

# SOUND SCATTERING BY LARGE EDDIES

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## 1. INTRODUCTION

Current sound propagation modelling predominantly addresses refraction and surface effects, but with increasing interest in turbulent scattering into barrier shadow regions. Scattering by isotropic turbulence in the ubiquitous inertial sub-range can be included in propagation models in a number of ways. But more recently there has been interest in scattering by larger scale eddies. These have been included as scattering by turbulent elements in an extended turbulent energy spectrum. In this paper we describe observations of turbulent structure and wind profiles, using ground-based metrological stations and acoustic profilers, to address questions relating to the coherent nature of many large eddy systems, as opposed to the random nature of turbulent eddies. In particular, convection, buoyancy oscillations and some topographically-driven flows exhibit periodicities which can strongly drive fluctuations in sound intensity. We discuss modelling of these structures.

## 2. SCATTERING THEORY

Turbulent scattering occurs from incoherent fluctuations having scales  $\chi$  from the inner scale  $\lambda_i$  ( $\sim 0.01\text{m}$ ) to the outer scale  $L_o$  ( $\sim 100\text{m}$ ). For smaller scales the scattering is negligible, whereas for larger scales the structures tend to be coherent and not treatable by statistical theories. Scattering is very strongly associated with acoustic wavenumber

$$k = \frac{1}{\chi\sqrt{2(1-\mu)}} \quad (1)$$

where  $\mu$  is the cosine of the scattering angle (angle between forward direction and scattering direction). Turbulence theory gives for the scattering cross-section per unit volume of air, when scattering is into solid angle  $d\Omega$ , as

$$\sigma_s d\Omega = \frac{1}{8} k^{\frac{1}{3}} \frac{\mu^2}{[2(1-\mu)]^{\frac{11}{6}}} [\Gamma_T + (1+\mu)\Gamma_V] d\Omega \quad (2a)$$

where

$$\Gamma_T = 0.033 \frac{C_T^2}{T^2} \quad (2b)$$

and

$$\Gamma_V = 0.76 \frac{C_V^2}{2\pi c^2} \quad (2c)$$

Here  $C_T^2$  and  $C_V^2$  are measures of the strength of temperature  $T$  and velocity  $V$  fluctuations and are obtained from time averages over spatially-separated measurements

$$\begin{aligned} C_T^2 &= \langle [T(x+\Delta x) - T(x)]^2 \rangle (\Delta x)^{-\frac{2}{3}} \\ C_V^2 &= \langle [V(x+\Delta x) - V(x)]^2 \rangle (\Delta x)^{-\frac{2}{3}} \end{aligned} \quad (3)$$

In path length  $s$ , scattering reduces the acoustic intensity from  $I_0$  to

$$I = I_0 e^{-k_s s} \quad (4)$$

where

$$k_s = \int \sigma_s d\Omega = 2\pi \int_{-1}^{1-\delta} \sigma_s d\mu \quad (5)$$

and

$$\delta = \frac{1}{2k^2 L_0^2} \quad (6)$$

An example of the scattering pattern is given in Fig. 1.

### 3. $L_0$ EFFECTS

It can be seen from the above equations that scattering is dominantly forward, and the strength of the forward peak is very dependent on  $L_0$ , the outer scale. This outer scale varies with distance  $z$  from the surface, being essentially equal to  $z$  near the surface, and increasing to a larger value determined by the scale of obstacle or shear which is the generator for the turbulence. This means that turbulent scattering is less forward scattering near the ground.

One way of modelling this scattering is to consider the scattering phase function as composed of two parts. For smaller scales and wider scattering angles we have a smooth scattering pattern approximately  $\cos^2$  in shape and this “fills in” shadow regions behind barriers or in low intensity regions. Scattering from larger scale turbulence, at forward angles, is essentially beam spreading. Both scattering regimes are potentially very important: in the second case observed discrepancies when turbulence is ignored could be due to the spreading of acoustic energy. Some examples of this will be given in the presentation.

### 4. Coherent Structures

The difficulty with proper treatment of turbulent beam spreading effects is that for scattering from scales larger than  $L_0$  the conventional scattering theories (e.g. Tatarski, 1971; Ostashev et al., 1999; Wilson et al., 1999; Ostashev and Wilson 2000) no longer apply because the scattering is from *coherent structures* rather than statistically random ones. Volume integrations are therefore quite different.

This transition from turbulence to organized wind structures is problematic. However, some insight can be gained through comparison between *scattering* from a large scale and *refraction* from a comparable scale coherent structure.

Typical coherent structures are gravity waves, convective plumes, and rotors. These structures have in common spatially periodic updrafts and downdrafts accompanied by periodic horizontal velocity component variations. The detail and spatial and temporal scales of course vary. However, as illustration, we consider a circulation which satisfies the equations of motion and is described by

$$\begin{aligned} u &= U \cos\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi z}{H}\right) \\ w &= U \frac{H}{L} \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi z}{H}\right) \end{aligned} \quad (7)$$

where  $(x,z)$  are spatial coordinates and  $(u,w)$  the corresponding velocity components. Quantities  $U$ ,  $H$ , and  $L$  are constants determining scales and intensities of motion. This circulation is a simple ground-based rotor, of ellipsoidal shape determined by  $H$  and  $L$ , and has similarities to sea-breeze circulations (although without the Coriolis forcing).

## 5. Discussion

In the presentation we consider simple ray tracing through this coherent structure, with  $U$  both positive and negative, and show that the results of surface acoustic intensity patterns are very different from those which would be obtained assuming turbulent scattering from similar scales.

This example in itself does not solve the problem of dealing with the transition region between large turbulent scales and coherent structures, but it does serve to show how important this problem is.

Also, we highlight the dangers of introducing simplistic turbulent scattering into existing propagation codes. For example, one method which has been reported as a method of including turbulence in a propagation model, is to add random fluctuations to the wind profiles. This approach could potentially work, but it is very important that the random scales be introduced with proper attention to coherency, and that all scales be represented. If this is not done properly, then volume integrals will not give the correct scattering patterns and the result may be somewhat between that of turbulent scattering and coherent structure refraction.

Finally, we would like to emphasise the desirability of meteorological measurements at scales encompassing the outer scale. This is not trivial to do: the Salford SODAR measures some quantities at much smaller scales and other quantities at larger scales, and balloon measurements are also generally inadequate.

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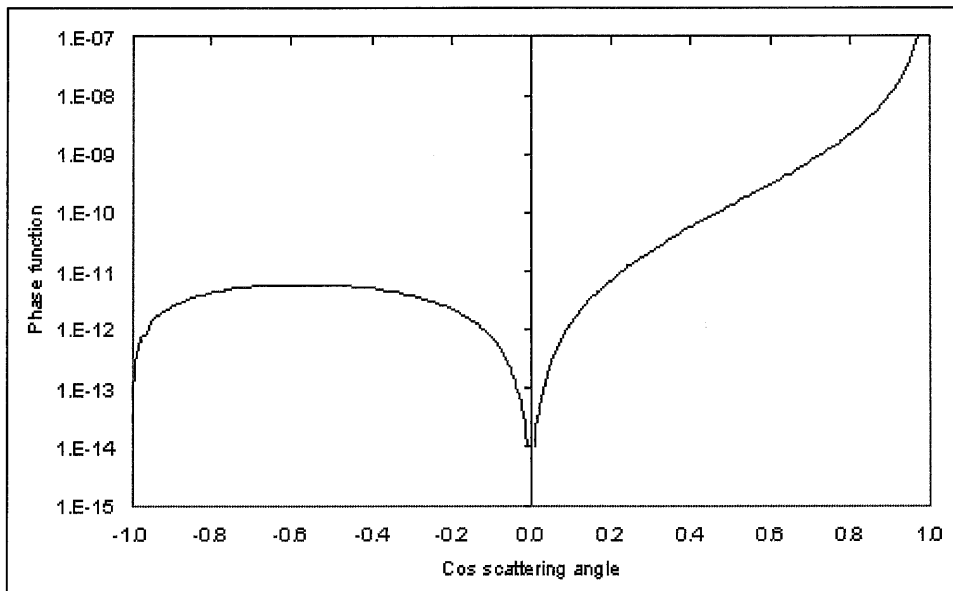


Figure 1. Turbulent scattering phase function for  $L_0=100$  m,  $C_T^2=10^{-5} \text{ m}^{-2/3}$ , and  $C_V^2=0.01$ .