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# AN EFFICIENT ADAPTATION STRUCTURE FOR HIGH SPEED TRACKING IN TONAL CANCELLATION SYSTEMS

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## INTRODUCTION

The filtered-X algorithm and other gradient descent based methods often provide an effective adaptation means for controlling periodic disturbances at multiple points. However, as the number of actuators and error sensors becomes large, the slowed convergence rate associated with the spread in singular values for the transducer-error path represents a significant issue in applications where phase drift exists between multiple disturbances at roughly the same frequency. Computational efficiency is also required for systems where a large number of actuators and error sensors are used. This paper is intended to provide an intuitive framework unifying a number of existing adaptation strategies. A computationally efficient algorithm is then presented which provides for convenient time-sharing between principle components, normalization of convergence and null-space constraint based on a singular value decomposition (SVD) of the transducer error path at frequencies of the disturbance.

## DESIRED CHARACTERISTICS

Consider the block diagram in Fig. 1 representing a system containing ***n*** actuators, and ***p*** error sensors. The three transforms, $\mathbf{H}_1$, $\mathbf{H}_2$ and $\mathbf{H}_3$ represent matrix transfer functions performing output, error and regressor filtering, respectively, at discrete frequencies of a disturbance. The method of filtering for $\mathbf{H}_1$ and $\mathbf{H}_2$ is direct, i.e. $\psi = \mathbf{H}_1\mathbf{y}$ and $\mathbf{e} = \mathbf{H}_2\varepsilon$. The filtering method implied by $\mathbf{H}_3$ is that corresponding to the filtered-X algorithm whereby the *i-th* error signal is correlated with a regressor filtered by $h_{3,ij}$, for adapting the *j-th* complex controller parameter. The update is then

$$\mathbf{y}(\tau+1) = \mathbf{y}(\tau) + \mu\mathbf{H}_3^H\mathbf{e}(\tau) = \mathbf{y}(\tau) + \mu\mathbf{H}_3^H\mathbf{H}_2\varepsilon(\tau) \tag{1}$$

Assuming a stationary and convergent system, the final controller output is obtained as the update term approaches zero or when

$$\mathbf{H}_3^H\mathbf{H}_2\varepsilon(\tau) = \mathbf{H}_3^H\mathbf{H}_2(\mathbf{C}\mathbf{H}_1\mathbf{y}(\tau) + \mathbf{d}) = \mathbf{0} \tag{2}$$

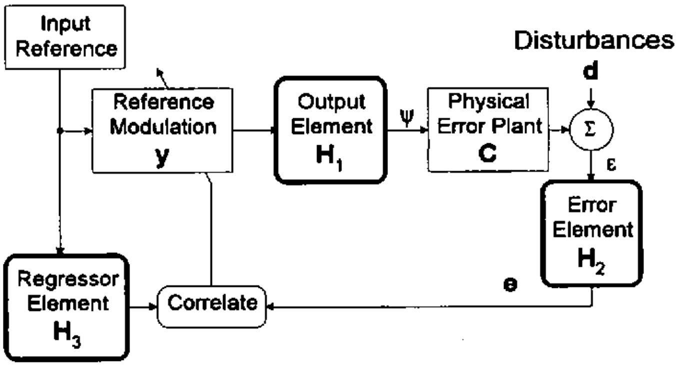


Figure 1 - Single Frequency Active Noise Cancellation System

so a sufficient condition for adaptation to cease is

$$\mathbf{y}(\infty) = -(\mathbf{H}_3^H\mathbf{H}_2\mathbf{C}\mathbf{H}_1)^{-1}\mathbf{H}_3^H\mathbf{H}_2\mathbf{d} \tag{3}$$

The uniqueness of $\mathbf{y}(\infty)$ depends on the invertability of the matrix product, $\mathbf{H}_3^H\mathbf{H}_2\mathbf{C}\mathbf{H}_1$. In the absence of constraint, zero singular values in the product $\mathbf{H}_3^H\mathbf{H}_2\mathbf{C}$ also allow for drift in $\psi$, since the physical output of actuators becomes unobservable. Finally, an ill-conditioned matrix inverse in Eq. 3 implies the potential for convergence to high output power levels [1].

The difference between the convergent value and present value of $\mathbf{y}$ can be found by substitution in the previous three equations resulting in

$$\mathbf{y}(\tau+1) - \mathbf{y}(\infty) = [\mathbf{I} + \mu(\mathbf{H}_3^H\mathbf{H}_2\mathbf{C}\mathbf{H}_1)][\mathbf{y}(\tau) - \mathbf{y}(\infty)] \tag{4}$$

which specifies a convergent solution as long as the eigenvalues of $[\mathbf{I} + \mu(\mathbf{H}_3^H\mathbf{H}_2\mathbf{C}\mathbf{H}_1)]$ have a magnitude less than unity. Based on this, a family of convergent adaptation methods become apparent [5]. Broadband filters may be used [2,3], filters may be selectively designed over frequency, or simple weighting matrices can be applied to the regressor, output or error signals in real time. Examples include: setting $\mathbf{H}_1$ or $\mathbf{H}_2$ to $\mathbf{C}^H$ yielding an output or error transform based gradient descent algorithm, setting $\mathbf{H}_3 = \mathbf{C}$, corresponding to the filtered-X update and a normalized or "Gauss-Newton" update results by setting $\mathbf{H}_2 = [\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{C}^H$ [2].

Consider the singular value decomposition for $\mathbf{C}$, resulting in a unitary ***pxp*** matrix $\mathbf{U}$, ***pxn*** matrix $\mathbf{S}$ and unitary ***nxn*** matrix $\mathbf{V}$ such that $\mathbf{C} = \mathbf{U}\mathbf{S}\mathbf{V}^H$. Construct a normalizing matrix, $\bar{\mathbf{S}}$, from the transpose of $\mathbf{S}$ by inverting only nontrivial singular values and deleting columns in $\bar{\mathbf{S}}$ for others and set

$$\mathbf{H}_1 = \mathbf{V}\bar{\mathbf{S}},\ \mathbf{H}_2 = -\mathbf{U}^H z^{k_d} \text{ and } \mathbf{H}_3 = \mathbf{I}z^{-k_d} \tag{5}$$

The delay term, $z^{-k_d}$, is useful for approximating delay characteristics in $\mathbf{C}$ for the purpose of broadening the bandwidth of single frequency solutions. Then the resultant solution for the applied output, $\psi$, from Eq. 3 becomes

$$\psi(\infty) = \mathbf{H}_1\mathbf{y} = -\mathbf{V}\bar{\mathbf{S}}\mathbf{U}^H\mathbf{d} \tag{6}$$

which, as long as no zero singular values exist in the error path, results in the following cases [2]:

Case 1: $p > n$, $\psi(\infty) = -(\mathbf{VS}^H\mathbf{U}^H\mathbf{USV}^H)^{-1}\mathbf{VS}^H\mathbf{U}^H\mathbf{d} = -(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{d}$
Case 2: $n > p$, $\psi(\infty) = -\mathbf{VS}^H\mathbf{U}^H(\mathbf{USV}^H\mathbf{VS}^H\mathbf{U})^{-1}\mathbf{d} = -\mathbf{C}^H(\mathbf{CC}^H)^{-1}\mathbf{d}$
Case 3: $n = p$, $\psi(\infty) = -\mathbf{V\bar{S}U}^H\mathbf{d} = -\mathbf{C}^{-1}\mathbf{d}$

Inspection reveals that the optimal solution depends on two elements: 1) the error path and 2) the phase and amplitude of the disturbance, $\mathbf{d}$. The approach of Eq. (5) is to solve for the dependency that the controller solution has in relation to characteristics of the transducer-error path ahead of time. This is possible because in many applications, characteristics of the transducer-error path are relatively time invariant or are very slowly varying. The phase and amplitude of disturbances is often unpredictable and rapidly time varying. It is only this dependency that remains to be resolved in order to solve for the desired controller solution. This is quickly achieved since the convergence speed of the system is normalized by setting all eigenvalues of the product $(\mathbf{H}_3^H\mathbf{H}_2\mathbf{CH}_1)$ to unity.

In cases where tracking of phase drift occurs, this process can be viewed as a form of nonlinear modulation being applied to the input reference for matching the instantaneous frequency of the disturbance. The principle limiting factor on the convergence rate now becomes the physical delays present in the transducer-error plant, rather than the eigenvalue spread associated with the product $\mathbf{C}^H\mathbf{C}$, which is the case with gradient descent based algorithms.

Figure 2 illustrates a simpler example for a singular value plot taken from a lined duct with 2'x6' rectangular cross section over a frequency range of 0 to 750 Hz where four independently excited loudspeakers were used to generate a sound field measured by four microphones. Singular values cut-on at frequencies roughly corresponding to the cut-on frequencies of each mode in the duct. The need for null-space constraint is apparent at lower frequencies where select singular values are zero.

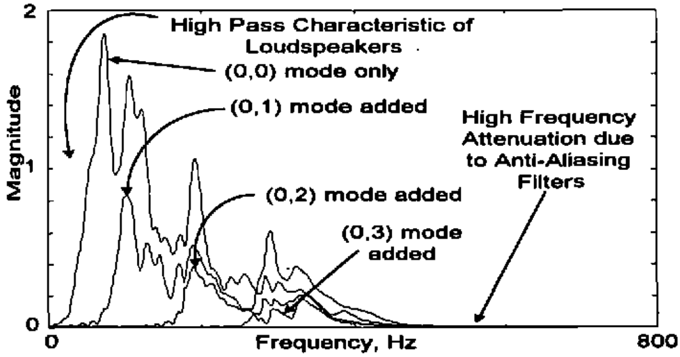


Figure 2 - First Four Singular Values vs. Frequency for a 2'x6' lined duct

This is also certainly the case when the number of actuators exceeds the number of error sensors. This problem is corrected by setting elements of $\bar{\mathbf{S}}$ to zero which correspond to singular values too small to invert. Coupling this with normalization provides for stable and rapid tracking of nonstationary disturbances.

## EFFICIENT METHOD OF ADAPTATION

In performing a real-time implementation of Fig. 1 based on the settings of Eq. (5), it becomes convenient to time-share with respect to the principle components of the system. This method provides the advantage of orthogonality or independence between the adaptation of these components, reducing the impact on convergence speed. The following time-domain equations specify a real-time implementation. Note the ***px1*** vector, $\rho$ accumulates correlation between the reference and error signals during each sample period, *k*, according to: $\rho_{k+1} = \rho_k + \varepsilon_{k+1} e^{j\omega(k-k_d)}$, $i = 1, .., p$ determines which components are being adapted and $e^{j\omega(k-k_d)}$ is a delayed complex representation for the in-phase and quadrature reference signals.

| | |
|---|---|
| $g = \mathbf{h}_{2,row-i}\rho$ | Extract component by dot prod. with *i-th* row $\mathbf{H}_2$ |
| $\rho = \rho - g\mathbf{h}^H_{2,row-i}$ | Eliminate component from accumulation |

The update (or constraint) is then performed for the selected coordinate:

| | |
|---|---|
| If $i \le n$ and $\lambda_i >$ *threshold*, | Test for magnitude of singular value |
| $\psi = \psi + \mu g \mathbf{h}_{1,col-i}$ | Update parameters with *i-th* col. of $\mathbf{H}_1$ |
| else if $i \le n$, | |
| $g_v = \mathbf{h}^H_{1,col-i}\psi$, | Extract component of null-space (*i-th* col of $\mathbf{V}$) |
| $\psi = \psi - \beta \mathbf{h}_{1,col-i} g_v$ | Leak or remove null-space coordinate |

The designer may selectively choose the rate at which coordinates are updated or the number of them updated during each sample period. The accumulator term, $\rho$, is bounded since the ***p*** orthogonal rows of the matrix $\mathbf{H}_2 = -\mathbf{U}^H z^{k_d}$ span its complex parameter space.

The reader is referred to [4] for experimental results where high speed tracking was successfully performed while canceling a 3-bladed lawnmower deck exhibiting beat frequencies due to varying belt tension around each pulley.

## REFERENCES

[1] S. J. Elliott, C. C. Boucher, and P. A. Nelson, "The Behavior of a Multiple Channel Active Control System," IEEE Trans. Acoust., Speech, Signal Processing, vol. 40, no. 5, pp.1041-1051, 1992.

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[5] The systems described in this paper utilize technologies that are the subject of various foreign and domestic patents of Digisonix Inc.