Frequency range* (Hz)	σ_e (Nsm^{-4})	α_e (m^{-1})	T_e
Two-param	11.3	300 – 2 k	7600	3	-
Three param	11.3	300 - 2 k	8200	3	0.3
Average			**7900**		

Table III: List of best-fit parameters corresponding to figure 4 using two- and three-parameter impedance models. The frequency range indicates the range at which the least-squares fitting was implemented.

The parameter values are given in Table III. The estimated values are broadly in line with expected values of flow resitivity and rate of change of porosity while the estimate for effective tortuosity is unreallistically low indicating that the extra term may not be significant in this case.

CONCLUSION

A fast and efficient algorithm for deducing specific impedance of ground surfaces was used to obtain surface impedance of sand and snow covered grounds. A simple Gaussian least squares fitting was implemented to obtain estimates for the ground impedance parameters for these surfaces. It was shown that a modified two-parameter Attenborough model yields better agreement with data at higher frequencies.
Further work is required to resolve the large variation in estimated values of the rate of change of porosity as well as phase measurement errors at high the frequencies.

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