DOPPLER PROCESSING USING COMB SPECTRUM TRANSMISSION PULSES

T Collins & P Atkins

University of Birmingham

1. INTRODUCTION

Conventionally, in order to process against reverberation an active sonar pulse designer is faced with a choice between two techniques. Either the effective length of the reverberation volume can be reduced using pulse compression with linear period modulation (LPM) chirps or the relative velocity between a target and the reverberation can be exploited by Doppler filtering using long continuous wave (CW) pulses [1]. Two novel classes of pulse designs have been proposed which offer a compromise between these extremes of range and Doppler resolution by virtue of their comb shaped power spectra which exploit both narrow-band Doppler sensitivity and broad-band pulse compression. These comb spectra are realised either by the use of a low frequency periodic modulation function (Newhall trains or sinusoidal FM [2]) or by adding several tonals together (Cox combs [3]).

To assess the applicability of these novel pulse designs, examples were transmitted during low-frequency sea trials. Results will be presented which compare the resulting reverberation levels with those observed using conventional LPM and CW pulses, showing that the novel Doppler sensitive pulse designs can achieve superior reverberation suppression over a wide range of target velocities.

2. REVERBERATION PROCESSING

In most long-range active sonar receivers, a matched filter is used to process against ambient noise. Assuming additive white Gaussian noise, the signal-to-noise ratio at the output of a matched filter is easily calculated [1] and can be shown to be proportional to the ratio between the total transmitted energy and the power spectral density of the noise. In theory, therefore, the probability of detecting targets does not depend on the pulse bandwidth, modulation function or any other features of the pulse design except the average power transmitted and the pulse duration.

The processing gain against reverberation, by comparison, is very much dependent on the pulse design and does not vary with transmitted power. Range and Doppler resolution, as well as the distribution of any sidelobes, all affect the reverberation level. Using conventional transmission waveforms, there are three ways in which a sonar pulse designer may attempt to suppress reverberation :

- Minimise the length of the reverberation volume by using as short a transmission pulse as possible (a 'short' CW pulse).
- Minimise the *effective* length of the reverberation volume using a wide-band FM chirp.
- Exploit the relative Doppler shift between reverberation and target echoes using a narrow-bandwidth waveform (a 'long' windowed CW pulse).

Each of these methods can be demonstrated graphically using the ambiguity function, as in figure 1.

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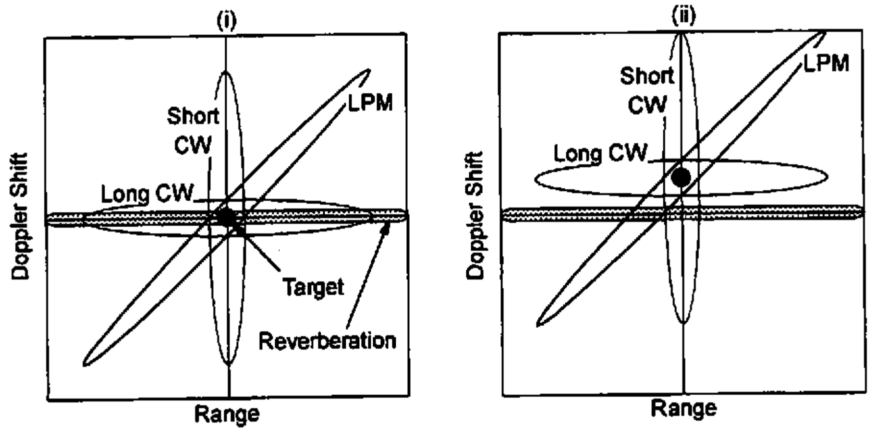

Figure 1. (i) A stationary target and (ii) a moving target, both surrounded by stationary reverberating scatterers. The ellipses indicate the ambiguity functions of target echoes using a long CW, short CW and LPM pulse designs.

The shaded area in figure 1 represents the range-Doppler extent of the reverberating scatterers. They are assumed to be uniformly distributed over a long range but confined to a narrow spread in Doppler. In figure 1(i) the target is a point scatterer which is stationary relative to the reverberation. The reverberation level interfering with target detection can be estimated from the proportion of the shaded region which is enclosed by the ambiguity function of the target echo. This is obviously a much smaller region for both the FM chirp and the short CW pulse than for the long CW pulse, indicating a corresponding suppression of the reverberation level. In figure 1(ii) the target is moving relative to the reverberation. The shaded region enclosed by the short CW and FM chirp pulses remains much the same as in figure 1(i), indicating the Doppler insensitivity of these pulses. By comparison, the ambiguity function of the long CW pulse is now completely separated from the shaded region, implying a negligible reverberation level. In fact the reverberation level will have a small non-zero value, depending on the Doppler sidelobe levels of the CW pulse.

In order to quantitatively compare the theoretical reverberation levels for different transmission waveforms, the Q-function can be used [2]. Under the assumptions that the reverberating scatterers are infinite in number, stationary and evenly distributed in range and that the target is a single point scatterer moving with a relative radial velocity, v, the expected signal-to-reverberation ratio after matched filtering is proportional to :

$$Q(\eta) = \int_{-\infty}^{\infty} \left|\chi(\tau,\eta)\right|^2 \mathrm{d}\tau \qquad 1$$

where, $$\chi(\tau,\eta) = \sqrt{\eta}\int_{-\infty}^{\infty} s(t)s^*\left[\eta(t-\tau)\right]\mathrm{d}t \qquad 2$$

η = Doppler compression factor, $1 + 2v/c$,
c = velocity of sound,
$s(t)$ = Transmitted pulse design,
$|\chi(\tau,\eta)|^2$ = Wide-band ambiguity function of $s(t)$.

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In other words, the Q-function is the area under a constant Doppler cross-section of the ambiguity function. For optimum reverberation suppression, the Q-function should be made as low as possible. As a results of the volume invariance property of the ambiguity function, however, the total area under the Q-function is fixed. All the pulse designer can do to suppress reverberation is to alter the Doppler distribution of this area.

Although the assumptions made in deriving the Q-function represent an unrealistically ideal scenario, it will be shown that it can still be used to predict the relative reverberation processing potential of different pulse designs in a practical scenario. N.B. Neither the ambiguity function nor, therefore, the Q-function can be inverted to directly synthesise an optimal pulse design. Instead, the Q-function will be used to compare the reverberation performance of different transmission pulses.

An alternative form of equation 1 can be derived from the Doppler Cross Power Spectrum (DCPS) of a pulse [4] which forms a Fourier pair with the ambiguity function and can be calculated from the spectrum of the transmitted waveform :

$$\Gamma_s(f,\eta) = \frac{1}{\sqrt{\eta}} S(f) S^*\left(\frac{f}{\eta}\right) \qquad 3$$

Noting that energy is conserved in a Fourier transform, it follows that

$$\int_{-\infty}^{\infty} \left|\Gamma_s(f,\eta)\right|^2 df = \int_{-\infty}^{\infty} \left|\chi_s(\tau,\eta)\right|^2 d\tau = Q_s(\eta) \qquad 4$$

So, to achieve a low Q-function, and hence a high reverberation processing gain, it is necessary to minimise the area under the square of the modulus of the DCPS along a line of constant Doppler scaling. Using an FM chirp, this is done by spreading the energy of the transmitted pulse design over as broad a bandwidth as possible. By contrast, a long CW pulse achieves a low Q-function by concentrating the energy into as narrow a bandwidth as possible so that only a small Doppler shift is needed to minimise the DCPS. A much lower Q-function is formed in this way, but only when the target is moving. To reduce the Q-function even further, a compromise between wide-band chirps and narrow-band CW pulses is required.

If the frequency modulation function of a waveform is periodic, the spectrum of the waveform will consist of many individual spectral lobes spaced at multiples of the repetition frequency of the FM function. The energy of the pulse will therefore be spread over frequency, in common with an FM chirp, although the narrow spectral lobes will cause it to be as sensitive to the Doppler effect as an equivalent length CW pulse. The ambiguity function of such a waveform has the shape shown in figure 2. The periodic large range sidelobes will enclose a smaller proportion of the reverberation distribution than a CW pulse would leading to a corresponding decrease in the reverberation level. The drawback of the comb-like spectrum, however, is the appearance of large sidelobes at Doppler scalings where adjacent spectral lobes overlap one another. This is an inevitable consequence of the volume invariance property of the ambiguity function [1]. The energy that is removed from the zero-Doppler ridge must reappear somewhere else on the range-Doppler plane. It should be noted that the large range ambiguities mean that comb-spectrum pulse designs cannot be used to accurately range targets in the way that chirps can. They will, however, be at least as unambiguous as a CW pulse with the advantage of a consistent improvement in reverberation suppression.

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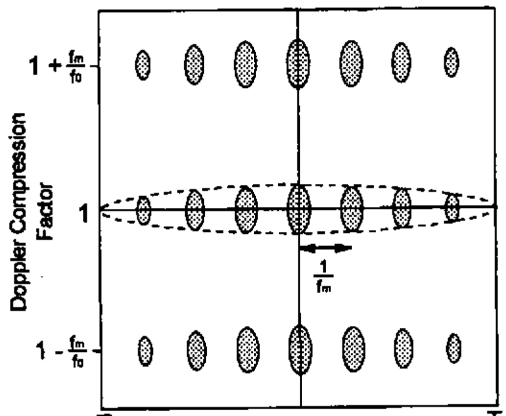

Figure 2. The ambiguity function formed using a periodic modulation function of frequency, f_m. The carrier frequency is f_0. The dotted outline shows the extent of the ambiguity function of a CW pulse of the same duration and carrier frequency.

The actual form of the periodic modulation function is largely arbitrary. For optimal zero Doppler reverberation suppression, however, it should cause the spectrum of the waveform to fill the available bandwidth as evenly as possible. The best candidate from this point of view is a series of linear frequency modulation (LFM) chirps forming a *Newhall train* [2]. A drawback of this option is that the modulation function is not continuous but steps suddenly at the end of each chirp. This can cause practical transmission difficulties with some power amplifiers. As an alternative, sinusoidal frequency modulation (SFM) can be used. This does not fill the transmission bandwidth quite as evenly although the resultant difference in reverberation processing will generally be insignificant.

A slightly more obvious way of creating a comb spectrum is to simply add together the individual sinusoids which make up the comb. The advantage is that the frequencies of the tones can now be placed arbitrarily. A special case is a *Cox comb* where the spacings between the tones form a geometric progression [3]. Using this scheme, a greater number of tones can often be squeezed into the transmission band, giving improved reverberation processing. Also, the large range sidelobes inevitable with periodic modulation functions are spread out, giving a lower mean level and allowing unambiguous ranging of point targets. The disadvantage is that the envelope of a Cox comb is no longer a smooth window function but has an impulsive, noise-like nature. Consequently the peak-to-mean power ratio is much higher which will result in poorer ambient noise processing in peak power limited systems.

3. EXPERIMENTAL TRIALS RESULTS

To evaluate the relative merits of Doppler sensitive pulse designs in a reverberation limited environment, several different waveforms were transmitted during two separate active sonar sea-trials. Both the transmitter and receiver were towed in this scenario and were constantly moving at around 4 knots.

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In the first trial (January 1997), four pulse designs were considered. Each was two seconds in duration and occupied the band from 1200 Hz to 1500 Hz. They were a Hamming windowed CW pulse, an LPM chirp a Newhall train of 18 LFM chirps and a Cox comb consisting of 29 tones (both Hamming windowed). The theoretical Q-functions are shown in figure 3(i).

The four pulse designs used in the second trial (August 1997) were only one second long and occupied the same band. They were a Hamming windowed CW pulse, a rectangular windowed LPM chirp a Hamming windowed sinusoidally frequency modulated (SFM) pulse (modulation frequency = 45 Hz) and a Hamming windowed Cox comb consisting of six geometrically spaced tones. The shorter pulse duration was used for this trial in order to minimise the 'dead-range' of the mono-static sonar. Q-functions of the pulses are shown in figure 3(ii). The functions for the SFM and Cox waveforms are indistinguishable and so are shown as a single curve.

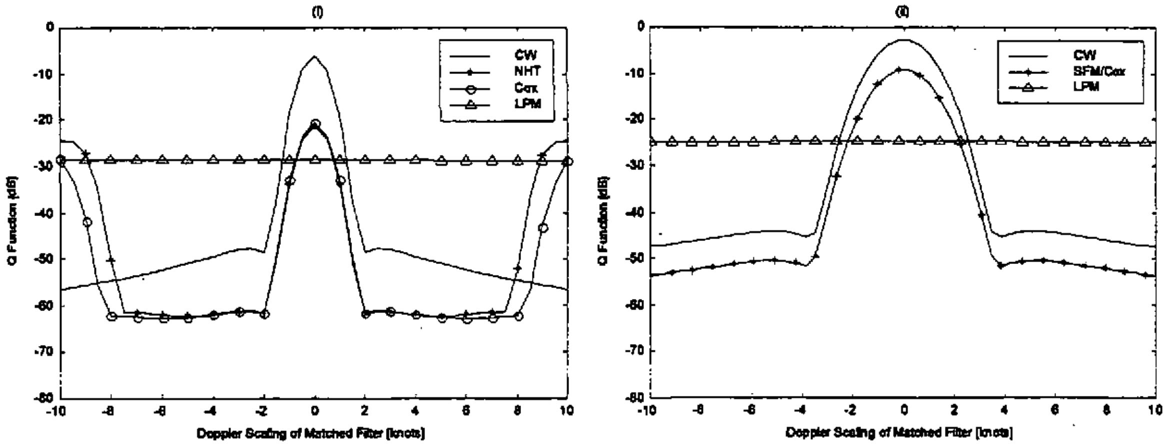

Figure 3. Theoretical Q-functions of the transmitted waveforms in (i) January 1997 trial, (ii) August 1997 trial.

The comb waveforms used in the January trial were designed to maximise the reverberation processing gain. To achieve this, the first major Doppler sidelobes were centred at ±10 knots. As such, the Q-functions are much lower than the CW curve although the range of detectable velocities is only around ±7 knots. In the August trial, more realistic waveforms were transmitted with the Doppler sidelobes placed at ±50 knots. This gives a huge range of detectable target velocities at the expense of a reduction in the reverberation processing advantage.

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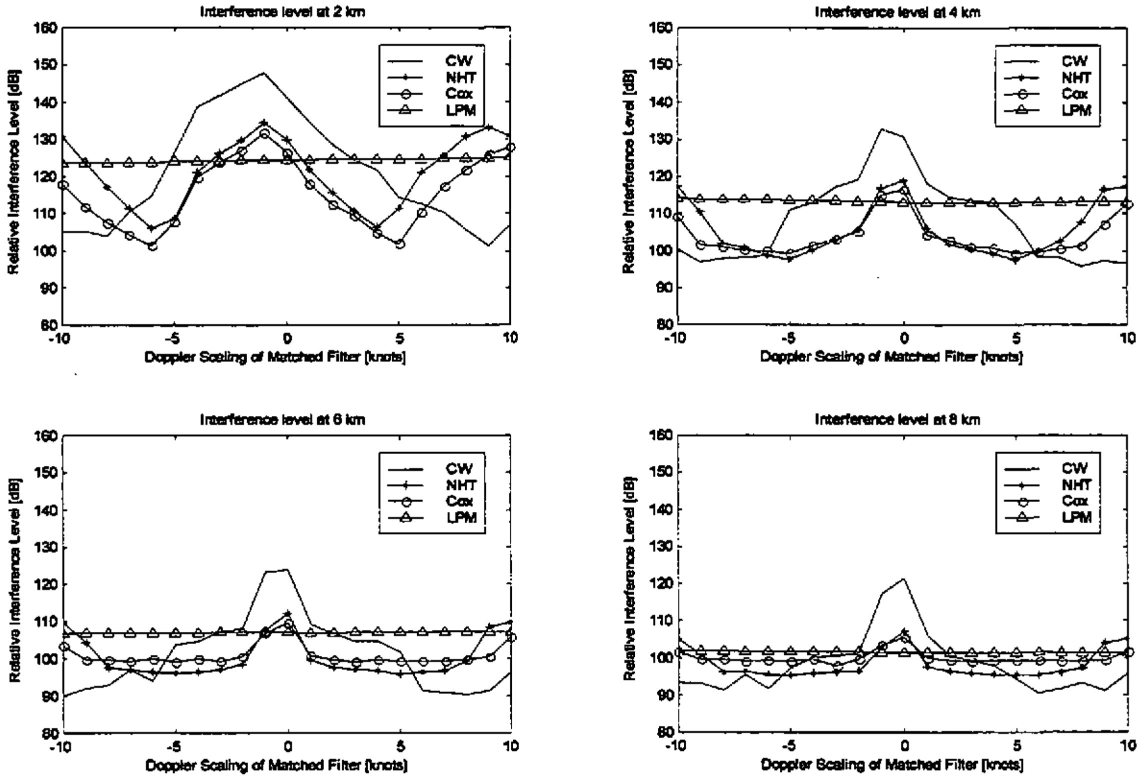

Figure 4. Measured interference levels at different ranges, January 1997 trial.

Figure 4 shows the reverberation levels observed for the January trial by passing the received signals though a bank of matched filters tuned to relative target velocities in the range ±10 knots. The vertical axes plot the total unwanted signal levels comprising reverberation and ambient noise, henceforth known as interference. The plots are normalised relative to an arbitrary reference level and averaged over 15 transmissions.

The results observed should be the convolution of the theoretical Q-functions and the Doppler distribution of the reverberation. All results shown were measured using the broadside beam of the receiver towed array, hence the zero mean velocity. The reverberation distribution extends to around ±4 knots (the tow ship velocity) as echoes from all directions are received through the array sidelobes. This is why the observed reverberation levels appear significantly more spread in Doppler than their theoretical counterparts.

At ranges of around 6 km and beyond, the interference levels for the CW pulse, Cox comb and Newhall train become noise limited. The CW pulse and Newhall train both contain around 4 dB more energy than the Cox comb and consequently reach their noise limits at a slightly lower interference level.

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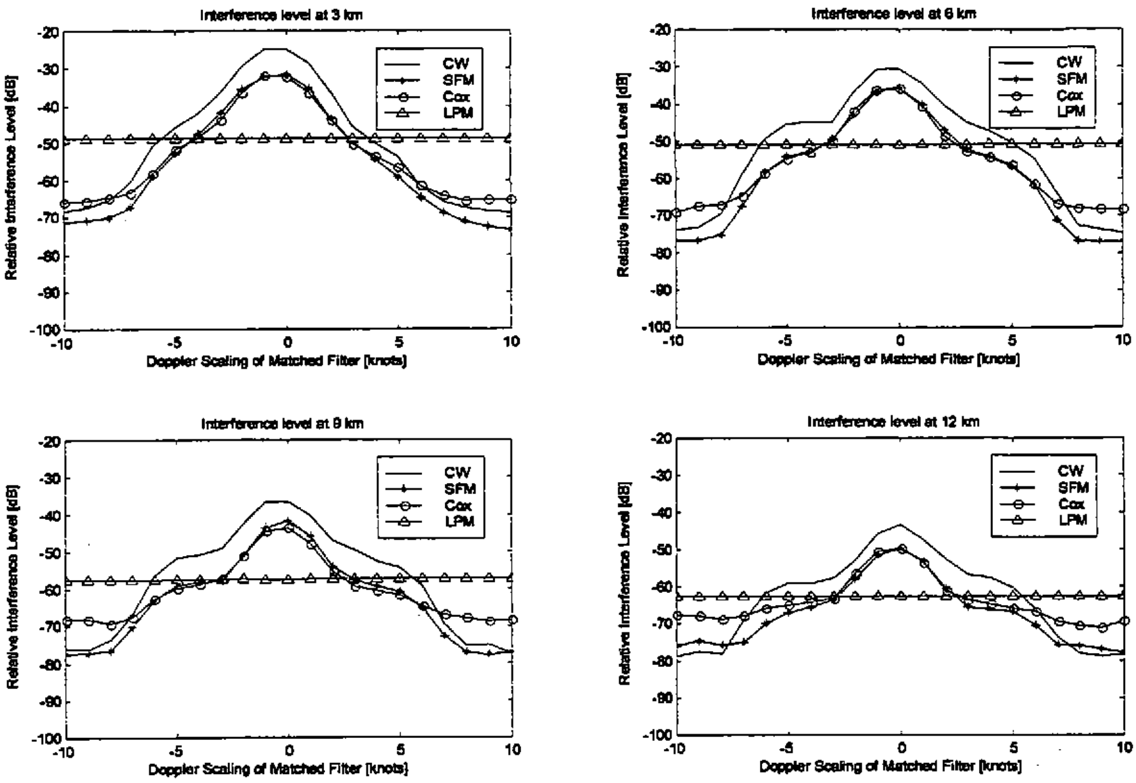

Figure 5. Measured interference levels at different ranges, August 1997 trial.

Figure 5 shows the observed interference levels during the August trial. The two trials were conducted in different areas under different conditions and so cannot be directly compared. Several similar effects can be observed, however, including the Doppler spreading due to the tow-ship velocity and the noise limiting effects at longer ranges. It is this noise limiting which appears to be the only significant difference between Cox combs and Newhall or SFM waveforms, the improved power efficiency of the latter pulses giving them a processing advantage at longer ranges.

4. CONCLUSIONS

In terms of reverberation suppression, two pulse designs emerge as consistent winners in these trials. At low target velocities and at very long ranges, it is impossible to better the chirp pulse. When the velocity of the target falls within the Doppler spread of the reverberation, pulse compression is the only tool available to a pulse designer which can suppress the interference. Of the Doppler sensitive pulses, however, it is the SFM and Newhall trains which appear superior. In the January trial, both the Newhall train and Cox comb outperform the CW pulse by a margin of up to 15 dB. In the August trial the margin is more consistent, but reduced to only 6 dB. The SFM and Newhall train have the edge over the Cox comb at longer ranges, however, due to their superior processing gain against ambient noise.

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Apart from a tiny theoretical improvement in the Q-function, the only advantage the Cox comb has over a comparable SFM pulse (or a Newhall train) concerns range ambiguities. SFM and Newhall waveforms display a series of ambiguous sidelobes in their matched filter response which are smeared and suppressed with a Cox comb. The sidelobe levels are still significant, however, and it is doubtful whether any real performance improvement could be expected in practice. On the evidence of these trial results, it is the improved ambient noise processing of SFM pulses which is more significant and would probably make them the Doppler sensitive waveform of choice.

The potential improvements in reverberation processing possible with comb-spectrum pulse designs have been illustrated both in theory, using the Q-function, and in a practical sea-trial. It has been shown that SFM, Newhall train or geometric comb waveforms can be employed as an alternative to CW pulses, providing useful reverberation suppression even at low target velocities.

The authors wish to acknowledge the financial support of DERA throughout this research.

5. REFERENCES

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