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INDIRECT MEASUREMENT OF VIBRATIONAL ENERGY FLOW BY RECIPROCAL METHODS

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1. INTRODUCTION

The description of structure-borne sound transmission between vibrating machines and supporting structures is complicated because all contacts and all components of vibration can contribute to the overall emission. It is often assumed that only vertical forces contribute to the total emission [5] but there is growing evidence that other components, such as moments, become important at high frequencies [2] and when the installation is close to a structure discontinuities such as a floor edges [4].

There remains a need to consider the full transmission process in order to establish a hierarchy of paths and thence, by elimination of least influential components, simplify the analysis within engineering accuracy.

It is difficult to measure the translational forces at the supports directly without disturbing the installed condition. In addition, there are no reported methods of registering moment directly. However the generalised forces can be obtained indirectly by reciprocal methods. This paper describes laboratory measurement of forces beneath an installed fan unit and the practical problems encountered.

2. MEASUREMENT PRINCIPLE

First, consider a simplest case where a machine is connected to structure only by a single mounting point at x_e where the machine can only excite a vertical translational force F_e during operation. Under the action of this force the response velocity at x_e is v_e. Following this, the power transmitted via this mounting point is [1],

$$P = \frac{1}{2}\mathrm{Re}(F_e v_e^*) \qquad \cdots\cdots\cdots\cdots(1)$$

Therefore power can be calculated directly if the cross spectrum of force and velocity at point x_e is known. In practice, direct measurement of force at the mounts is difficult and often impossible. If equation (1) is rearranged, it gives :

$$P = \frac{1}{2}\mathrm{Re}(F_e v_e^*) = \frac{1}{2}\mathrm{Re}(\frac{F_r v_e^* v_r}{v_r}) = \frac{1}{2}\mathrm{Re}(\frac{v_e^* v_r}{Y_{v_r F_e}}) \qquad \cdots\cdots\cdots(2)$$

where v_r is the response velocity of the supporting plate due to excitation force F_e at point x_e measured at an arbitrary remote point r when the machine is in operation. The ratio $Y_{v_r F_e}$ is termed the transfer mobility from x_e to x_r. By principle of reciprocity [6], $Y_{v_r F_e} = Y_{v_e F_r}$ and equation (2) can be written as:

$$P = \frac{1}{2}\mathrm{Re}(\frac{v_e^* v_r}{Y_{v_e F_r}}) \qquad \cdots\cdots\cdots\cdots(3)$$

The numerator in equation (3) is obtained from the cross spectrum of v_e and v_r. The reciprocal of transfer mobility from x_e to point x_r, $Y_{v_e F_r}$, can be measured without much difficulty. Thus this arrangement converts the problem of direct force measurement to a simpler floor transfer mobility measurement.

With the machine idle but still on the structure, a force F_r is applied at the remote point x_r to obtain the transfer function between the two points, $\tilde{Y}_{v_e F_r}$. If $Y_{v_e F_r}$ can be replaced by $\tilde{Y}_{v_e F_r}$, then the power imparted by the machine to the structure can be calculated without removing the existing installed machine. The difference between $Y_{v_e F_r}$ and $\tilde{Y}_{v_e F_r}$ is that the former is measured without machine loading and thus x_e is under the condition of free movement. Whereas for the latter case, mobility is obtained when point x_e is constrained by the loading of machine, for light-weight machines on heavy floors, both values are approximately equal. In practice heavy weight machines are usually resiliently mounted and therefore the effect of dynamic loading is negligible due to large mobility mismatch. This is an important requirement for characterisation of the machine as a structure-borne source from measurements in the installed condition.[3].

3. EXPERIMENTAL RESULTS

Measurements were carried out on a 120mm thick concrete floor structure. A force transducer was inserted directly below the footing of a centrifugal fan to obtain the force spectrum for comparison with the indirectly measured force in the later stage. Initial measurements were of vertical forces only and vibrational force from the fan was imparted into the structure via a slender driving rod to eliminate moment excitation. In mobility measurement, the structure was excited by impulse hammer method and the response velocity was acquired by mean of accelerometers.

For realistic examination of the combined effect of machine loading and the reciprocal measurement on the transfer mobility, note that $Y_{v_r F_e}$ is the transfer mobility due to the excitation at the mounting point when the structure was unloaded whilst $\tilde{Y}_{v_e F_r}$ is the reciprocal when machine is rigidly bolted to the structure. In Figure 1 is shown the spectrum of $Y_{v_r F_e}$ and $\tilde{Y}_{v_e F_r}$. The deviation is within ±3dB, except in the region between 1040 and 1336Hz which is possibly due to the excitation at a nodal point on the plate.

The power transmitted into the structure is calculated using equation (1) and (3) from the directly and indirectly obtained forces respectively. It can be seen in Figure 2 that the agreement is in general within ±5dB.

4. MULTIPLE DEGREES OF FREEDOM

The above method can be expanded to the problem of two degrees of freedom at a single mounting point. Consider a translational and rotational component at one point. Power is expressed as:

$$P = \frac{1}{2}\mathrm{Re}\left(F_e v_e^* + M_e w_e^*\right) = \frac{1}{2}\mathrm{Re}\left(\begin{bmatrix} v_e^* & w_e^* \end{bmatrix}\begin{pmatrix} F_e \\ M_e \end{pmatrix}\right) \qquad \cdots\cdots\cdots\cdots(4)$$

F_e and M_e are respectively the force and moment excited at the mounting point by the machine. The translational and rotational velocities, v_e and w_e at the mounting point are the results of force and moment contribution. Again, two remote points x_{r_1} and x_{r_2} are selected for measurement of their translational velocities v_{r_1} and v_{r_2} when machine is in operation. It follows that:

$$\begin{pmatrix} F_e \\ M_e \end{pmatrix} = \begin{bmatrix} Y_{v_{r_1}F_e} & Y_{v_{r_1}M_e} \\ Y_{v_{r_2}F_e} & Y_{v_{r_2}M_e} \end{bmatrix}^{-1} \begin{pmatrix} v_{r_1} \\ v_{r_2} \end{pmatrix}$$

substitute into equation (4),

$$P = \frac{1}{2}\mathrm{Re}\left(\begin{pmatrix} v_e^* & w_e^* \end{pmatrix}\begin{bmatrix} Y_{v_{r_1}F_e} & Y_{v_{r_1}M_e} \\ Y_{v_{r_2}F_e} & Y_{v_{r_2}M_e} \end{bmatrix}^{-1} \begin{pmatrix} v_{r_1} \\ v_{r_2} \end{pmatrix}\right) \qquad \cdots\cdots\cdots\cdots(5)$$

Consider reciprocity relationships yield $Y_{v_{r_1}F_e} = Y_{v_e F_{r_1}} = \ddot{Y}_{v_e F_{r_1}}$ and etc., where the superscript $(\cdot\cdot)$ represents the same conditions described in section 2. In order to obtain phase between the velocities, x_{r_1} is selected as the reference point here. With the machine in operation, the transfer function between v_{r_1} and all others point are measured:

$$v_e^* = \varphi^*(v_{r_1}, v_e)\, v_{r_1}^* \quad ; \quad w_e^* = \varphi^*(v_{r_1}, w_e)\, v_{r_1}^* \quad \text{and} \quad v_{r_2}^* = \varphi^*(v_{r_1}, v_{r_2})\, v_{r_1}^*$$

where φ is the transfer or cross transfer functions between velocities and angular velocities of points indicated in the parentheses and asterisk denotes conjugate. From equation (5),

$$P = \frac{1}{2}\mathrm{Re}\left(|v_{r_1}|^2\left(\varphi^*(v_{r_1}, v_e) \quad \varphi^*(v_{r_1}, w_e)\right)\begin{bmatrix} Y_{v_{r_1}F_e} & Y_{v_{r_1}M_e} \\ Y_{v_{r_2}F_e} & Y_{v_{r_2}M_e} \end{bmatrix}^{-1}\begin{pmatrix} 1 \\ \varphi(v_{r_1}, v_{r_2}) \end{pmatrix}\right)$$

The mobilities required in the above matrix are not difficult to measure ; the main point is that direct measurement of moment mobility is avoided.

If the above conditions is expanded to a more common practical case where there are n number of contact points and each has two components, then the number of remote points required will be 2 x n. As before, choose point 1 as the reference:

$$P = \frac{1}{2}\mathrm{Re}\left(|v_{r_1}|^2\left(\varphi^*(v_{r_1}, v_{e1}), \varphi^*(v_{r_1}, w_{e1}) \ldots \varphi^*(v_{r_1}, v_{en}), \varphi^*(v_{r_1}, w_{en})\right)\left[\Upsilon\right]^{-1}_{2n \times 2n}\begin{pmatrix} 1 \\ \varphi(v_{r_1}, v_{r_2}) \\ \vdots \\ \varphi(v_{r_1}, v_{r2n}) \end{pmatrix}\right)$$

In principle six degrees of freedom can be considered for each point and this will require 6 x n remote points.

5. CONCLUDING REMARKS

The reciprocity principle when applied in conjunction with the simple impulse hammer method, allows force at a machine mounting foot to be measured indirectly with an deviation of less than ±5dB. In the absence of a direct method for measurement of moment, this method is particularly useful to quantify the contribution from moment excitation.

It remains to explore the practicalities of the method by considering vertical forces at multiple supports. This, again will allow comparison of direct and indirect measurements before considering moments where such comparisons are not possible.

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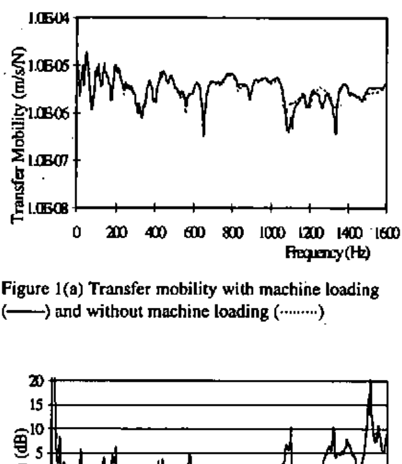

Figure 1(a) Transfer mobility with machine loading (——) and without machine loading (·········)

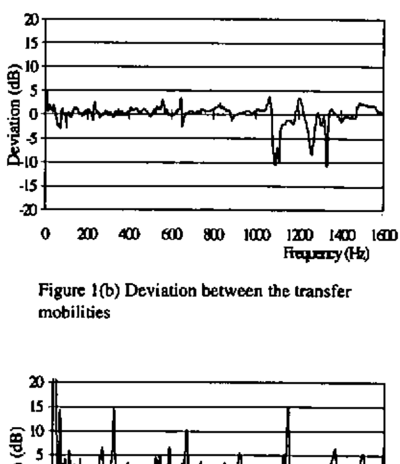

Figure 1(b) Deviation between the transfer mobilities

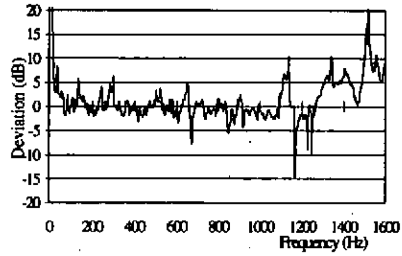

Figure 2(a) Deviation between directly and indirectly measured force

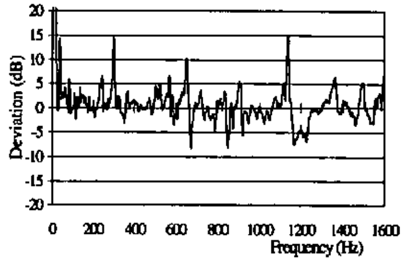

Figure 2(b) Deviation between powers