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# POINT MOBILITY OF A SEMI-INFINITE BEAM WITH CURVATURE

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## 1. INTRODUCTION

In a flat structure such as a straight beam the flexural and longitudinal wave motions are uncoupled. For a curved beam, however, there is interaction between the longitudinal and bending deformations leading to coupled extensional-flexural wave propagation. In this paper coupled extensional-flexural wave propagation is investigated by considering the mobility of a 'semi-infinite' beam with a constant radius of curvature. In section two the theoretical response of a curved "semi-infinite" beam when excited at its free end is developed. In section three the apparatus used for an experimental study on a curved mild steel beam is described and a comparison between the measured and predicted mobility of the structure is presented in section four.

## 2. THEORY

Assume that a "semi-infinite" beam with curvature is excited at its end by a point harmonic force, $Fe^{i\omega t}$, acting in the circumferential direction. At a given position along the beam the total flexural or extensional displacement will be given by the sum of the displacements due to individual waves in the beam. It can be shown[1] that for a curved beam of constant radius of curvature, R, there are two different frequency regions separated by the ring frequency $\Omega = 1$, where $\Omega = \omega R/c_0$, and $c_0$ is the speed of purely extensional waves in a straight rod. Above the ring frequency, $\Omega = 1$ there are three wave types: (i) a predominantly flexural travelling wave; (ii) a predominantly flexural near field wave; and (iii) a predominantly extensional travelling wave. Below the ring frequency the predominantly flexural travelling and near field waves still exist, however the predominantly extensional travelling wave is replaced by a predominantly extensional near field wave. It should be remembered that because of the coupling effect of curvature each wave type will consist of both extensional and flexural motion. To determine the response of the structure the boundary conditions at the free end of the beam need to be evaluated. These are assumed to be: (i) that the axial force is equal to the externally applied force; (ii) that the bending moment is zero; and (iii) that the shear force is zero. Substituting expressions for the dispacement into expressions for the resultant forces [1] gives a set of three simultaneous equations in the unknown wave amplitudes. For a given excitation frequency, $\omega$, these equations can be solved to find the unknown wave amplitudes and hence, the point receptance and cross receptance can be predicted.

## 3. EXPERIMENTAL APPARATUS AND METHOD

The test structure consisted of a 5m long curved beam 50 x 6 mm cross section with a constant radius of curvature of 1.0m. To obtain 'semi-infinite' conditions one end of the beam was inserted into an anechoic termination. The excitation force was obtained by striking the beam with an instrumented hammer at the free end. Whilst the response acceleration was measured using one accelerometer mounted in the circumferential direction and another mounted in the radial direction. The applied force and resulting accelerations were recorded simultaneously on a multi-channel spectrum analyser and the point and cross mobility calculated directly from a single measurement by dividing the response velocity by the applied force.

## 4. RESULTS

For a given curved beam the relative strength of the flexural and extensional waves will depend upon the particular geometry of the beam and the nature of the excitation force. Using the actual dimensions of the experimental curved beam and assuming a purely circumferential excitation force, the boundary conditions at the free end of the beam were applied to find the point receptance and cross receptance at the excitation location. By differentiating with respect to time the corresponding mobilities were obtained. The predicted point mobility is compared with the measured point mobility in Fig. 1 over the frequency range $\Omega = 0.01$ to $\Omega = 10.0$. For the experimental beam this corresponds to a dimensional frequency range of 8.2 to 8200Hz. Fig. 1(a) shows the modulus of the point mobility which indicates that there are two frequency regions in the point mobility separated by the ring frequency, $\Omega = 1$. Above the ring frequency the predicted value of the point mobility asymptotes to a constant value which corresponds to the point mobility of purely extensional waves in a "semi-infinite" straight bar given by[2] as:

$$Y_{sF} = \frac{c_0}{ES} \qquad (1)$$

where E is Young's modulus and S the cross sectional area of the bar.

The corresponding measured data show resonant behaviour, with the resonant frequencies corresponding to those of purely extensional waves in a straight rod of length 5m. Below the ring frequency the measured data do not exhibit resonant behaviour and thus, near-field extensional waves are indicated. This is confirmed by inspection of Fig. 1(b) which shows the corresponding phase angle and indicates that below the ring frequency the velocity is 90° out of phase with the applied force. Above the ring frequency the velocity is in phase with the force which indicates that travelling waves are being generated. Below the ring frequency both the measured and predicted data exhibit the characteristics of a "mass line" and by considering the mobility of a simple mass element it can be shown that the beam acts as a mass of length equal to the radius of curvature, R. Further, in terms of the non-dimensional frequency, $\Omega$, the point mobility can be expressed as:

$$Y_{sF} = \frac{-i}{\Omega S \rho c_0} \qquad (2)$$

where $\rho$ is the density of the material.
Fig. 2 shows the measured and predicted cross mobility (flexural velocity per unit circumferential force). The measured data indicate resonant frequencies corresponding to flexural waves in a free-free beam of length 5m. This is surprising as the point mobility indicated that the response of the beam below the ring frequency consisted of near field extensional waves. An explanation for this discrepancy may be that the wavelength of the extensional near field waves is greater than the length of the experimental beam (5m). Thus, at the end of the beam within the anechoic termination extensional near field waves were converted into flexural travelling waves which gave the resonant behaviour shown in Fig. 2. Comparison of the predicted values of the cross mobility shown in Fig. 2 with the predicted values of the point mobility shown in Fig. 1 indicates that below the ring frequency both mobilities have the same modulus but have a phase difference of 180°. Thus, from Eq. (2) the cross mobility can be expressed as:

$$Y_{zF} = \frac{i}{\Omega S \rho c_0} \qquad (3)a$$

In terms of radian frequency, $\omega$, this becomes:

$$Y_{zF} = \frac{i}{\omega R\, S\rho} \qquad (3)b$$

Thus, from Eq. (3)b it can be seen that the cross mobility is dependent upon the frequency, $\omega$, the radius of curvature, R, and the mass per unit length, $S\rho$, of the beam. It can also be seen that the flexural velocity is 90° out of phase with the applied circumferential force.

## 5. SUMMARY

In this paper, formulae for the point and cross mobilities of a "semi-infinite" beam with a constant radius of curvature have been presented. The point mobility has two frequency regions: (i) above the ring frequency, and (ii) below the ring frequency. Above the ring frequency the point mobility is dominated by predominantly extensional travelling waves and has a value which asymptotes to the point mobility of purely extensional waves in a straight "semi-infinite" bar. Below the ring frequency the point mobility is dominated by predominantly extensional near field waves and the beam acts as a mass of length equal to the radius of curvature. The cross mobility is dependent upon frequency, the radius of curvature and the mass per unit length of the beam and the flexural velocity is 90° out of phase with the applied circumferential force.

**Acknowledgement**

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**References**

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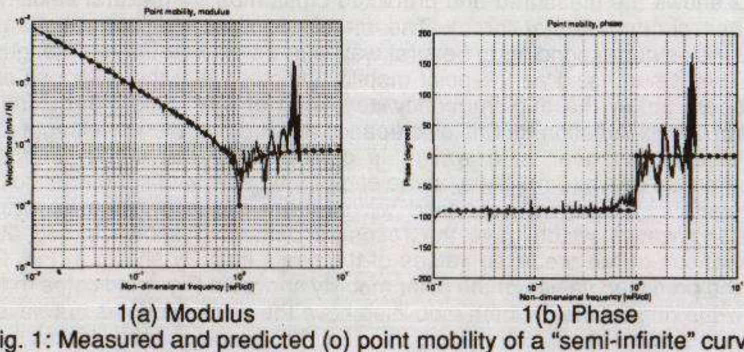
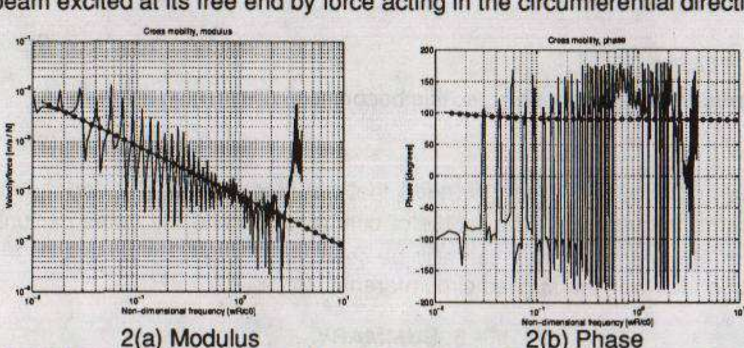



1(a) Modulus 1(b) Phase

Fig. 1: Measured and predicted (o) point mobility of a "semi-infinite" curved beam excited at its free end by force acting in the circumferential direction




2(a) Modulus 2(b) Phase

Fig. 2: Measured and predicted (o) cross mobility of a "semi-infinite" curved beam excited at its free end by force acting in the circumferential direction