FEASIBILITY STUDY FOR THE LABORATORY SIMULATION OF RANDOM PRESSURE FIELDS

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1. INTRODUCTION

In many aeronautical applications, there is a considerable interest in reproducing experimentally the wall-pressure field due to a fully-developed Turbulent Boundary Layer (TBL). Such an excitation is a random process stationary both in space and time and is a significant source of noise within the cabin of a high-speed jet aircraft under typical cruise conditions. Flight tests measurements performed by Bhat and Wilby [1] on a commercial airplane at Mach 0.85 have shown that the TBL wall-pressure fluctuations efficiently excite the fuselage vibrations above about 500 Hz and up to 2 kHz. Relevant experimental studies performed over a broad frequency range are required in order to reduce the flow-induced noise within the cabin of jet aircrafts. Since representative full-scale testing set up to measure the response of aircraft structures to TBL pressure fluctuations is a very expensive and time-consuming process, an alternative would be to reproduce a pressure field with the same statistical properties as a TBL using an array of loudspeakers in a laboratory. It is one of the objectives of this study to consider the feasibility of simulating in a laboratory a TBL pressure field with a given spatial correlation structure.

Another application is concerned with the laboratory simulation of a diffuse acoustic pressure field, as the one that excites an aircraft structure tested in a sound transmission suite in order to determine its transmission properties. In this testing environment, the pressure field exciting the structure approximates a diffuse acoustic field which can be modelled as a random process stationary both in space and time [2, 3]. Although the sound field observed in reverberant rooms can be reasonably approximated at high frequency by a large number of independent plane waves uniformly distributed among propagation directions, phase relationships between these plane waves have to be satisfied near the walls of the room [3]. Moreover, below about the Schroeder's frequency, the acoustic response of the room starts to exhibit a modal behaviour and the diffuse approximation is no more valid. It is therefore of interest to investigate in the low-frequency domain the feasibility of simulating a wall-pressure field having the same spatial correlation function as an acoustic diffuse field and using an array of loudspeakers in the laboratory. This simulation technique would allow to perform transmission testing of structures over a broad frequency range without being limited in the low-frequency domain by the poor reverberant characteristics of the testing room.

Although a large part of the literature to date has been focussed on the active control of tonal and stochastic disturbance, surprisingly little attention has been devoted to the use of signal processing techniques in order to assess the accuracy to which the statistics of random pressure fields can be well reproduced. Indeed, both problems are very similar in terms of the signal processing methods that can be used. However, some relevant studies have been pointed out. Previously, Fahy [4] has considered the problem of simulating TBL pressure fluctuations using jet noise, a siren tunnel or a single loudspeaker, but concluded that none of these could reproduce the decay characteristics or lateral correlation properties of a TBL. Although a brief discussion is included in this paper about the use of an array of shakers, the electrical and mechanical difficulties of implementing such a system were considered, in 1966, to be such that it would be a difficult practical proposition.

Robert and Sabot [5] have taken another approach to the laboratory simulation of TBL excited structures, by noting that the modes of a structure are excited almost independently by such an excitation, and that there is generally a relatively small number of significantly-excited structural modes. They thus considered the use of an array of shakers acting on the panel driven so that each structural mode was excited independently by uncorrelated random signals. This approach assumes that the mode shapes of the structure are accurately known and so in practice some preliminary modal analysis would have to be performed before the drive signals to the shakers could be determined.

The TBL simulation could be made completely independent of the properties of the structure if a sufficiently dense array of loudspeakers were used to reproduce the spatial correlation properties of the TBL pressure field. The price that must be paid for such a general approach, however, is the large number of loudspeakers required for an accurate simulation, particular at high frequencies, as we shall see below.

This paper considers the feasibility of generating a random pressure field having the same spatial correlation function as either an acoustic diffuse field or a TBL pressure field using an array of actuators for laboratory simulations. In Section 2, models for the correlation structure assumed for an acoustic diffuse field excitation and a TBL excitation are first presented. In Sections 3 and 4, criteria are derived on the minimum number of independent components required for the generation of an acoustic diffuse field and a TBL pressure field, respectively. Section 5 focuses on convergence issues for the simulation of a TBL pressure field when using a set of independent sources. Sections 6 and 7 present some new results which demonstrate the acoustic effects of a loudspeaker array in order to simulate respectively an acoustic diffuse field and a TBL pressure field. In these sections, criteria are also derived to determine the number of independent loudspeakers required to reproduce the correlation structure of the random pressure fields at a given accuracy.

2. MODELS FOR THE GENERATED RANDOM PRESSURE FIELDS

The way in which a structure is excited and subsequently radiates sound can be very different when excited by a TBL pressure field and an acoustic diffuse field. This can be illustrated by the results of simulations, shown in Figure 1, from recent theoretical work by Maury *et al.* [6]. This graphs shows the power spectral density of the sound power radiated on one side of a tensioned aluminium panel when subject to either a pressure field whose correlation structure corresponds to a TBL, or a diffuse acoustic field. The sound power radiated by the panel and represented in Figure 1 has been normalised by the power spectrum of the excitation so that the differences of 10 to 20 dB in the levels of the radiated pressure are due to the different correlation structures of the two forms of excitation, and therefore the way they couple with the panel structural modes. The correlation structures assumed for the acoustic diffuse field excitation and the TBL excitation are detailed below.

2.1 Spatial Correlation Function of an Acoustic Diffuse Field

The spatial correlation structure of the acoustic diffuse field [2, 3] takes the form:

$$S_{AB}(r;\omega) = S_{pp}(\omega)\,\frac{\sin k_0 r}{k_0 r}\,, \qquad (1)$$

where r is the distance between the two measurement points, A and B, in any direction and k_0 is the acoustic wavenumber given by $k_0 = \omega/c_0 = 2\pi/\lambda_0$, where c_0 is the speed of sound and λ_0 is the acoustic wavelength. A typical correlation function of the acoustic diffuse field is plotted in Figure 2.

2.2 Spatial Correlation Function of a TBL Pressure Field

The correlation structure assumed for the TBL excitation was that of the Corcos model [7], so that the cross spectral density of the pressure at two points, A and B, which are a distance r_x apart in the spanwise direction and r_y apart in the streamwise direction is given by

$$S_{AB}(r_x, r_y; \omega) = S_{pp}(\omega)\, e^{-|r_x|/L_x}\, e^{-|r_y|/L_y}\, e^{-j\omega r_y/U_c}, \qquad (2)$$

where $S_{pp}(\omega)$ is the power spectral density at any point

L_x is the correlation length in the spanwise direction

L_y is the correlation length in the streamwise direction

and U_c is the convection velocity.

In the simulations used to produce Figure 1, the correlation lengths were assumed to be inversely proportional to frequency, as suggested by Corcos, and have the form

$$L_x = \frac{\alpha_x U_c}{\omega}, \quad L_y = \frac{\alpha_y U_c}{\omega}, \qquad (3)$$

where α_x and α_y are constants which were taken to be 1.2 and 8 in the simulations below, and the convection velocity was assumed to be 135ms^{-1}, which corresponds to a free stream velocity of about 225ms^{-1} and a Mach number of M $\approx$ 0.72. A typical correlation function of the TBL pressure field is plotted in Figure 3.

In the next sections the random pressure field is specified in terms of the spectral density matrix S_{dd} of the N_S outputs d of a sensor array, which are microphones in this case. The matrix S_{dd} has been calculated from the models described above. First, a reduced rank approximation [8] for the spectral density matrix of the microphone array has been considered. If this matrix can be reproduced to acceptable accuracy with a model of rank N_R, if the number of actuators is also equal to N_R and if we assume for the time being that the "plant" matrix G between the sensor outputs $\hat{d}$ and the actuator inputs has a perfect pseudo-inverse, $G^\dagger$, such that $G\,G^\dagger \approx I$, then N_R also defines the lower bound on the number of actuators required to reproduce the random pressure fields to the specified accuracy. In order to quantify the accuracy of the reduced rank approximation, a normalised mean-square error has been defined as the ratio of the resulting mean-square value of the errors between d and $\hat{d}$, $J_{N_S} - J_{N_R}$, to the mean-square value of d, J_{N_S}, and is equal to:

$$\frac{J_{N_s} - J_{N_R}}{J_{N_s}} = \frac{\sum_{i=N_R+1}^{N_s} \lambda_i}{\sum_{i=1}^{N_s} \lambda_i} \qquad (4)$$

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In practice, of course, it may not be possible to realise such a set of actuators, and when practical actuators are used, a larger number may be required. This latter case has also been explored by considering acoustic actuators. The reduced rank approximation of the spectral density matrix does, however, provide a theoretical lower bound on the required number of actuators to achieve a given accuracy of simulation. Criteria on the number of independent actuators required to achieve a given accuracy are now discussed for the two kinds of random pressure field considered in this Section.

3. CRITERIA ON THE NUMBER OF INDEPENDENT COMPONENTS FOR THE GENERATION OF AN ACOUSTIC DIFFUSE FIELD

The spectral density matrix for the pressure in an acoustic diffuse field has been calculated at a single frequency using the model described in paragraph 2.1 for the individual power and cross spectral densities. A linear array of 100 microphones was initially used, which was evenly spaced over a distance of 10 acoustic wavelengths. Figure 2 shows the cross spectral density between a microphone near the centre of the array and all the others. The cross-spectral densities are entirely real in this case and the power spectral density has been normalised to unity. The spectral density matrix has been calculated using equation (1).

A theoretical formulation for the normalised mean-square error can be obtained by integrating the spatial Fourier transform of the correlation function (1) over a finite range of $k_x \lambda_0$, to give:

$$\left.\frac{J_\infty - J_{N_R}}{J_\infty}\right|_{th} (N_R) = 1 - \frac{N_R}{N_\lambda}, \quad N_R \le N_\lambda$$
$$= 0, \quad \text{otherwise}, \qquad (5)$$

where N_λ is the number of acoustic wavelengths in the analysis window and k_x is the dual variable of the separation distance after Fourier transform. Figure 4 shows the eigenvalues of the matrix S_{dd} generated using the simulation above together with the theoretical prediction. From Figure 4, it can be observed that the amplitudes of the successive eigenvalues decay rapidly after the 10 first eigenvalues, in accordance with the theoretical predictions. And so, it can be seen that only a finite number of independent sources are required to achieve almost perfect reproduction of an acoustic diffuse field.

The number of eigenvalues corresponds to the number of independent sources used in the acoustic diffuse field simulation and in Figure 5 this has been divided by the number of acoustic wavelengths in the simulation, to give a graph of the normalised mean-square error against the number of independent sources per acoustic wavelength, and again compared with the theoretical prediction. This simple simulation predicts that only 1 independent source per acoustic wavelength is required to reduce the mean-square error by a factor of 10, i.e. –10 dB in Figure 5. Moreover, if a slightly higher number of independent sources per acoustic wavelength is considered, the normalised mean-square error drops down very quickly and so, it is interesting to note that for a diffuse acoustic field almost perfect reproduction of the correlation structure can, in principle, be achieved with at least 1 independent source per acoustic wavelength.

Figure 6 shows the spatial variation of the cross spectral density due to the original acoustic diffuse field model and the one due to the least squares approximation to the acoustic diffuse field having 0.8, 1 and 1.2 independent components per acoustic wavelength. With 1 independent component the spatial correlation structure is reproduced very well at all positions except near the edges. Larger deviations

are seen with 0.8 independent components whereas the spatial correlation structure reproduced with 1.2 independent components almost coincide with the original model.

Because the acoustic diffuse field can be almost perfectly reproduced with 1 independent source per acoustic wavelength, the total number of independent reference signals per square metres is thus:

$$N_{ref} = \frac{4}{\pi \lambda_0^2} \quad . \tag{6}$$

This result uses the property according to which the space correlation structure of a diffuse field has circular symmetry about the origin and has a sinc-like decay along a radius, with length $\lambda_0 /2$, of the circular correlation area. This formula predicts that the number of independent sources required to reproduce an acoustic diffuse field up to a frequency of 1kHz is about 12 per square metre. The dependence of N_{ref} on λ_0^2, however, means that about 1,210 independent sources per square metre would be required to reproduce the diffuse field up to a frequency of 10kHz, but such high frequency range is not of particular interest for the laboratory simulations we are concerned with. Indeed, as explained in the Introduction, the generation of an acoustic diffuse field with an array of actuators is only of interest in the very-low frequency range in order to compensate for the acoustic modal response of reverberant rooms for sound transmission measurements.

4. CRITERIA ON THE NUMBER OF INDEPENDENT COMPONENTS FOR THE GENERATION OF A TBL PRESSURE FIELD

The spectral density matrix for the TBL pressure field has been calculated at a single frequency using the model described in paragraph 2.2 for the individual power and cross-spectral densities. As in the previous section, a linear array of 100 microphones is initially used, which is evenly spaced over a distance of 10 correlation lengths. The results presented in the first part of this section are only related to the correlation function in the spanwise direction, and results related to the streamwise direction will be discussed at the end of this section. Figure 3 shows the cross spectral density between a microphone near the centre of the array and all the others. The cross-spectral densities are entirely real in this case and the power spectral density has been normalised to unity. The spectral density matrix has been calculated using equation (2).

A theoretical formulation of the normalised mean-square error, obtained by integrating the spatial Fourier transform of the correlation function (2) in the spanwise direction over a finite range of $k_x L_x$, is given by the following expression:

$$\left.\frac{J_\infty - J_{N_R}}{J_\infty}\right|_{th}(N_R) = 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{2\pi N_R}{N_{CL}}\right), \quad (7)$$

where N_R/N_{CL} is the number of independent sources per correlation length, which is equal to the maximum value of $k_x L_x/2\pi$, over which the integral is calculated. The theoretical prediction is plotted as the solid line in Figure 7 together with the eigenvalues of the matrix S_{dd} generated using the simulation above as discrete points (dots). From Figure 7, it can be observed that the amplitudes of the first few eigenvalues are significantly larger than the others. They are also almost unchanged if a larger

number of microphones are used in the simulation. We also observe that the first 30 eigenvalues follow very well the theoretical exponential decay, the discrepancy observed for the higher order eigenvalues is attributed to the finite number of independent sources taken into account.

Figure 8 shows the normalised mean square error associated with the assumed cross spectral density of the TBL pressure field between the center microphone and the other microphones, as defined by Eq. (7), and that associated with a reduced rank approximation to the spectral density matrix calculated from a 100 microphone array, as defined by Eq. (4). All these quantities are plotted as a function of the number of reference signals per correlation length. From Figure 8, it can be seen that in order to reduce the normalised mean-square error by a factor of 10, i.e. –10dB, the reduced-rank approximation predicts that 2 independent reference signals per correlation length are required whereas the theoretical model only predicts that 1 independent component per correlation lengths is sufficient to achieve this level of accuracy. This is due to the fact that the theoretical model assumes that the spatial correlation function which is defined over an infinite domain is known at every points of this domain, i.e. that we have an infinite number of microphones, whereas the simulation assumes that the correlation function is only define over 10 correlation lengths and is only known at a finite number of points (100 in our case). If the number of correlation lengths we aim to reproduce is increased, the agreement will get better for a larger number of independent sources per correlation lengths. These convergence properties will be examined in Section 5.

Although it is observed from Figures 7 and 8 that the first few eigenvalues or independent sources capture the dominant components of the random process, it is interesting to understand the influence of neglecting the less dominant eigenvalues on the spatial reproduction of the TBL correlation function. Figure 9 shows the spatial variation of the cross spectral density due to the original Corcos TBL model and the one due to the least squares approximation to the TBL pressure field having 1, 2 and 3 independent components per correlation length. With 2 independent components the spatial correlation structure is reproduced very well at all positions except near the origin, where it is about 10% too small. Larger deviations are seen with fewer independent components, and the peak value is about 20% too small with 1 independent source per correlation length. From these simulations, it can be seen that neglecting the less dominant components mainly affects the accuracy with which the peak value is reproduced, the exponential decay being essentially unchanged.

If a similar series of simulations are performed using data from a linear array of microphones in the streamwise direction, almost identical results are obtained once the number of independent sources is again normalised by the correlation length. Although the phase shift due to convection now has to be accounted for, it seems that a simple phasing of the independent sources can reproduce this aspect of the TBL pressure field without any additional problems. The required number of independent sources is thus again determined by the correlation length, which at a given frequency is, of course, longer in the streamwise direction that the spanwise direction and hence relatively fewer independent sources are needed in this direction. These simulations have also been performed in two dimensions, with a microphone array which extends in both the spanwise and streamwise directions. The total number of independent signals is found to be closely approximated by the product of the numbers required in each direction individually, as calculated from the one-dimensional simulations above.

If we assume that the TBL is sufficiently well reproduced if the mean-square error is reduced by 10dB, then 2 independent sources per correlation length are required in both the spanwise and the streamwise directions. The total number of independent reference signals per square metres is thus

$$N_{\mathrm{ref}} = \frac{4}{L_x L_y}. \tag{8}$$

These results only assume that the correlation structure is separable in the streamwise and spanwise directions and is exponentially decaying in these two directions. The number of independent sources required to reproduce a TBL pressure field to a given accuracy over a structure thus scales with the product of the two correlation lengths, but these will depend on frequency. The original Corcos model assumes that the correlation lengths have the form of equation (3), and using these expressions the number of independent reference signals per square metre is predicted to be

$$N_{\mathrm{ref}} = \frac{4\omega^2}{\alpha_x \alpha_y U_c^2} \approx 0.4\left(\frac{\omega}{U_c}\right)^2, \tag{9}$$

where the final expression has been obtained by assuming that α_x = 1.2 and α_y = 8. If U_∞ = 225ms^{-1} (M = 0.72) and so U_c = 135ms^{-1}, this formula predicts that the number of independent sources required to reproduce a TBL to this degree of accuracy up to a frequency of 1kHz is about 900 per square metre. The dependence of N on ω^2, however, means that about 90,000 independent sources per square metre would be required to reproduce the TBL up to a frequency of 10kHz.

5. CONVERGENCE ISSUES FOR THE GENERATION OF A TBL PRESSURE FIELD

We have noted in he previous section that if the number of correlation lengths we aim to reproduce is increased, the agreement between the theoretical prediction and the reduced-rank approximation of the spectral density matrix is likely to get better when considering a larger number of independent sources per correlation length. The convergence properties of the simulation of a TBL pressure field will be examined in this section when considering the independent components of the random field required to simulate a large number of correlation lengths over a large number of microphones. Further convergence results will be given in Section 7 when accounting for the acoustic effect of a loudspeaker array over the microphone domain.

In the following, the results will only be presented in the spanwise direction since similar results have been obtained in the streamwise direction. Figure 10 shows the number of independent components required for a 10dB reduction in the normalised mean-square error and related to the theoretical prediction (1 independent component per correlation length), to the reduced-rank approximation of the cross-spectral density matrix when considering a small number of correlation lengths and extrapolated to a larger number of them (2 independent components per correlation length) and to the reduced-rank approximation when using 100 microphones, 200 microphones and 400 microphones, plotted as a function of the number of correlation lengths we aim to reproduce in the analysis window. It can be seen that the criterion (8) initially established in order to simulate at a given accuracy a small number of correlation lengths is still valid to a certain extent when we aim at reproducing a larger number of correlation lengths, i.e. up to about 30, 50 and 70 correlation lengths to be generated respectively over 100, 200 and 400 microphones. In order to reproduce a large number of correlation lengths over a fixed number of microphones, it is shown that the required number of independent components necessary to achieve a 10dB reduction in the normalised mean-square error underestimates the criterion (8) and tends towards an asymptotic value which is a fraction of the total number of microphones used in the simulation (about 90 and 180 independent components respectively for 100 and 200 microphones). This is due to the limited number of microphones used in the simulations, and for a fixed number of correlation lengths to be reproduced over an increasing number of microphones, the required number of independent components for a 10 dB reduction error tends towards the criterion (8). Therefore, it

appears that the main parameters governing the convergence of the approximation are the assumed number of correlation lengths and the total number of microphones.

Figure 11 clearly shows the influence of an increased number of microphones on the relative mean-square error between the theory and the simulation when we aim to reproduce either 3 or 30 correlation lengths. It has been observed that a better convergence is observed when we aim to simulate a smaller number of correlation lengths since then the slope discontinuity at the origin of the assumed correlation function we aim to reproduce is much less important. However, in both cases, the limiting values of the relative mean-square error between the theory and the simulation do not tend towards zero when the total number of microphones increases. This is due to truncation effects since the theory assumes a spatial correlation function defined over an infinite domain whereas the correlation function we aim to reproduce in the simulation is only defined over a fixed number of correlation lengths. The larger is the number of correlation lengths we aim to simulate, the smaller is the highest wavenumber or eigenvalue component accounted for, and since it is the first independent components capturing most of the energy of the random process that are the less sensitive to an increase in the total number of microphones, we can understand why the error between the theory and the simulation converges faster towards a non-zero limit for a larger number of correlation lengths to simulate.

6. THE GENERATION OF AN ACOUSTIC DIFFUSE FIELD WITH AN ARRAY OF INDEPENDENT LOUDSPEAKERS

In this section, we consider an array of loudspeakers as one practical actuator arrangement which could be used to reproduce the correlation structure of an acoustic diffuse field. First, a series of computer simulations has been performed in which the loudspeakers have been modelled as acoustic monopole sources radiating over a rigid surface, and their number and height above the microphone array has been varied. Figure 12 shows the variation of the residual mean-square error as a function of the height of the loudspeaker array, normalised by the acoustic wavelength, for different numbers of loudspeakers in the array, N_L. These reductions are almost independent of the height of the loudspeaker array provided it is greater than about one quarter the acoustic wavelength and lower than one acoustic wavelength. The control filters are clearly able to compensate for the matrix of acoustic responses between the loudspeakers and microphones provided the loudspeakers are not too close, in which case some microphone outputs are overwhelmingly dominated by the nearfield responses of the adjacent loudspeakers. There may be numerical conditioning problems if the loudspeakers are too far from the microphones, however, and a reasonable height, *h*, appears to be about twice the separation distance between the loudspeakers. It can also be observed from Figure 12 that when using 12 loudspeakers in order to reproduce the spatial correlation function over 10 acoustic wavelengths, the residual mean-square error falls down below 10dB very rapidly as the distance from the loudspeakers to the microphones increases and already achieves its minimum value when the loudspeakers are half an acoustic wavelength apart from the microphones.

The number of loudspeakers required to achieve a given accuracy of an acoustic diffuse field simulation is now investigated. Figure 13 shows a graph of the normalised mean-square error obtained when using an array of loudspeakers positioned twice as far from a 100 microphones array as the distance they were apart from each other, plotted against the number of independent sources per acoustic wavelength. In Figure 13, the normalised mean-square error is compared with the reduced-rank approximation error given by Eq. (4), and again with the theoretical prediction given by Eq. (5). It can be
seen that the results with loudspeakers are very similar to those obtained by considering the minimum possible number of actuators, of any type, required to achieve a 10 dB reduction in the normalised mean-square error for the acoustic diffuse field simulation since, in this case, only 1 loudspeaker per acoustic wavelength is also required.

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From Figure 13, we note that in order to achieve further reduction in the mean-square error, the number of loudspeakers required per acoustic wavelength does not decay as rapidly as the minimum possible number of actuators. Figure 14 shows the assumed spatial correlation function of the acoustic diffuse field, that of the approximate pressure field generated using 0.8, 1 and 1.2 loudspeakers per acoustic wavelength, as a function of normalised separation, the distance from center microphone divided by acoustic wavelength. As it can be seen by comparing Figs. 6 and 14, below 10 dB reduction error, an excellent approximation of the spatial correlation function is already achieved in this configuration whatever the nature of independent sources used in the simulation.

7. THE GENERATION OF A TBL PRESSURE FIELD IN THE SPANWISE AND IN THE STREAMWISE DIRECTION WITH AN ARRAY OF INDEPENDENT LOUDSPEAKERS

Once again, a practical actuator arrangement which could be used to generate a pressure field that reproduces a TBL correlation structure is an array of loudspeakers. The number of loudspeakers required to achieve a given accuracy of TBL simulation in the spanwise and streamwise directions will be investigated by considering a one-dimensional array of N_L evenly spaced loudspeakers at a height h above the surface in which an array of 100 evenly spaced microphones are positioned. The loudspeakers array is assumed to be driven by the outputs of a matrix control filters, whose responses are adjusted to minimise the sum of the mean-square differences between the microphone outputs and those due to a corresponding TBL excitation, as discussed in [9].

As in the previous section, a series of computer simulations has first been performed in which the number of loudspeakers and their height above the microphone array has been varied. Figure 15 shows the variation of the residual mean-square error for simulations either in the spanwise and in the streamwise direction, as a function of the height of the loudspeaker array, normalised by the correlation length, for different numbers of loudspeakers in the array, N_L. With very few loudspeakers in the array and for heights which are small compared to the correlation length, the mean-square error is smaller in the spanwise direction that in the streamwise direction and very little attenuation in the mean-square error is achieved in this case. When larger number of loudspeakers are used, which achieve a more appreciable reduction in mean-square error, these reductions are, in both cases (spanwise and streamwise), almost independent of the height of the loudspeaker array provided it is greater than about one quarter the correlation length and lower than one correlation length. As in the previous section, the control filters are clearly able to compensate for the matrix G of acoustic responses between the loudspeakers and microphones provided the loudspeakers are not too close. Numerical conditioning problems have been observed if the loudspeakers are too far from the microphones. Indeed, the condition number of the matrix G^HG which has to be inverted to find the optimal solution reaches values up to 10^{18} for heights above one correlation length. However, a reasonable height appears to be about twice the separation distance between the loudspeakers. This justifies the loudspeakers arrangement used for the simulation results presented below.

Comparisons have been made in Figure 16 on the normalised mean-square error associated to the generation of a TBL pressure field either in the spanwise or in the streamwise direction, and for 3 and 10 correlation lengths to be reproduced over a 100 microphones array, as a function of the number of loudspeakers per correlation length. The loudspeakers have been positioned as described above, i.e. twice as far from the microphone array as the distance they were apart from each other. From the simulations performed in the spanwise direction in order to reproduce 3 and 10 correlation lengths, it is instructive to note that the results obtained for the minimum possible number of actuators, of any type, required to achieve a given error level in the TBL simulation, are very similar to the results obtained with an array of loudspeakers whatever the number of sources per correlation length.

In the streamwise direction, however, the two graphs are similar only if more than 10 sources per correlation length are used, in which case the normalised mean-square error is about –20dB and –35dB respectively for 3 and 10 correlation lengths to be reproduced over the 100 microphones domain and an

extremely good reproduction of the TBL correlation structure would be achieved. For smaller number of loudspeakers, it can be seen that the attenuations in mean-square error are much less than with the corresponding minimum number of actuators. In order to achieve a 10dB reduction in the error, for example, about 5 loudspeakers per correlation length are required, whereas we found in Section 4 that, in principle, only 2 sources per correlation length were required if these have the correct spatial response.

Assuming that the TBL is sufficiently well reproduced if the mean-square error is reduced by 10dB, then 2 loudspeakers per correlation length are sufficient to reproduce the TBL pressure field in the spanwise direction whereas 5 loudspeakers per correlation length are required in the streamwise direction. The total number of independent loudspeakers required per square metres is thus

$$N_{\text{louspeakers}} = \frac{10}{L_x\, L_y}. \qquad (11)$$

These results only assume that the correlation structure is separable in the streamwise and spanwise directions and is exponentially decaying in these two directions. The original Corcos model assumes that the correlation lengths have the form of equation (3), and using these expressions the number of loudspeakers per square metre is predicted to change with frequency as:

$$N_{\text{louds[eakers}} = \frac{10\omega^2}{\alpha_x\, \alpha_y\, U_c^{\,2}} \approx \left(\frac{\omega}{U_c}\right)^2 \qquad (12)$$

If again we assume that U_c = 135ms^{-1}, this formula predicts that about 23 loudspeakers per square metre would be required to reproduce the TBL pressure field with this accuracy up to 100Hz, whereas 2,300 loudspeakers per square metre would be required for reproduction up to 1kHz.

Simulations with an array of loudspeakers have also been performed over a large number of correlation lengths and with an increasing number of microphones. Figure 17 shows the number of independent components required for a 10dB reduction in the normalised mean-square error related to the reduced-rank approximation of the cross-spectral density matrix and to the approximate pressure field generated using an array of loudspeakers when using at most 100 microphones and 200 microphones, plotted as a function of the number of correlation lengths we aim to reproduce in the analysis window either in the spanwise or in the streamwise direction. By comparison with Figure 10, it can be seen that the criterion of 2 independent sources per correlation length, initially established for the minimum possible number of actuators, of any type, in order to simulate at a given accuracy a small number of correlation lengths in the spanwise direction, is still valid when we aim at reproducing with an array of loudspeakers an approximate pressure field with up to 20 correlation lengths over an array of 100 or 200 microphones. However, in order to generate an approximate pressure field in the streamwise direction with an array of loudspeakers, 5 loudspeakers per correlation length appears to be a criterion valid in order to simulate up to 16 correlation lengths in the analysis window and again becomes dependent on the total number of microphones that are used for a large number of correlation lengths to be reproduced.

8. CONCLUSIONS

Results have been presented that investigate the feasibility of simulating spatially random pressure fields when using an array of actuators driven independently. Results are presented for simulations over a large number of correlation lengths. The pressure field is specified in terms of the spectral density matrix of the outputs of an array of sensors which could be calculated from empirical models, either for an acoustic diffuse field or for a TBL pressure field.

This formulation allows a lower bound to be calculated on the possible number of actuators required, which is equal to the number of independent reference signals used in the simulation. For the acoustic diffuse field and for the Corcos TBL pressure field, this number scales with the correlation length, and for a 10% mean square error in the simulation, about 1 and 2 independent reference signals per correlation length are required for the Corcos TBL pressure field, this number is the same in both the streamwise and spanwise direction.

When simulations are performed with acoustic monopoles as sources to reproduce both types of random pressure fields, the number of sources again scales with the correlation length but now 5 sources per correlation length are required to achieve the same level of accuracy as above for the TBL pressure field in the streamwise direction, the criteria obtained for the minimum possible number of actuators are still valid when using an array of loudspeakers, in order to reproduce an acoustic diffuse field or a TBL pressure field in the spanwise direction. The empirical models we have considered predict that the number of sources required per unit area to reproduce the assumed model rises as the frequency is squared.

In the TBL case, if we aim to reproduce a larger number of correlation lengths, it has been shown that these criteria overestimate the required number of independent sources unless the number of sensors is increased. It also appears that the main parameters governing the convergence of the approximation of the TBL pressure field when using independent sources of any type, including loudspeakers, are the assumed number of correlation lengths and the total number of microphones. It has been shown that better convergence is observed when we aim to simulate a smaller number of correlation lengths since then the slope discontinuity at the origin of the assumed correlation function is much less important.

9. REFERENCES

[1] Bhat W. V. and Wilby J. F., "Interior Noise Radiated by an Airplane Fuselage Subjected to Turbulent Boundary Layer Excitation and Evaluation of Noise Reduction Treatments", Journal of Sound and Vibration, Vol. 18(4), 1971, 449-464.

[2] Cook R.K., Waterhouse R. V., Berendt R. D., Edelman S. and Thompson M. C. Jr., "Measurements of Correlation Coefficients in Reverberant Sound Fields", Journal of the Acoustical Society of America, Vol.27, 1955, 1072-1077.

[3] Pierce A. D., "Acoustics: an introduction to its Physical Principles and its Applications", McGraw-Hill Book Company, 1981.

[4] Fahy F. J., "On simulating the transmission through structures of noise from turbulent boundary layer pressure fluctuations", Journal of Sound and Vibration, Vol. 3, 1966, 57-81.

[5] Robert G. and Sabot J., "Use of random forces to simulate the vibroacoustic response of a plate excited by a hydrodynamic turbulent boundary layer", ASME Winter Meeting, 1984, Symposium on Flow-induced Vibrations, Vol. 5: Turbulence-induced noise and vibration of rigid and compliant surfaces (Conference code 06196).

[6] Maury C., Gardonio P. and Elliott S. J., "A Wavenumber Approach for the Response of Aircraft Sidewalls to Random Pressure Fluctuations", 17th International Congress on Acoustics, 2001, Vol. I: Vibrations and Structural Acoustics, Measurement Techniques in Structural Acoustics.

[7] Corcos G.M., "The resolution of turbulent pressures at the wall of a boundary layer", Journal of Sound and Vibration, Vol.6, 1967, 59-70.

[8] Elliott S.J. and Gardonio P., "Multichannel simulation of structural response to stochastic pressure fields", ISVR Technical Memorandum N° 848, 1999, University of Southampton.

[9] Elliott S.J., Gardonio P. and Maury C., "A feasibility study for the laboratory simulation of turbulent boundary layer pressure fields", Proceedings of the 7th AIAA/CEAS Aeroacoustics Conference, AIAA 2001-2194, 2001.

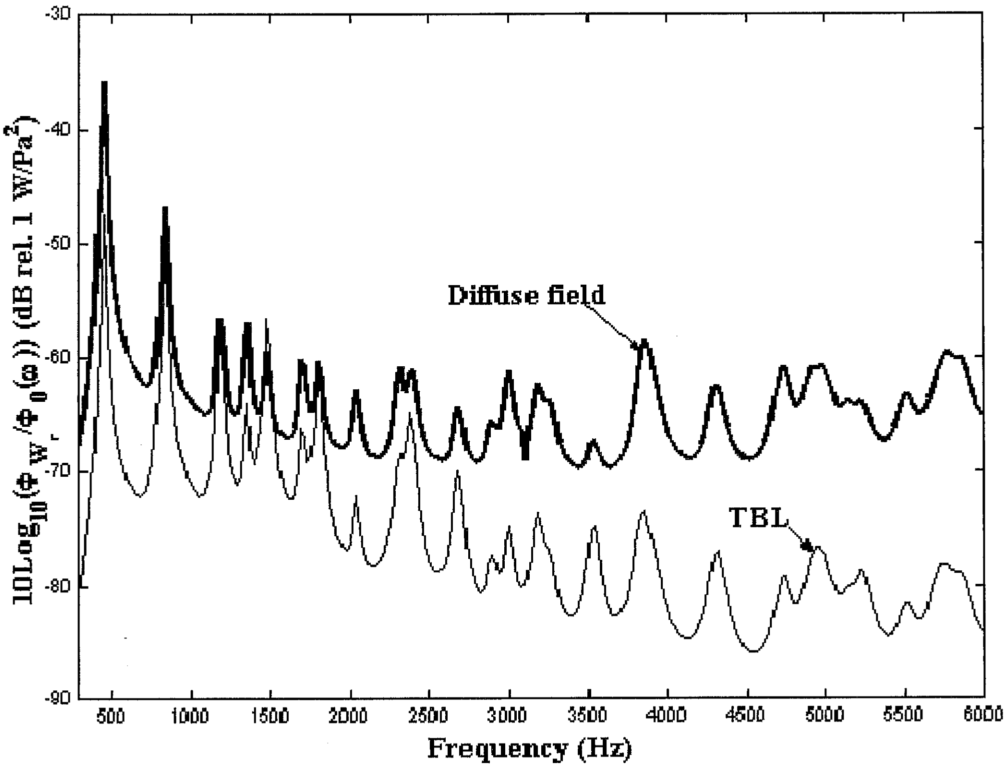

Fig. 1 The sound power inwardly radiated by a fuselage panel normalised by the excitation spectrum. Bold curve: acoustic diffuse excitation; thin curve: TBL excitation.

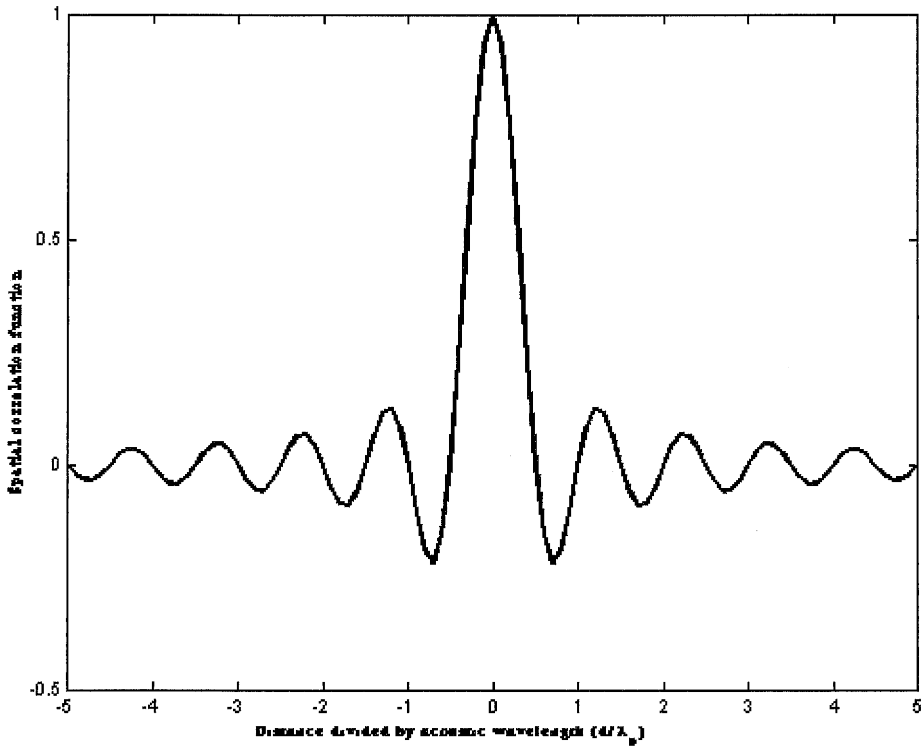

Fig. 2 Spatial correlation function of the acoustic diffuse field, plotted as a function of normalised separation, the distance from center microphone divided by acoustic wavelength.

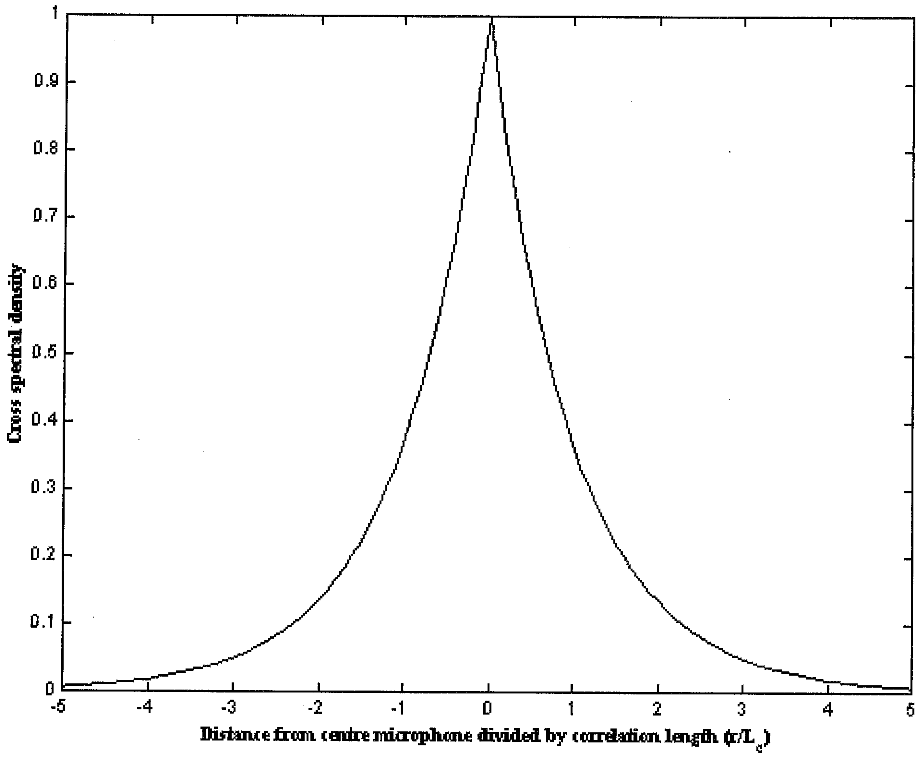

Fig. 3 Spatial correlation function over 10 correlation lengths in the spanwise direction of the Corcos model of the TBL pressure field, plotted as a function of normalised separation, the distance from centre microphone divided by correlation length

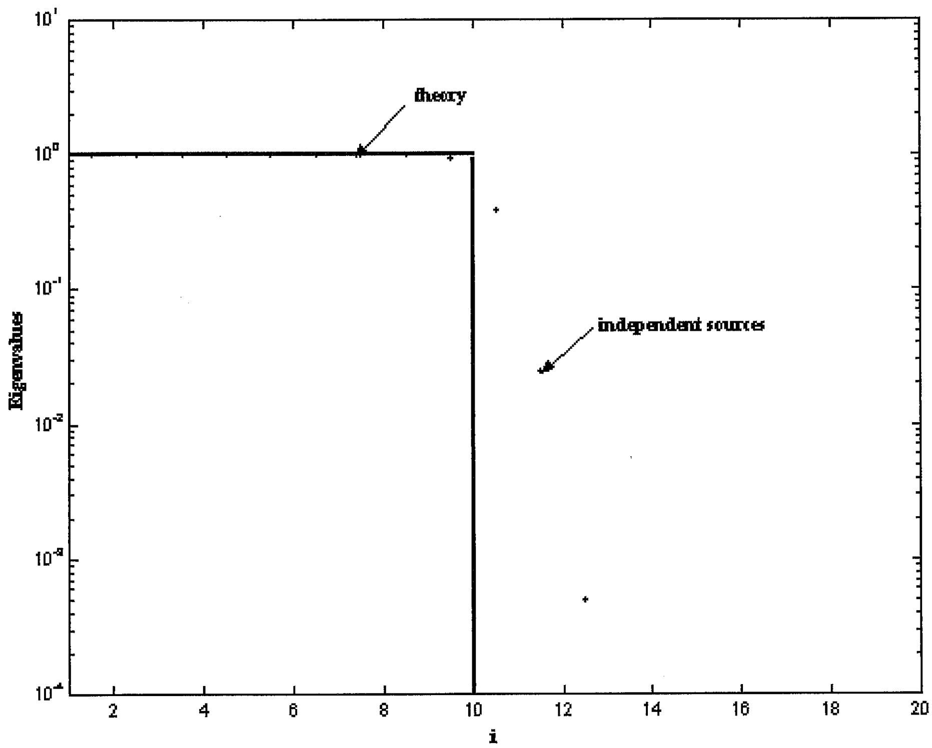

Fig. 4 The spatial Fourier transform of the assumed cross spectral density of the acoustic diffuse field between the center microphone and the other microphones (solid line) and the first eigenvalues of the spectral density matrix calculated for the 100 microphones (dots).

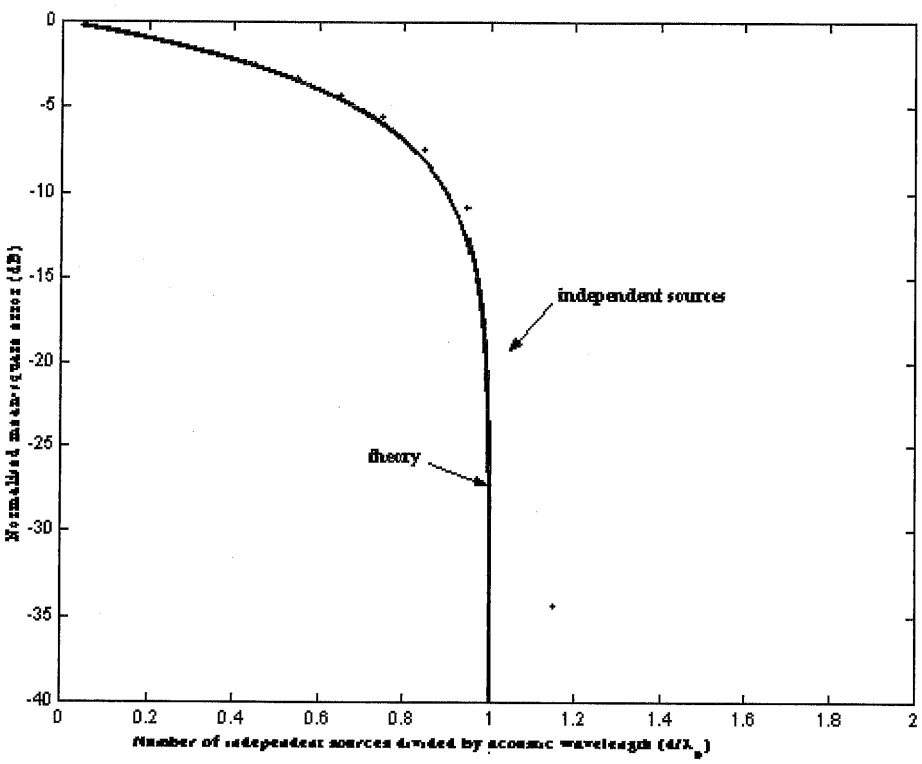

Fig. 5 The normalised mean square error associated with the assumed cross spectral density of the acoustic diffuse field between the center microphone and the other microphones (solid line) and that associated with a reduced rank approximation to the spectral density matrix calculated from a 100 microphone array, which is equal to the number of independent reference signals per acoustic wavelength used in the simulation (dots).

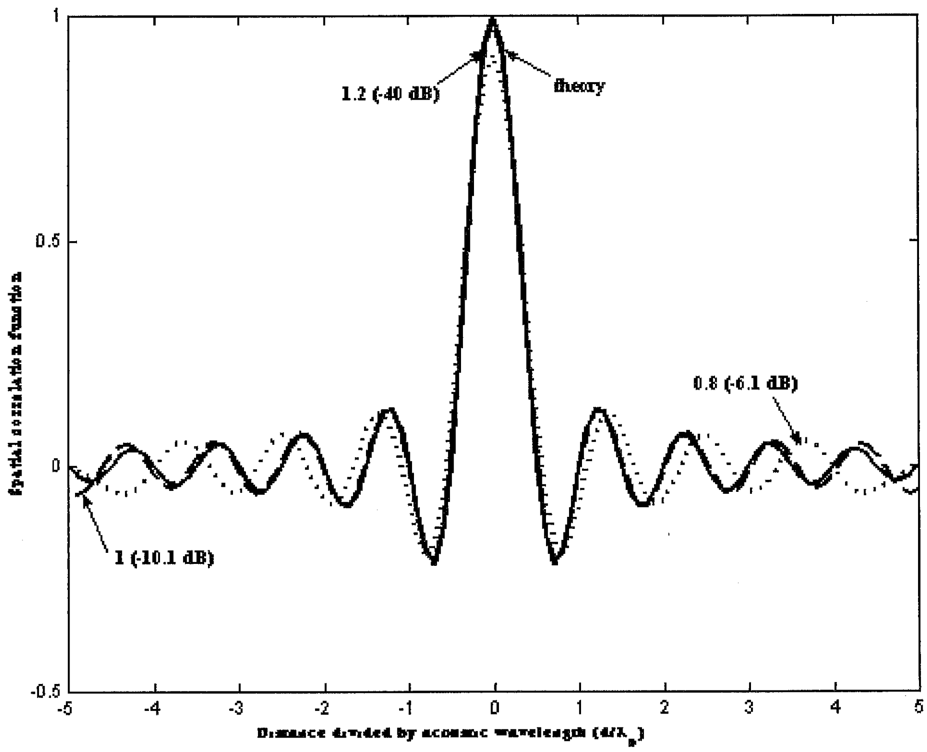

Fig. 6 The assumed spatial correlation function of the acoustic diffuse field (solid line), that of the reduced rank approximation using 0.8 reference signal per acoustic wavelength (dotted line), 1 reference signal per acoustic wavelength (dashed line) and 1.2 reference signal per acoustic

wavelength (thin solid line), plotted as a function of normalised separation, the distance from center microphone divided by acoustic wavelength.

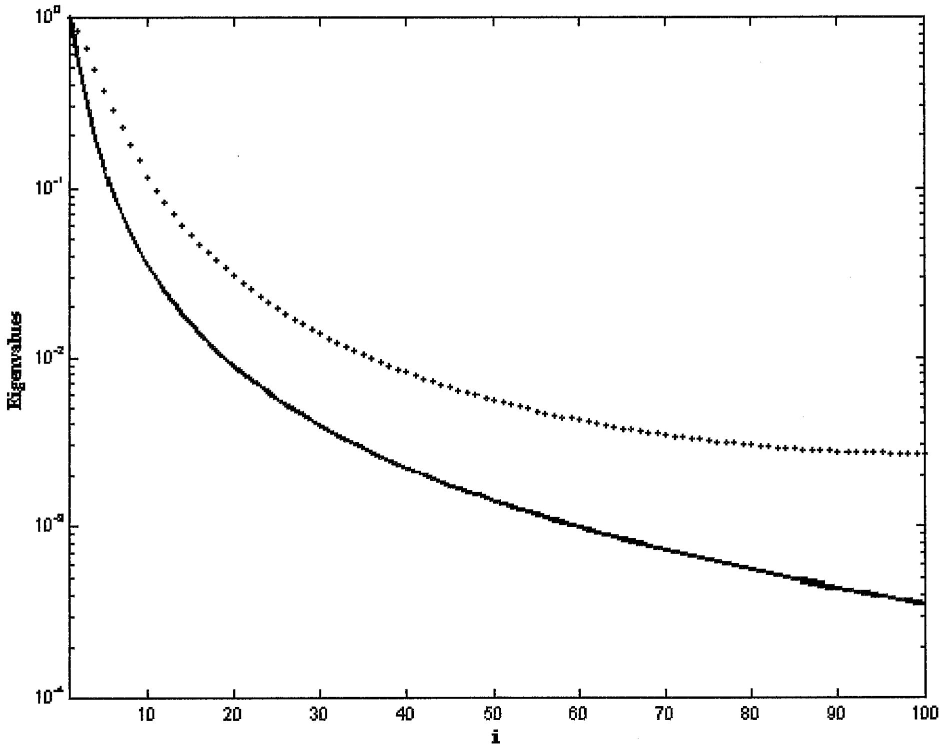

Fig. 7 The spatial Fourier transform of the assumed cross spectral density of the TBL pressure field between the center microphone and the other microphones (solid line) and the first eigenvalues of the spectral density calculated for the 100 microphones (dots).

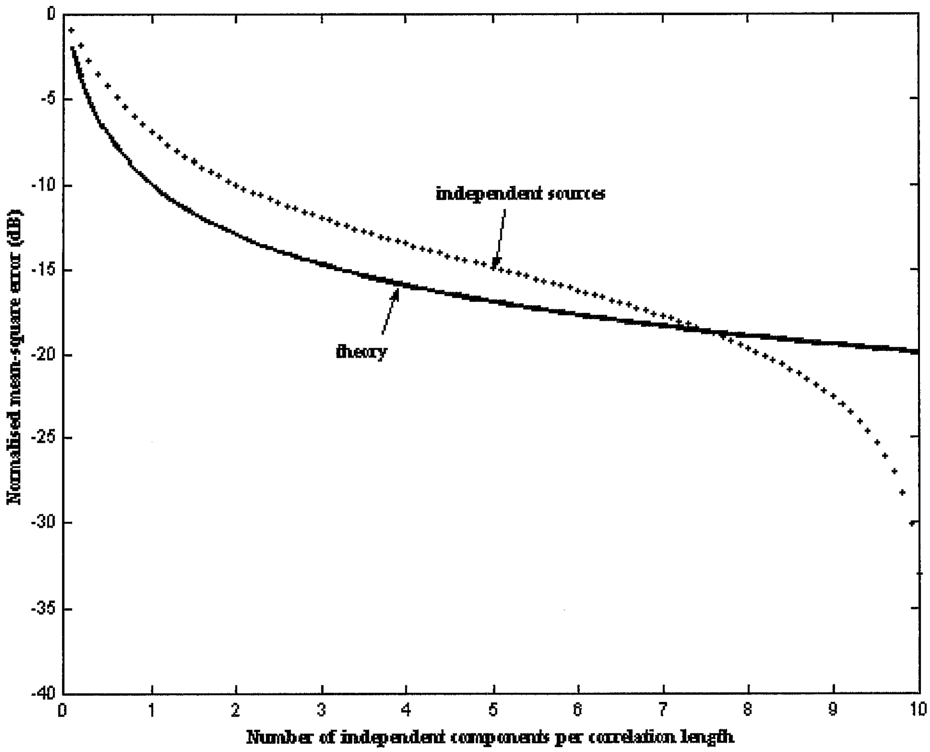

Fig. 8 The normalised mean square error associated with the assumed cross spectral density of the TBL pressure field between the center microphone and the other microphones (solid line) and that associated with a reduced rank approximation to the spectral density matrix calculated from a 100 microphone array, which is equal to the number of independent reference signals per correlation length used in the simulation (dots).

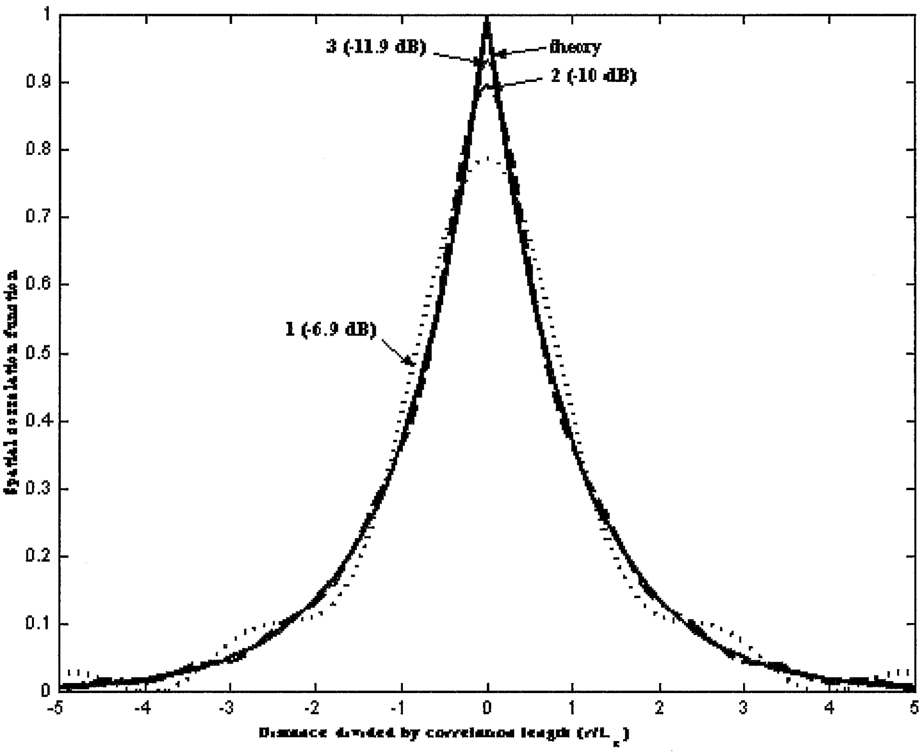

Figure 9 The assumed spatial correlation function of the TBL pressure field (solid line), that of the reduced rank approximation using 1 reference signal per correlation length (dotted line), 2 reference signals per correlation length (dashed line) and 3 reference signals per correlation length (thin solid line), plotted as a function of normalised separation, the distance from center microphone divided by correlation length.

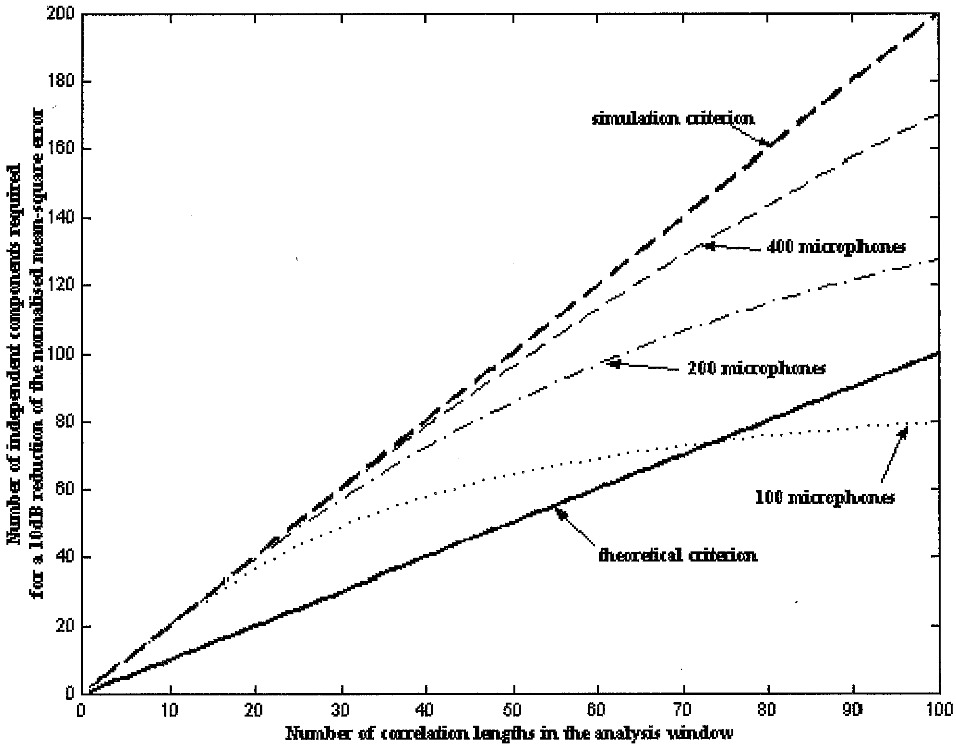

Fig. 10 The number of independent components required for a 10dB reduction in the normalised mean-square error and related to the theoretical prediction (bold solid line), to the reduced-rank approximation of the cross-spectral density matrix when considering a small number of correlation lengths and extrapolated to a larger number of them (bold dashed line) and to the reduced-rank approximation when using 100 microphones (dotted line), 200 microphones (dash-dotted line) and 400 microphones (dashed line), plotted as a function of the number of correlation lengths we aim to reproduce in the analysis window.

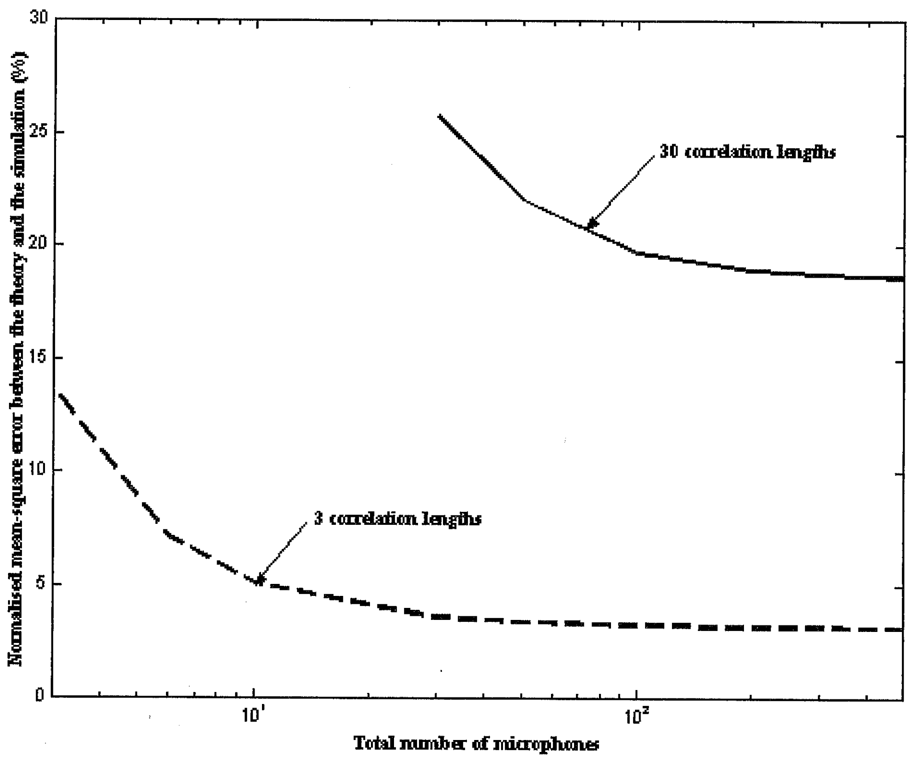

Fig. 11 Influence of an increased number of microphones on the normalised mean-square error between the theory and the simulation when we aim to reproduce 3 correlation lengths (dashed line) and 30 correlation lengths (solid line) in the spanwise direction.

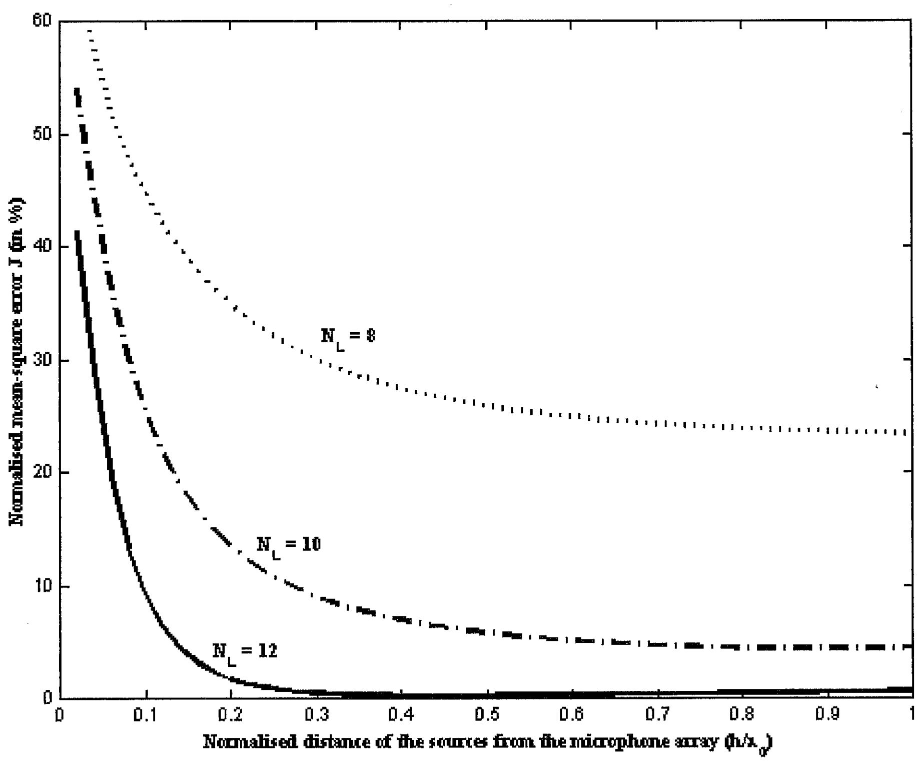

Fig. 12 The normalised mean square error for the simulations in which various numbers of acoustic sources, N_L, were used to reproduce the acoustic diffuse field as a function of the normalised distance of the sources from the microphone array.

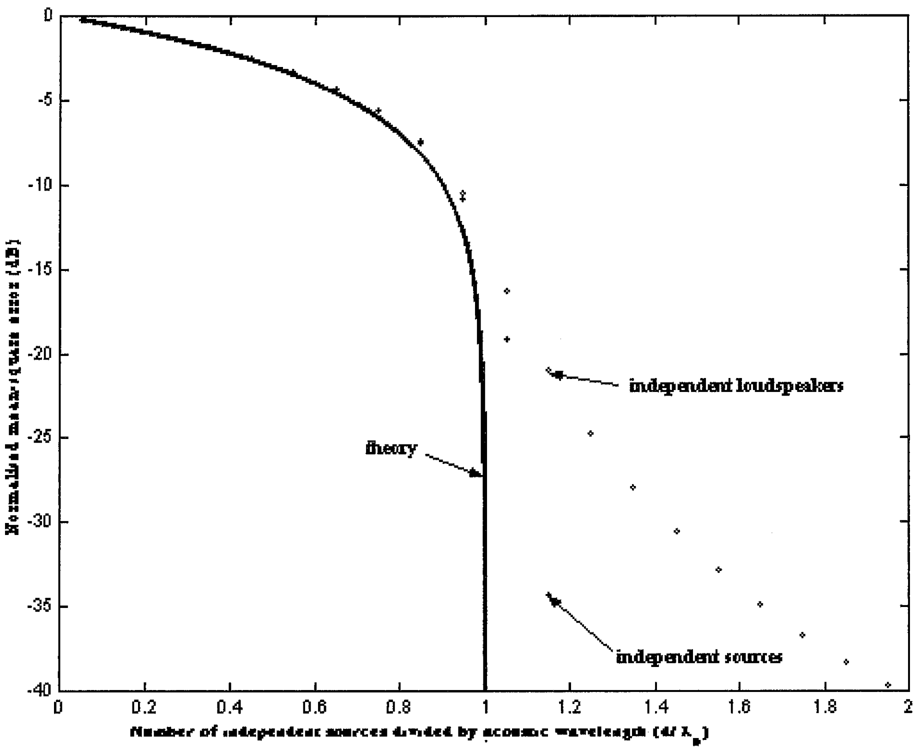

Fig. 13 The normalised mean square error associated with the assumed cross spectral density of the acoustic diffuse field between the center microphone and the other microphones (solid line), that associated with a reduced rank approximation to the spectral density matrix calculated from a 100 microphone array, which is equal to the number of independent reference signals per acoustic wavelength used in the simulation (dots) and that obtained when using an array of loudspeakers positioned twice as far from the microphone array as the distance they were apart from each other (circles), plotted as a function of the number of reference signals per acoustic wavelength.

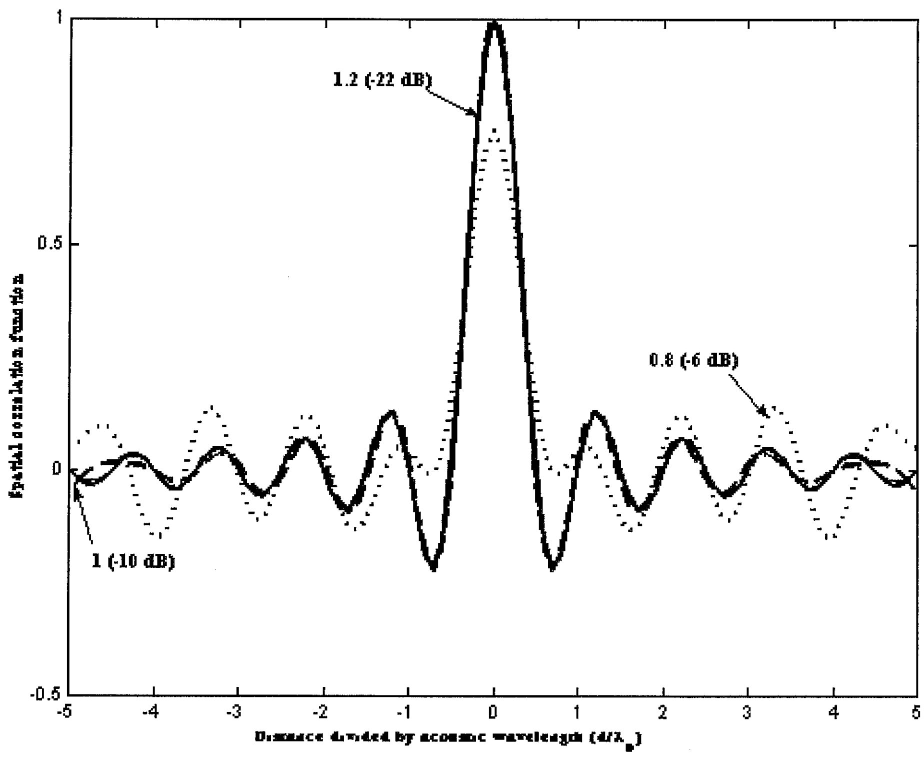

Fig. 14 The assumed spatial correlation function of the acoustic diffuse field (solid line), that of the approximate pressure field generated using 0.8 loudspeakers per acoustic wavelength (dotted line), 1 loudspeaker per acoustic wavelength (dashed line) and 1.2 loudspeaker per acoustic wavelength (thin solid line), plotted as a function of normalised separation, the distance from center microphone divided by acoustic wavelength.

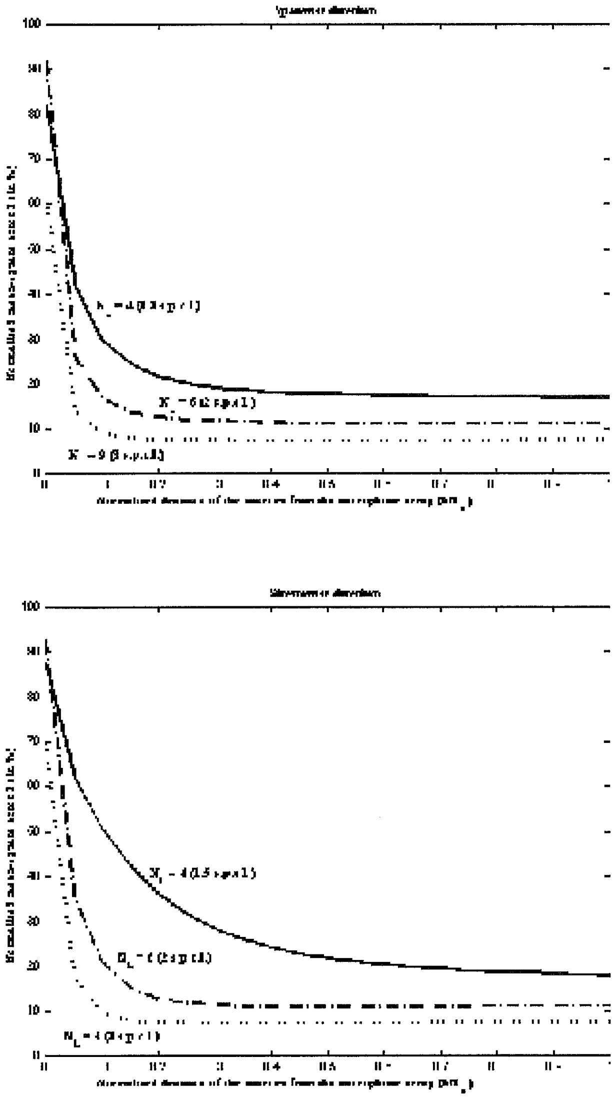

Fig. 15 The normalised mean square error for the simulations in which various numbers of acoustic sources, N_L, were used to reproduce the TBL pressure field over 3 correlation lengths in the spanwise direction (top figure) and in the streamwise direction (bottom figure) as a function of the normalised distance of the sources from the microphone array.

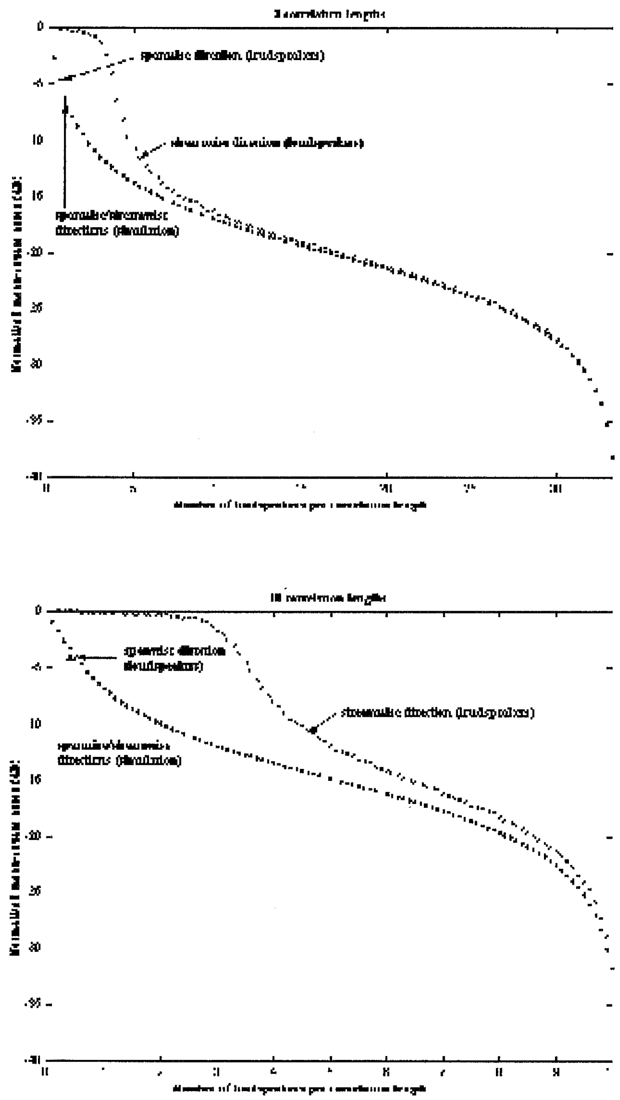

Fig. 16 The normalised mean square error associated with a reduced rank approximation to the spectral density matrix calculated when we aim to reproduce 3 correlation lengths (top figure) and 10 correlation lengths (bottom figure) over a 100 microphone array and that of the approximate pressure field generated using an array of loudspeakers, plotted as a function of the number of sources per correlation length.

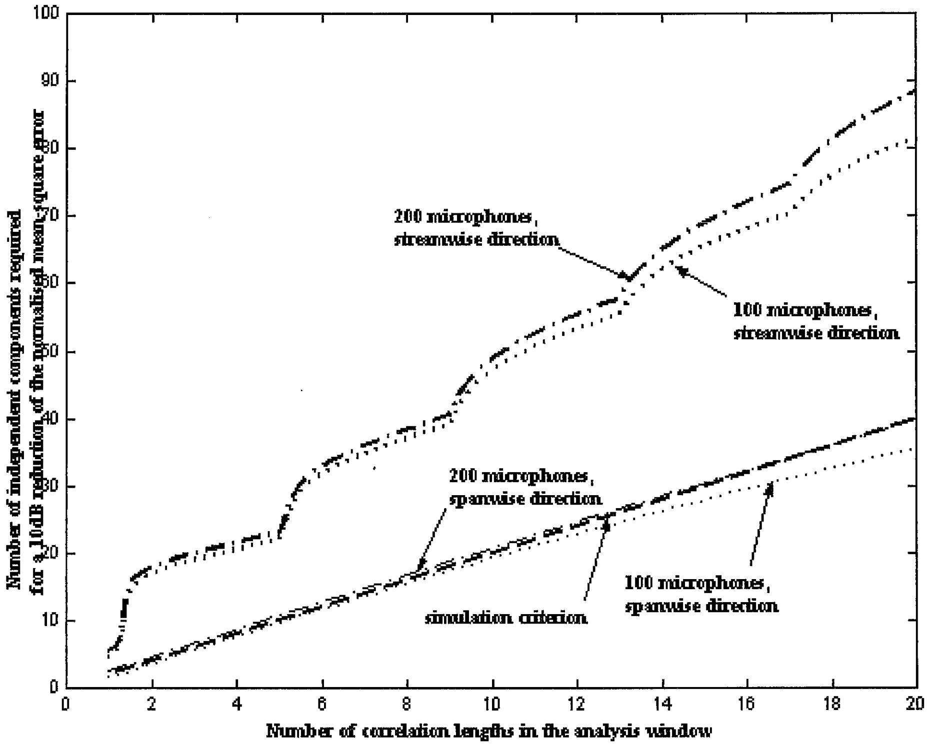

Fig. 17 The number of independent sources required for a 10dB reduction in the normalised mean-square error related to the reduced-rank approximation of the cross-spectral density matrix and to the approximate pressure field generated using an array of loudspeakers when using at most 100 microphones (dotted line) and 200 microphones (dash-dotted line), plotted as a function of the number of correlation lengths we aim to reproduce in the analysis window either in the spanwise or in the streamwise direction.