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THE EFFECT OF EXCITATION LOCATION UPON THE MEASUREMENT OF VIBRATIONAL POWER IN CURVED BEAMS

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1. INTRODUCTION

Unwanted vibration in ships, aircraft and buildings if often caused by the operation of machinery installed within the structure. The best way to reduce the unwanted vibration is to modify the source and isolate it from the supporting structure. However, if the problem persists the vibration transmission characteristics through the structure from the source connection points to the area of unacceptable vibration levels must be examined. For example, a typical machinery installation in a ship consists of a machine mounted on a suspension system which is attached to the main structure of the vessel. In addition to the primary connection at the machinery seating, there will also be structural connections through the pipework, control linkages and exhaust system. Each of these connections provides a flanking path for the vibrational energy. Thus to minimise the vibration transmission and ultimate noise radiation from a machine will involve the investigation of several parallel transmission paths. Vibrational power transmission analysis techniques allow the direction of propagation of vibrational energy to be determined, and a magnitude to be assigned to each path. Previous research into structural transmission paths has shown it is possible to measure vibrational power transmission in simple beam and plate structures. More recently transmission through pipes with bends, branches and discontinuities has been studied[1], which has led to useful design rules concerning the position and size of pipe supports for minimum power transmission. However, in many practical structures transmission paths are composed of more complex curved elements. Therefore, there is a need to extend power transmission analyses to this class of structure.

In a straight beam flexural and longitudinal (extensional) wave motions are uncoupled. For a curved beam, however, there is interaction between the longitudinal and bending deformations leading to coupled extension-flexural propagation. In an elementary theory, Love[2] assumed that the centre-line remains unextended during flexural motion, whilst flexural behaviour is ignored when considering extensional motion. Using these assumptions the vibrational behaviour of complete or incomplete rings has been considered by many researchers who are interested in the low frequency behaviour of arches and reinforcing rings[3]. In the same reference[2] Love presented equations for thin shells which include the effects of extension of the mid-surface during bending motion. With some manipulation these can be reduced to equations applicable to a curved beam of constant radius of curvature. Graff derives these equations from first principles[4] and also presents frequency versus wave number and wave speed versus wave number data. In this paper the governing equations are based upon a reductions of Flügges thin shell equations[5]. These reduced equations have been shown to predict the same vibrational behaviour as the Love based equations[6] but have the advantage of a more concise mathematical form.

Experimental studies of wave motion have been reported for a 'semi-infinite' beam with constant radius of curvature excited in the circumferential direction at its free end, with both the point mobility[7] and input power[8] characteristics investigated. More recently a strategy to measure the vibrational power transmited along a curved beam has been presented[9]. In this paper these analyses are extended by studying the effect upon the transmitted power of exciting the beam under four different excitation conditions: i) in the circumferential direction at the free end; ii) in the radial direction at the free end; iii) in the circumferential direction at a point 0.6m from the free end; and iv) in the radial direction at a point 0.6m from the free end. In section two the measurement strategy derived in[9] is re-presented and in section three the experimental apparatus described. In section four the results of the experimental studies are reported. Based upon these results some recommendations are made to improve the measurement of transmitted power in a curved beam

2. THEORY

2.1 **Wave Motion in a Curved Beam** Flügge based equations of motion for a curved beam have previously been presented[6]. From the dispersion curves and associated extensional to flexural wave amplitude ratios, two different frequency regions were identified: **above** and **below** the ring frequency $\Omega=1$. Where $\Omega=\omega R/C_0$, and C_0 is the phase velocity of extensional waves in a straight bar. Above the ring frequency three wave types can exist: a flexural travelling wave, a flexural near field wave and an extensional travelling wave. Below the ring frequency the flexural travelling and near field waves still exist, however, the extensional wave is now evanscent. In a real structure the relative strength of each wave type will depend upon the nature of the excitation and any boundary conditions.

2.2 **Vibration Power Transmission in a Curved Beam** Using the Flügge based expressions for stresses and displacements in a curved beam, equations for vibrational power transmission were previously derived[9]. By applying finite difference approximations the following expressions for time-averaged vibrational power transmission were obtained:

$$< P_e(\text{straight}) >_t = \frac{ES}{\omega^3}\frac{1}{\Delta u}\int_0^\infty \mathrm{Im}(G(a_{u2},a_{u3}))df \tag{1}$$

$$\begin{aligned} < P_e(\text{curved}) >_t = \frac{ES}{\omega^3}\Bigg[& -\frac{1}{4R}\int_0^\infty \mathrm{Im}\{G(a_2,a_{u2})\}df \\ & -\frac{1}{4R}\int_0^\infty \mathrm{Im}\{G(a_2,a_{u3})\}df \\ & -\frac{1}{4R}\int_0^\infty \mathrm{Im}\{G(a_3,a_{u2})\}df \\ & -\frac{1}{4R}\int_0^\infty \mathrm{Im}\{G(a_3,a_{u3})\}\Bigg] \end{aligned} \tag{2}$$

$$< (P_{bm} + P_{sf})(\text{straight}) >_t = -\frac{EI}{\omega^3 \Delta_f^3}\int_0^\infty Im\{G(a_1,a_3)\}df + \frac{4EI}{\omega^3 \Delta_f^3}\int_0^\infty Im\{G(a_2 a_3)\}df + \frac{EI}{\omega^3 \Delta_f^3}\int_0^\infty Im\{G(a_4,a_2)\}df \quad (3)$$

$$< (P_{bm} + P_{sf})(\text{curved}) >_t = -\frac{8EI}{\Delta_f \omega^3 R^2}\int_0^\infty Im\{G(a_2,a_3)\}df \quad (4)$$

where a_1, a_2, a_3, a_4 represent signals from an array of accelerometers mounted in the radial direction and a_{u1}, a_{u2}, represent those from an array of accelerometers mounted in the circumferential direction. A complete list of notation is given in the appendix.

By analogy to power transmission in a straight beam[1], these equations are expressed in terms of an extensional component, P_e, and a combined bending moment P_{bm}, and shear force P_{sf} component. These two components have been further separated into 'straight' and 'curved' components. The 'straight' component represents the equivalent straight beam expression and the 'curved' component contains additional terms due to curvature. These four components will be referrred to as the extensional straight component, the extensional curved component, the flexural straight component and the flexural component, respectively. The power input to the structure can be calculated from the real part of the cross spectral density between the force and resulting velocity at that point[10]

$$P_{in} = Re\{G_{FV}\} \quad (5)$$

3 EXPERIMENTAL APPARATUS AND METHOD

The test structure consisted of a curved 5m long mild steel beam, 50 x 6.5mm section, with a constant radius of curvature of 1.0m. A 'semi-infinite' beam was obtained by inserting one end of the beam into an anechoic termination consisting of a 1m long box filled with sand to dissipate the energy of the wave motion. A detailed description of a similar test structure differing only in its radius of curvature has previously been published[8]. The excitation force was obtained by striking the beam with an instrumented hammer at two different locations: i) at its free end; and ii) at a position 0.6m from the free end. A purely circumferential excitation was obtained by striking a hard steel triangular block glued to the beam. The applied force was measured with the instrumented hammer whilst the response acceleration was measured with an accelerometer mounted in the circumferential direction. For radial excitation the beam was struck directly, and the response measured with a radially mounted accelerometer. A schematic representation of the experimental apparatus is shown in Fig 1. In each case the power input to the structure was calculated using Eqn (5).

Transmitted power was measured at a point, s_x, approximately 2m from the free end of the beam using six accelerometers to implement Eqn's (1) to (4). Four accelerometers were used to detect radial motion whilst two accelerometers were used to detect extensional motion. To minimise finite difference and phase mis-match errors the accelerometers were spaced 0.2 of a wavelength apart[11]. Thus, the flexural motion accelerometers were positioned 10cm apart and the extensional motion accelerometers 1.0m apart. A schematic representation of the transducer configuration is shown in Fig 2. All data were recorded simultaneously in a multi-channel FFT analyser. This was connected to a personal computer which performed the necessary post processing to calculate input and transmitted power.

4. RESULTS

The input power characteristics of a 'semi-infinite' beam with curvature having the same material properties but a larger radius of curvature than the test structure have previously been reported[8]. In (8) the beam was excited by a circumferential force acting at the free end, and it was shown that there were two distinct frequency regions : **above** and **below** the ring frequency Ω=1.0. Above the ring frequency power was input to the structure at resonant frequencies corresponding to the finite length of the beam, whilst below the ring frequency no power was input to the beam because of the evanescent nature of the predominantly extensional waves generated by the circumferentially acting force.

Fig 3 shows the power input to the current test structure by a force acting in the circumferential direction at a point *0.6m from the free end*. The frequency range is presented on a non-dimensional logarithmic scale from Ω=0.1 to Ω=10.0, which corresponds to a dimensional frequency range of 82Hz to 8200Hz. The input power is shown on a linear scale from -2×10^{-9} to 2×10^{-9} Watts. A similar result is obtained to that reported in (8): above the ring frequency Ω=1 power was input to the beam, whilst at frequencies higher than Ω=2 the excitation force was negligible and hence there was no input power; below the ring frequency the measurement of input power was corrupted by noise due to transducer cross-sensitivity (ie. the circumferentially mounted accelerometer moving in the radial direction). This arises because of the difficulty in obtaining a purely circumferential excitation of the test structure, however, it seems clear that no travelling extensional wave was induced and hence no power was input to the structure.

Transmitted power was evaluated using Eqns (1) to (4) and is shown in Fig 4. Fig 4a shows the extensional straight component (Eqn (1)). Below the ring frequency the data is corrupted by measurement noise, but following the statement above, that the input power is zero, the extensional straight component of transmitted power must also be zero. Above the ring frequency the extensional straight component indicates some transmitted power with a peak occurring at approximately Ω=1.5. Fig 4b shows the extensional curved component evaluated using Eqn (2). Below the ring frequency the data are corrupted by measurement noise whilst above the ring frequency the extensional curved component shows some transmitted power with a peak occurring just above the ring frequency. These two results (Fig 4a and 4b) can be explained by considering the nature of a predominantly extensional wave. It was noted in (8) that above the ring frequency a curved beam behaves essentially as a straight beam, ie. a predominantly extensional wave consists entirely of extensional motion. However, as the frequency reduces towards the ring frequency the predominantly extensional wave exhibits increasing flexural motion. This coupled extensional-flexural motion is indicated by the extensional curved component which is calculated from the cross spectrum of signals from circumferentially and radially mounted accelerometers. The extensional straight component, however, only indicates the power transmitted by purely extensional waves and hence is more significant at higher frequencies. Fig 4c shows the flexural straight component evaluated using Eqn (3). Above the ring frequency there was no transmitted power, whilst below the ring frequency some transmitted power was measured although this is likely to result from a slightly off-axis circumferential excitation, since it was shown in Fig 3 that no power is input to the structure below the ring frequency. Evaluation of the flexural curved component (Eqn (4)) revealed this component to be negligible and, thus, the data are not shown. Fig 4d shows the total power transmitted along the beam. This quantity was obtained by adding together the extensional straight, extensional curved and flexural straight components. Comparison with the input power shown in Fig 3 reveals that the power transmitted along the beam has been successfully measured.

The input and transmitted power characteristics of the beam when excited by a force acting in the radial direction at the free end are shown in Fig 5. Fig 5a shows the power input to the structure presented on a linear scale from -100×10^{-9} to 100×10^{-9} Watts. In contrast to the case of circumferential excitation it can be seen that the ring frequency has no significance for radial excitation and that power was input to the beam at frequencies corresponding to the resonant frequencies of flexural waves in a beam of length 5m. Fig 5b shows the extensional straight component which is negligible above Ω=0.4 and below this frequency is corrupted by accelerometer cross-sensitivity. Fig 5c shows the extensional curved component which is also negligible above Ω=0.4 and corrupted by measurement noise below this frequency. Fig 5d shows the flexural straight component which by comparison to the input power shown in Fig 5a has successfully indicated the transmitted power. The flexural curved component (Eqn (4)) is negligible over this frequency range and has not been shown. These results are not surprising since a force acting in the radial direction is likely to induce predominantly flexural motion with very little coupled flexural-extensional motion. Thus, for radial excitation of the beam the transmitted power should be measured using the flexural straight component only; addition of the extensional components will corrupt the final values of transmitted power.

The input and transmitted power characteristics of the beam when excited by a force acting in the radial direction at a point 0.6m from its free end contain the effect of waves reflected from the free end as well as those induced directly at the force location. However, the conclusions regarding the transmitted power measuring strategy remain the same as that for radial excitation at the free end and, hence, will not be discussed further.

5. SUMMARY AND RECOMMENDATIONS

This paper has reported upon accelerometer-based measurements of the power transmitted along an experimental 'semi-infinite' beam with a constant radius of curvature when excited at its free end and at a position 0.6m from the free end. A previously derived measuring strategy was introduced which by analogy with power transmission in a straight beam expressed the equations in terms of extensional and flexural (bending moment and shear force) components. These equations were further separated into straight beam and curved beam components. Analysis and measurements of transmitted power in terms of these straight and curved components showed that the location of the excitation force was unimportant in terms of the measurement strategy. However, the direction in which the applied force acted was important and the following recommendations can be made:

a for circumferential excitation the measurement strategy depends upon the frequency region:

i above the ring frequency, Ω=1, the transmitted power should be measured using the extensional straight and extensional curved components, Eqns (1) and (2), respectively. The extensional curved component is significant close to the ring frequency. The flexural straight component, Eqn (3), is negligible and may be ignored.

ii below the ring frequency very little power is input to the structure and measurement noise tends to dominate all three components. The value of transmitted power, should be assumed zero.

b for radial excitation the transmitted power should be measured using the flexural straight component only. The extensional components are dominated by accelerometer cross sensitivity effects and will corrupt the final values of transmitted power.

In all cases the flexural curved component, Eqn (4), is negligible and may be neglected.

ACKNOWLEDGEMENT

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APPENDIX : NOTATION

E	Young's Modulus
G	Single sided spectral density function
I	Second moment of area of beam cross-section
P_{bm}	Bending moment component of transmitted power
P_e	Extensional component of transmitted power
P_{in}	Power input to structure
P_{sf}	Shear force component of transmitted power
R	Radius of curvature
S	Cross sectional area of beam
a_i	radial transducer number
a_{ui}	circumferential transducer number
c_o	wave speed of extensional waves in a straight bar
f	frequency
k	wave number
s	coordinate in circumferential direction
s_x	transmitted power measurement position
Δ_f	distance between flexural transducers
Δ_u	distance between extensional transducers
Ω	non-dimensional frequency
ω	radian frequency

Special symbols

$\mathrm{Im}\{\ \}$	imaginary part
$\mathrm{Re}\{\ \}$	real part
$\langle\ \rangle_t$	time averaged

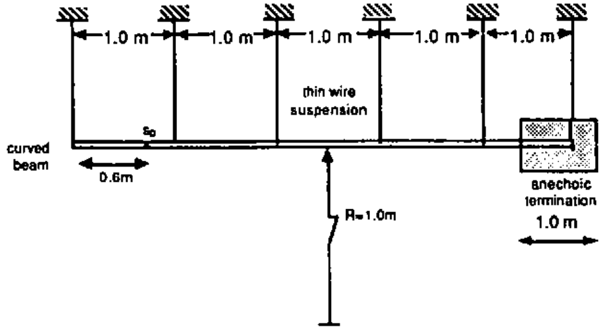

Figure 1 Schematic representation of the experimental apparatus, side view

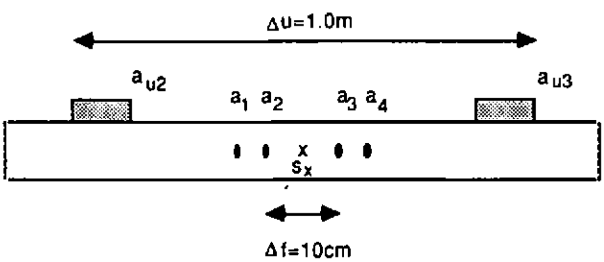

Figure 2 Schematic representation of the transducer configuration used to measure transmitted power

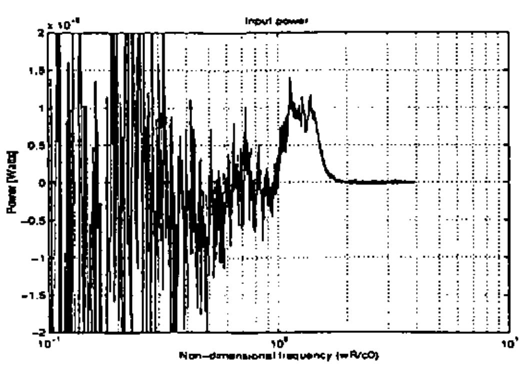

Figure 3 Power input to a *semi-infinite* beam with curvature excited by force acting in the circumferential direction at a point 0.6m from its free end

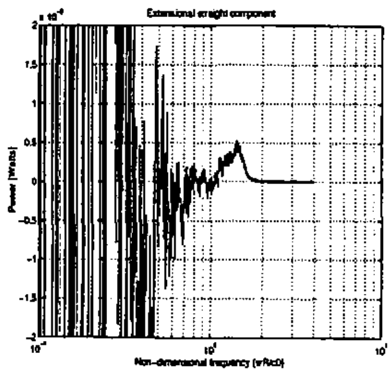

Fig 4a Extensional straight component

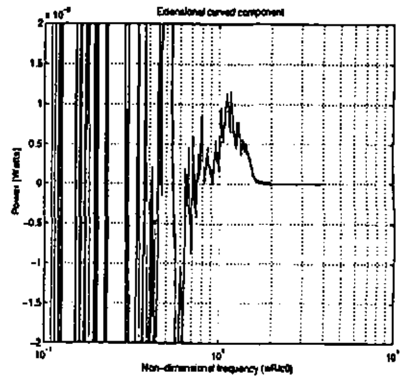

Fig 4b Extentional curved component

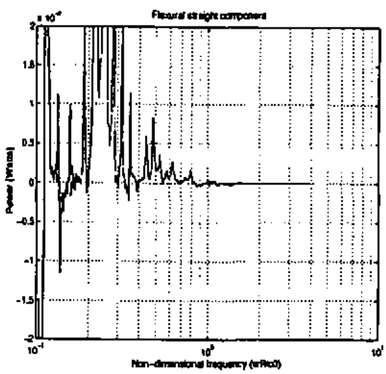

Fig 4c Flexural straight component

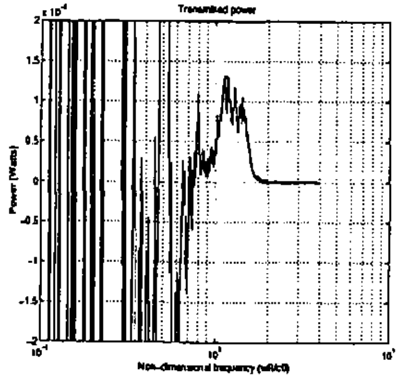

Fig 4d Total transmitted power

Figure 4 Power transmitted along a *semi-infinite* beam with curvature excited by a force acting in the circumferential direction at a point 0.6m from its free end

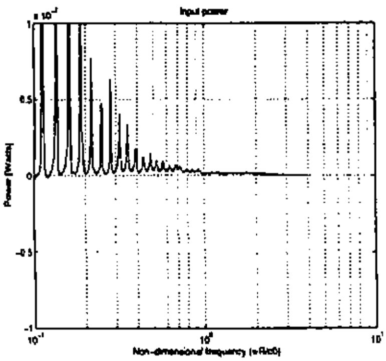

Fig 5a Input power

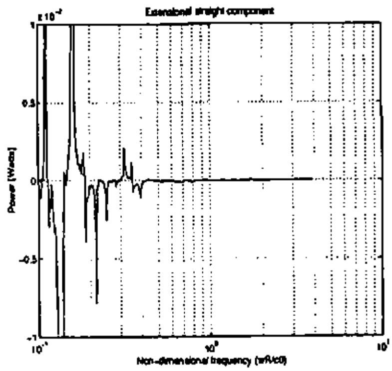

Fig 5b Extentional straight component

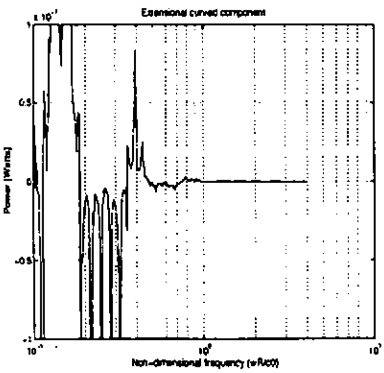

Fig 5c Extensional curved component

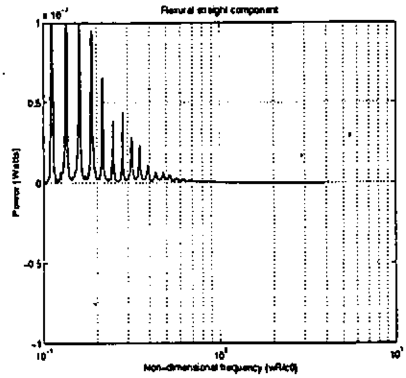

Fig 5d Flexural straight component

Figure 5 Comparison of the input and transmitted power characteristics of a *semi-infinite* beam excited by a force acting in the radial direction at its free end