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# ON THE SOUND PRESSURE OF A QUADRUPOLE ABOVE AN IMPEDANCE GROUND

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## 1. INTRODUCTION

This paper is concerned with examining sound field from a quadrupole source near an impedance boundary. This problem is of some importance since noise generated by jet flow and general unsteady fluid flow are quadrupole in nature.[1-5] All the discussions so far have concentrated on the quadrupole field in an unbounded medium or near a rigid solid surface.

In this paper we discuss the sound field of an arbitrarily-oriented quadrupole above an impedance surface. We will derive the Green's function of the Hankel transform for a quadrupole. We will also derive an asymptotic expression for a quadrupole field above a rigid porous ground. We will compare the two models with superposition of two out of phase dipoles.

## 2. GREEN'S FUNCTION REPRESENTATION

Sound pressure due to a point monopole source can be written in an integral form as:

$$P(z,r)=\int \varphi_m(z,k_r)J_0(rk_r)k_r dk_r \qquad (1)$$

where $J_0()$ is the Bessel function of zero order and $\varphi_m$ is the range-independent Green's function. The form of the Green's function for a monopole source in a homogeneous medium above a boundary with an admittance of $\beta$ is well known:

$$\varphi_m = \frac{1}{k_z} e^{ik_z|z-h|} + \frac{k_z - k_0\beta}{k_z + k_0\beta} \times \frac{1}{k_z} e^{ik_z(z+h)} \tag{2}$$

where h is the source height, $k_z = +\sqrt{k_0^2 - k_r^2}$. Here the first term represents the direct wave and the second term the reflected wave which is the product of the image source term and the plane wave reflection coefficient. The pressure field for a dipole and a quadrupole field are defined as[6]

$$P_{dipole} = (\mathbf{d} \cdot \nabla) P_{monopole} \tag{3}$$

$$P_{quadrupole} = (\mathbf{m} \cdot \nabla)(\mathbf{d} \cdot \nabla) P_{monopole} \tag{4}$$

where $\mathbf{d}$ and $\mathbf{m}$, may be regarded as direction cosines of the dipole and the quadrupole axes respectively. A dipole can be thought of as superposition of two closely-spaced, out-of-phase monopoles. Similarly, a quadrupole is a superposition of two out-of phase dipoles. Depending on whether the axes of the two constituent dipoles are parallel or perpendicular to one another, the quadrupole can be a longitudinal or a transverse one. The axes of a quadrupole is defined to be parallel to the direction of its constituent dipoles for longitudinal quadrupole and normal to the dipole axes for transverse quadrupole.
Performing the differentiation on the direct wave term of the dipole field, the Greens function of a dipole source ($\varphi_{d1}$) is given as

$$\varphi_{d1} = (-i)\frac{e^{ik_z|z-h|}}{k_z}\{\mathrm{sign}(z-h) ik_z \cos\mu + k_r \sin\mu \cos\varepsilon\} \tag{5}$$

where $\mu$ and $\varepsilon$ are the polar and azimuthal angles defining the direction cosine vector, $\mathbf{d}$. The field of a quadrupole is then defined as

$$\varphi_{q1} = (\mathbf{m} \cdot \nabla)\varphi_{d1} \tag{6}$$

where $\mathbf{m} = (\sin\lambda\cos\gamma, \sin\lambda\sin\gamma, \cos\lambda)$, $\lambda$ and $\gamma$ being the polar and azimuthal angles of the quadrupole axis.
The explicit expression for $\varphi_q$ in cylindrical co-ordinates can be determined by substituting eqn. (5) into eqn. (6) to give:

$$\varphi_{q1} = \frac{e^{ik_z|z-h|}}{k_z}\{k_z^2 \cos\mu\cos\lambda - k_r^2 \sin\mu\cos\varepsilon\sin\lambda\cos\gamma$$
$$+ \mathrm{sign}(h-z) k_r k_z [\cos\mu\sin\lambda\cos\gamma - \sin\mu\cos\varepsilon\cos\lambda]\} \tag{7}$$

The quadrupole field can be divided into three components. The first term in curly brackets represents an axial vertical quadruple component, the second term represent an axial horizontal quadrupole and the third component is a radial quadrupole component. Therefore an arbitrary quadrupole is sum of these three components with appropriate scaling factors due to the orientation of the quadrupole. The reflected wave term is evaluated in a similar fashion resulting in an image source term multiplied by the plane wave reflection coefficient.

Once the Green's function is calculated for a given geometry, the integration can be carried out by the Fast Field Program (FFP) method. In FFP, the Bessel function in the integrand is approximated by its large argument asymptotic form. This transforms the Hankel integral to a pair of Fourier Integrals. These in turn are approximated by Discrete Fourier sums which can be evaluated very efficiently by Fast Fourier Transform (FFT) method. To avoid the pole in the integrand, the integration path is deformed and a correction term is added[7]. This ensures a stable solution free of the so-called Gibbs oscillations[7].

## 3. ASYMPTOTIC EXPRESSIONS FOR QUAD FIELD ABOVE AN IMPEDANCE GROUND

The integral representation of a quadrupole field derived in section 2 is normally evaluated numerically by FFT method or other numerical schemes[8]. In this section we will derive asymptotic forms for the quadrupole field above a rigid porous ground by differentiating the corresponding field for a dipole as described before and using known approximate forms for the resulting terms. Here we give explicitly the evaluation of the first component in the quadrupole Green's function which is the vertical longitudinal term.

The vertical longitudinal component of a quadrupole, ignoring the directional scaling, is:

$$P_{v,quad}=\int\left[\varphi_{q1}-\frac{k_z-k_0\beta}{k_z+k_0\beta}\times k_z^2\times\frac{e^{ik_z(z+h)}}{k_z}\right]J_0(rk_r)k_r dk_r \tag{8}$$

The reflected term can be rearranged to give

$$P_{v.quad}=\int\left\{-k_z^2\frac{e^{ik_z|z-h|}}{k_z}-k_z^2\frac{e^{ik_z(z+h)}}{k_z}+\right.$$
$$\left.\left[-2ik_0\beta(ik_0\cos\theta)-2k_0^2\beta^2+k_0^2\beta^2\frac{2\beta}{\cos\theta+\beta}\right]\frac{e^{ik_z(z+h)}}{k_z}\right\}J_0(rk_r)k_r dk_r \tag{9}$$

where $\cos\theta=k_z/k_0$ .is the, possibly complex, angle of incidence. It can be seen that the integrand consists of five terms. The first and second terms have well known closed form solutions and are identified as direct and image quadrupole source terms. The third term represents an image dipole source[9] multiplied by a constant coefficient *($2ik_0\beta$)*. The fourth term is an image monopole term with a coefficient *($2k^2\beta^2$)*. Finally, the last term represents the boundary wave term[10,11] which has a closed-form approximate solution. The resulting expression for the sound field of an longitudinal quadrupole positioned vertically above an impedance boundary is:

$$P_{v.quad} = \left\{ \left[ 3(1 - ik_0R_1) - k_0^2R_1^2 \right]\cos^2\theta_1 - \left[1 - ik_0R_1\right] \right\} \frac{e^{ik_0R_1}}{R_1^3} + \\ \left\{ \left[ 3(1 - ik_0R_2) - k_0^2R_2^2 \right]\cos^2\theta_2 - \left[1 - ik_0R_2\right] \right\} \frac{e^{ik_0R_2}}{R_2^3} + \\ 2ik_0\beta(1 - ik_0R_2)\frac{e^{ik_0R_2}}{R_2^2} \times \cos\theta_2 - 2k_0^2\beta^2\left[\cos\theta_2 + \beta F(w)\right]\frac{e^{ik_0R_2}}{R_2} \tag{10}$$

where $R_1$ and $R_2$ are the distance from the source and the image source to the receiver respectively and $\theta_1$ & $\theta_2$ are the polar angles of the lines joining the source and the image to the receiver. The term *F(w)* is known as the boundary loss factor, given by:

$$F(w) = 1 + i\sqrt{\pi}\, we^{-w^2}\, \mathrm{erfc}(-iw) \tag{11}$$

and *w* is the numerical distance defined by $w = \dfrac{+\sqrt{ik_0R_2/2}}{\cos\theta_2 + \beta}$. Care should be taken in calculating the numerical distance to ensure that the correct phase has been used[12]. The form for $w$ given above with the positive real root of the square root ensures that this is always the case. The other two components of the quadrupole can be evaluated in a similar fashion.

Li & Taherzadeh[13] have given a more rigorous derivation of the quadrupole field and arrived at the following compact form for sound field of an arbitrarily-oriented quadrupole:

$$P_{quad} = \frac{-k_0^2}{4\pi}(\mathbf{d}\cdot\hat{\mathbf{R}}_1)(\mathbf{m}\cdot\hat{\mathbf{R}}_1)\frac{e^{ik_0R_1}}{R_1} \\ + \frac{-k_0^2}{4\pi}\left\{ \left[ (\mathbf{d}\cdot\hat{\mathbf{R}}_2)(\mathbf{m}\cdot\hat{\mathbf{R}}_2)R_p + (\mathbf{d}\cdot\hat{\mathbf{R}}_s)(\mathbf{m}\cdot\hat{\mathbf{R}}_s)(1 - R_p)F(w) \right]\frac{e^{ik_0R_2}}{R_2} \right\} \\ + \frac{1}{4\pi}\left[ (\mathbf{d}\cdot\mathbf{m}) - 3(\mathbf{d}\cdot\hat{\mathbf{R}}_1)(\mathbf{m}\cdot\hat{\mathbf{R}}_1) \right] \times \frac{ik_0R_1 - 1}{R_1} \times \frac{e^{ik_0R_1}}{R_1} \\ + \frac{1}{4\pi}\left[ (\mathbf{d}\cdot\mathbf{m}) - 3(\mathbf{d}\cdot\hat{\mathbf{R}}_2)(\mathbf{m}\cdot\hat{\mathbf{R}}_2) \right] \times \frac{ik_0R_2 - 1}{R_2} \times \frac{e^{ik_0R_2}}{R_2} \tag{12}$$

where $R_p = \dfrac{\cos\theta_2 - \beta}{\cos\theta_2 + \beta}$ is the plane wave reflection coefficient, and the vector $\hat{\mathbf{R}}_s \equiv (\sin\mu_s\cos\varepsilon_s, \sin\mu_s\sin\varepsilon_s, \cos\mu_s)$, may be regarded as the unit vector that characterises the direction of the ground wave term. The complex angle $\mu_s$, is determined by $\cos\mu_s + \beta = 0$.

The vector quantities $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ represent unit vectors pointing radially outward from the quadrupole source and image source to the receiver.

## 4. DISCUSSION AND NUMERICAL EXAMPLES

In what follows, the admittance of the rigid porous ground is modelled by a two-parameter impedance model[14] given by:

$$\frac{1}{\beta} = 0.436(1+i)\sqrt{\frac{\sigma_e}{f}} + 19.48i\left(\frac{\alpha_e}{f}\right) \tag{13}$$

where $\sigma_e$ and $\alpha_e$ are the effective flow resistivity and effective rate of change of porosity with depth respectively. The values for these parameters, used in the following calculations, are 40 kPa s m$^{-2}$ and 20 m$^{-1}$ respectively and $f$ is the frequency.

The sound field from a longitudinal quadrupole positioned horizontally above an impedance plane is similar to that of a monopole source and attenuates at the same rate, except at short distances comparable to the source and receiver height above the surface. On the other hand, the same quadrupole placed vertically behaves differently.

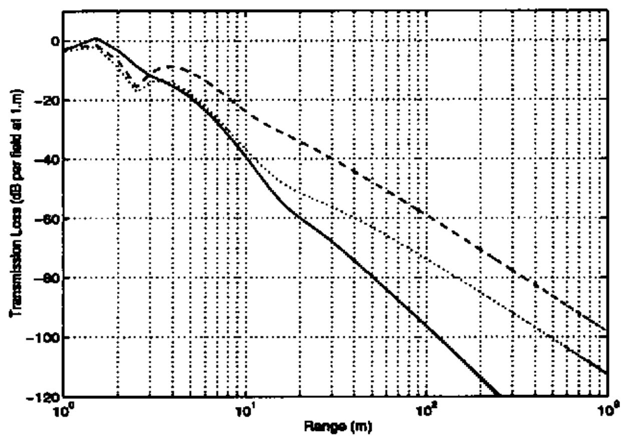


**Fig. 1 transmission Loss of a vertical quadrupole (solid line), vertical dipole (dotted line) and a monopole (dashed line). Frequency is 500 Hz, and source and receiver heights are 1.5 and 1.0m respectively.**

Figure 1 shows plot of Transmission Loss of a vertical longitudinal quadrupole field. The source and receiver heights are 1.5 and 1.0m respectively and the frequency is 500 Hz. Also plotted are the Transmission Loss of dipole and monopole sources. It indicates that a quadruple field attenuates very fast while the dipole source attenuation rate tends to that of a monopole at long range. The Transmission Loss values are calculated with reference to field at 1.m range. Figure 2 shows prediction of excess attenuation spectra of longitudinal quadrupole (solid liine), vertical dipole (dotted line) and monopole (dashed line) sources and receiver close to a porous ground 2.0m away. The source and receiver heights are 0.1 and 0.025m respectively.

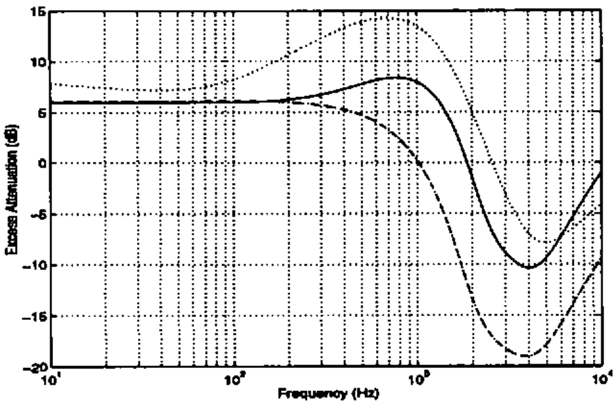


**Fig. 2 Excess Attenuation of sound due to a vertical quadrupole (sollid line), a vertical dipole (dotted line) and a monopole (dashed line) above an impedance ground. The source and receiver heights are 0.1m and 0.025m respectively and their separation is 2.0m.**

It can be seen, in this geometry, that the predicted ground wave contribution due to a longitudinal quadrupole is lower than that due to a vertical dipole but it is still higher than that of a monopole.

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